The Trouble With Budgeting to the 80th Percentile

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Background

- *Defense Science Board/Air Force Scientific Advisory Board Task Force on Acquisition of National Security Space Programs*, May 2003 (aka *The Young Report*)
- Chartered by USD(AT&L), SECAF, and USECAF/DNRO.
- Investigated systemic issues related to space systems acquisition:
  - Requirements definition
  - Budgetary planning
  - Staffing
  - Program execution
- Recommended improvements to the acquisition of space programs.

A Key Finding

"The space acquisition system is strongly biased to produce unrealistically low cost estimates throughout the acquisition process. These estimates lead to unrealistic budgets and unexecutable programs. We recommend, among other things, that the government budget space acquisition programs to a most probable (80/20) cost…"

-From the *Report of the Defense Science Board/Air Force Scientific Advisory Board Joint Task Force on Acquisition Of National Security Space Programs*, May 2003.

In other words…budget to the 80th percentile.

Will This Solve the Problem?

- Assuming the space cost community is *systematically* underestimating cost, then this approach is merely a band-aid.
  - Artificially increasing cost estimate because the system is broken.
  - It would be better to improve our cost estimating methods.
- On the other hand, if we assume the system is *not* broken then what impact does this technique have on budgeting?

The Portfolio Problem

A Simplistic Example

- Suppose a program has the following cost distribution (same as the roll of a die):

Cost	1	2	3	4	5	6
Probability	1/6	1/6	1/6	1/6	1/6	1/6
Cumulative Probability	1/6	2/6	3/6	4/6	5/6	6/6
	16.67%	33.33%	50.00%	66.67%	83.33%	100.00%

- So, if I want an 80% chance that my budget will not be exceeded, I need to budget for a possible cost of $5.
- Now, suppose I have another program with the same cost distribution, then I need to budget $5 for that one also.

A Simplistic Example

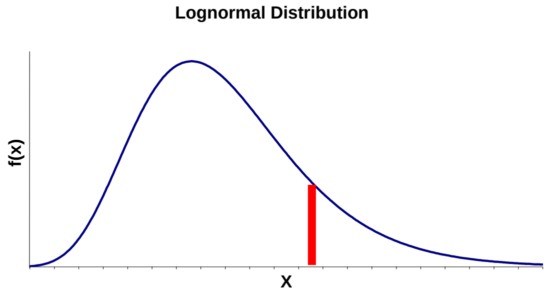
- But, what if we considered the cost of both programs *together*. The *joint* distribution is (same as roll of 2 dice):

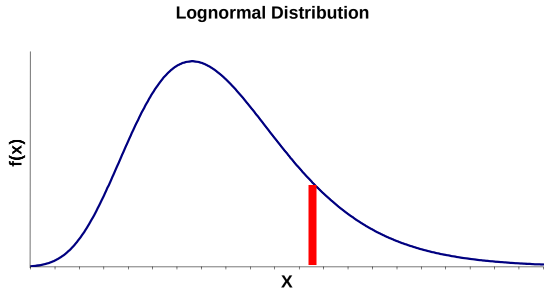
Cost	2	3	4	5	6	7	8	9	10	11	12
Probability	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36
Cumulative Probability	1/36	3/36	6/36	10/36	15/36	21/36	26/36	30/36	33/36	35/36	36/36
	2.78%	8.33%	16.67%	27.78%	41.67%	58.33%	72.22%	83.33%	91.67%	97.22%	100.00%

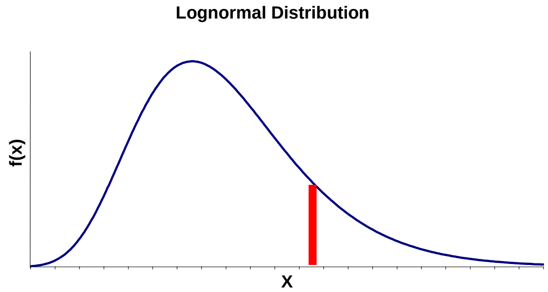
- So, now if I want an 80% chance that my joint budget will not be exceeded, I only need to budget for a total cost of $9.
- But, budgeting each program individually, I've budgeted for a total cost of $10 (which corresponds to the 91.7th percentile of the joint distribution).

A More Realistic Example

- Suppose an acquisition decision-maker desires to budget all of his programs at the 80th percentile.
  - Implies cost probability distributions exist.







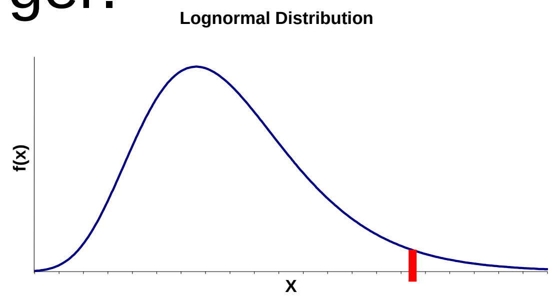
- Goal is to ensure programs have a good chance of avoiding cost overruns.

However, this philosophy constrains the total number of programs the decision-maker can fund.

If all programs are budgeted this way, then the total budget will be larger than necessary to achieve success on a portfolio of programs.

Consequences of 80th Percentile Budgeting

- Suppose each of *N* programs has a cost probability distribution with a mean and variance.
  - *Suppose further that these distributions are accurate reflections of reality!*
- Question: "If all *N* programs in the portfolio are budgeted at the 80th percentile, what is the corresponding percentile for the portfolio cost distribution?"
- In general, the answer is *not* the 80th percentile.
- In fact, it is usually significantly larger.
  - 95th percentile or more.



Portfolio Cost

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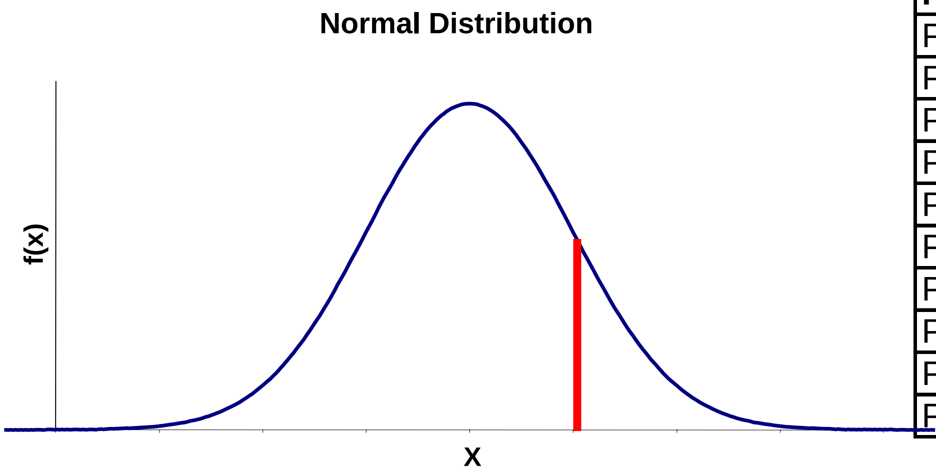
Example

- Consider a portfolio of N = 10 programs, each with uncorrelated, normally distributed cost estimates with mean μ_i and standard deviation σ_i.



Program	μ	σ	80th %ile
Program 1	$ 1,696	$ 539	$ 2,150
Program 2	$ 1,481	$ 404	$ 1,821
Program 3	$ 1,395	$ 435	$ 1,761
Program 4	$ 874	$ 288	$ 1,116
Program 5	$ 840	$ 219	$ 1,024
Program 6	$ 1,449	$ 371	$ 1,761
Program 7	$ 1,638	$ 537	$ 2,090
Program 8	$ 1,031	$ 259	$ 1,249
Program 9	$ 1,271	$ 323	$ 1,543
Program 10	$ 1,937	$ 602	$ 2,444

- The 80th percentile has been calculated for each program as follows:

$$P(X_i \leq x_{i,0.8}) = P\left(Z_i \leq \frac{x_{i,0.8} - \mu_i}{\sigma_i}\right) = \Phi\left(\frac{x_{i,0.8} - \mu_i}{\sigma_i}\right) = 0.8$$

Example

- The mean *portfolio* cost estimate, μ_T, is:

$$\mu_T = \sum_{i=1}^{N} \mu_i = \$13{,}612$$

- The standard deviation of the *portfolio* cost estimate, σ_T, is:

$$\sigma_T = \sqrt{\sum_{i=1}^{N} \sigma_i^2} = \$1{,}317 \quad \text{(assuming uncorrelated)}$$

- And the 80th percentile of the *portfolio* cost estimate is:

$$\Phi\left(\frac{x_{T,0.8} - \$13{,}612}{\$1{,}317}\right) = 0.8 \quad \Rightarrow \quad \Phi(0.8416) = 0.8 \quad \text{(ref. standard normal tables)}$$

$$\Rightarrow \quad \frac{x_{T,0.8} - \$13{,}612}{\$1{,}317} = 0.8416$$

$$\Rightarrow \quad x_{T,0.8} = (0.8416)(\$1{,}317) + \$13{,}612 = \$14{,}720$$

Example

- The 80th percentile of the portfolio cost estimate is $14,720.
- But the sum of the individual 80th percentiles is $16,959.

Program	μ	σ	80th %ile
Program 1	$ 1,696	$ 539	$ 2,150
Program 2	$ 1,481	$ 404	$ 1,821
Program 3	$ 1,395	$ 435	$ 1,761
Program 4	$ 874	$ 288	$ 1,116
Program 5	$ 840	$ 219	$ 1,024
Program 6	$ 1,449	$ 371	$ 1,761
Program 7	$ 1,638	$ 537	$ 2,090
Program 8	$ 1,031	$ 259	$ 1,249
Program 9	$ 1,271	$ 323	$ 1,543
Program 10	$ 1,937	$ 602	$ 2,444
Total	**$ 13,612**	**$ 1,317**	**$ 16,959**

- This is a difference of 15%!

Fundamental Question

- Does the decision-maker *really* want to budget all programs at the 80th percentile?
  - Or is it that he wants to ensure a reasonable probability that his portfolio budget will not be exceeded?
- If he budgets individual programs at 80th percentile, then he ends up budgeting far more than necessary for the overall portfolio.
- In fact, the sum of the 80th percentiles exceeds the 99th percentile of the cost of the portfolio!

$$\Phi\left(\frac{\$16{,}959 - \$13{,}612}{\$1{,}317}\right) = \Phi(2.5414) = 0.9945 \text{ (the } 99.45^{\text{th}} \text{ percentile)}$$

Example

- On the other hand, if he wants to ensure, say, an 80% probability that his portfolio budget will not be exceeded, then he needs to determine the individual percentiles that, when summed, correspond to the 80th percentile of the portfolio cost.

Program	μ	σ	**61st %ile**
Program 1	$ 1,696	$ 539	$ 1,846
Program 2	$ 1,481	$ 404	$ 1,594
Program 3	$ 1,395	$ 435	$ 1,516
Program 4	$ 874	$ 288	$ 954
Program 5	$ 840	$ 219	$ 901
Program 6	$ 1,449	$ 371	$ 1,552
Program 7	$ 1,638	$ 537	$ 1,788
Program 8	$ 1,031	$ 259	$ 1,103
Program 9	$ 1,271	$ 323	$ 1,361
Program 10	$ 1,937	$ 602	$ 2,105
Total	**$ 13,612**	**$ 1,317**	**$ 14,720**

In this example, the sum of the 61st percentiles is equal to the 80th percentile of the sum of the cost distributions.

Example

- The 80th percentile of sum of distributions is $14,720.
- The sum of 61st percentile of individual distributions is also $14,720.
- Thus, it is inefficient to budget each individual program at their 80th percentiles.
  - Too much money gets tied up.
  - Moreover, given a limited budget, the decision-maker would likely have no choice but to cut programs that would probably do just fine if budgeted at a lower percentile.
  - After all, by definition, each program has an 80% chance of coming in at or below its 80th percentile.
- The next chart displays the results we might expect for portfolios of different sizes.

Portfolios of Different Sizes

- The following two tables give, for different values of *N*, (1) the percentile that is necessary for each individual program in order that the portfolio is budgeted at the 80th percentile, and (2) the percentile of the portfolio budget that is realized when each individual program is budgeted at the 80th percentile.
- These tables assume individual program costs are uncorrelated.
- These tables also assume the individual programs' variances are similar to that of a recent space program.

N	(1) Individual percentile equivalent to 80th percentile total
1	80.0%
2	73.6%
3	70.3%
4	68.2%
5	66.7%
6	65.6%
7	64.7%
8	64.0%
9	63.4%
10	62.9%
20	60.0%
30	58.7%
40	58.0%
50	57.4%
100	56.1%
1000	53.8%
10000	53.1%

N	(2) Percentile of Total equivalent to sum of 80th percentiles
1	80.0%
2	87.3%
3	91.5%
4	94.2%
5	96.0%
6	97.2%
7	98.0%
8	98.6%
9	99.0%
10	99.3%
20	100.0%
30	100.0%
40	100.0%
50	100.0%
100	100.0%
1000	100.0%
10000	100.0%

Summary and Conclusions

- Should all programs in a large acquisition organization be budgeted at their 80th percentiles?
  - Probably not. If cost estimates are realistic, then doing so is inefficient.
  - And, if cost estimates are systematically low, then it is nothing more than a guess.
- Budgeting each individual program at it's 80th percentile cost estimate is equivalent to budgeting the entire portfolio of programs at a much higher percentile.
  - Exceeding the 99th percentile in most cases.
- A better alternative may be to choose a desired percentile at which to budget the cost of the entire portfolio.
  - Then determine the corresponding percentiles required of the individual programs.
  - Budget the individual programs at this percentile.

Further Consideration Needed

- Assumes cost distributions are not systematically low.
  - Might not be the case
- These examples were relatively homogeneous
  - No extreme cost estimates
  - Variances were proportionally similar
  - Need to study what happens with larger ranges of estimates
- Assumes all programs are budgeted at the same time
  - Need to study what happens when programs are budgeted at different times

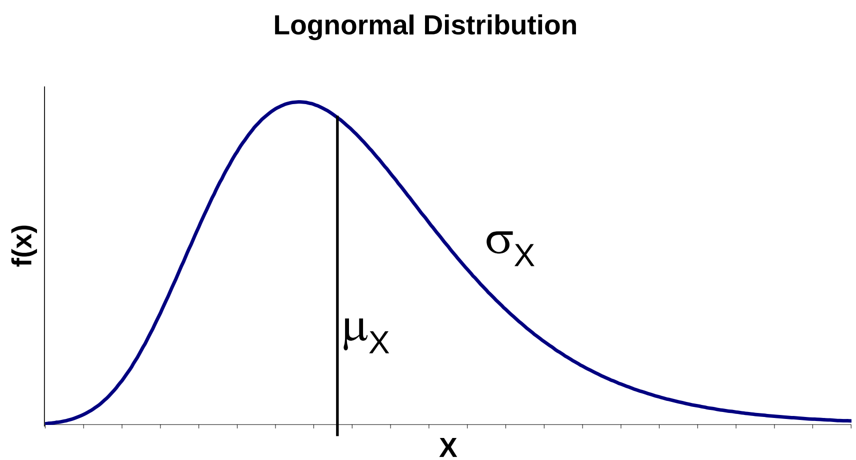
References

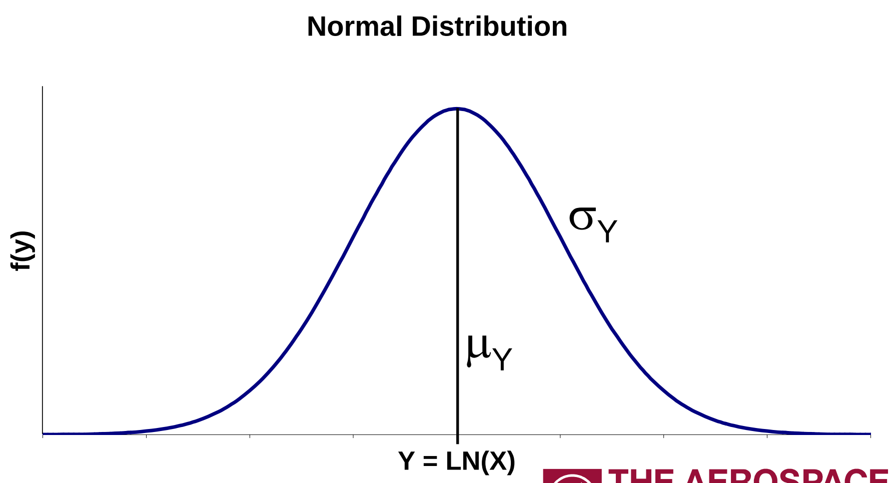
- Devore, Jay L., *Probability and Statistics for Engineering and the Sciences, 4th Edition*, Duxbury Press, 1995.
- Garvey, Paul R., *Probability Methods for Cost Uncertainty Analysis: A Systems Engineering Perspective*, Marcel Dekker, Inc., 2000.

Backups

What About Lognormal Distributions?

- In actual practice, cost probability distributions tend to have long right tails, and are often modeled as lognormal distributions.
- How do these results differ when costs are modeled as lognormal distributions?
- First, some background on the lognormal distribution.
  - Suppose X is a non-negative random variable where the natural logarithm of X, denoted by $Y = ln(X)$, follows a normal distribution.
  - Then, X is said to have a *lognormal* distribution.





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Lognormal Distributions

- Probability theory has shown that if *X* has a lognormal distribution, then the expected value and variance of *X* are related to the expected value and variance of *Y* = *ln(X)* as follows:

$$E[X] = \mu_X = e^{\mu_Y + \frac{1}{2}\sigma_Y^2}$$

$$Var[X] = \sigma_X^2 = e^{2\mu_Y + \sigma_Y^2}\left(e^{\sigma_Y^2} - 1\right)$$

$$E[Y] = \mu_Y = \tfrac{1}{2}\ln\left[\frac{\mu_X^4}{\mu_X^2 + \sigma_X^2}\right]$$

$$Var[Y] = \sigma_Y^2 = \ln\left[\frac{\mu_X^2 + \sigma_X^2}{\mu_X^2}\right]$$

Lognormal Distributions

- Calculating percentiles from a lognormal distribution is straightforward.
- Since $Y = \ln(X) \sim \mathrm{Normal}\left(\mu_Y, \sigma_Y^2\right)$

  then $\dfrac{\ln(X) - \mu_Y}{\sigma_Y} \sim \mathrm{Normal}(0,1)$

- Therefore, percentiles can be calculated as follows:

$$P(X \le x_p) = F(x_p) = P\left(Z \le \frac{\ln(x_p) - \mu_Y}{\sigma_Y}\right) = \Phi\left(\frac{\ln(x_p) - \mu_Y}{\sigma_Y}\right) = p$$

Lognormal Distributions

- The 80th percentile of a lognormal distribution is determined as:

$$\Phi\left(\frac{\ln(x_{0.8})-\mu_Y}{\sigma_Y}\right)=0.8 \quad \Rightarrow \quad \Phi(0.8416)=0.8 \quad \Rightarrow \quad \frac{\ln(x_{0.8})-\mu_Y}{\sigma_Y}=0.8416$$

$$\ln(x_{0.8})=0.8416\sigma_Y+\mu_Y \quad \Rightarrow \quad \boxed{x_{0.8}=e^{0.8416\sigma_Y+\mu_Y}}$$

- So, to calculate the 80th percentile of a lognormal distribution, it is necessary only to determine μ_Y and σ_Y, and plug them into the equation above.

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Example

- Now suppose the table below contains ten individual programs whose cost estimates follow lognormal distributions.
- Their 80th percentiles are calculated as discussed previously.

Program	μ_X	σ_X	μ_Y	σ_Y	80th %ile
1	$ 6,457	$ 660	8.768	0.102	$ 6,999
2	$ 9,741	$ 1,766	9.168	0.180	$ 11,151
3	$ 4,359	$ 188	8.379	0.043	$ 4,516
4	$ 1,535	$ 181	7.329	0.118	$ 1,683
5	$ 9,579	$ 1,338	9.158	0.139	$ 10,664
6	$ 1,759	$ 283	7.460	0.160	$ 1,987
7	$ 2,978	$ 414	7.989	0.138	$ 3,314
8	$ 5,063	$ 819	8.517	0.161	$ 5,722
9	$ 8,486	$ 1,203	9.036	0.141	$ 9,461
10	$ 3,235	$ 432	8.073	0.133	$ 3,586
Total	**$ 53,192**				**$ 59,083**

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Example

- The mean and standard deviation of the *portfolio* are:

$$\mu_{XT} = \sum_{i=1}^{N} \mu_{Xi} = \$53{,}192$$

$$\sigma_{XT} = \sqrt{\sigma_{XT}^2} = \sqrt{\sum_{i=1}^{N} \sigma_{Xi}^2} = \$2{,}823 \qquad \text{(uncorrelated)}$$

- And the 80th percentile of the *portfolio* is derived as:

$$\mu_{YT} = \tfrac{1}{2}\ln\left[\frac{\mu_{XT}^4}{\mu_{XT}^2 + \sigma_{XT}^2}\right] = \tfrac{1}{2}\ln\left[\frac{(\$53{,}192)^4}{(\$53{,}192)^2 + (\$2{,}823)^2}\right] = 10.880$$

$$\sigma_{YT} = \sqrt{\sigma_{YT}^2} = \sqrt{\ln\left[\frac{\mu_{XT}^2 + \sigma_{XT}^2}{\mu_{XT}^2}\right]} = \sqrt{\ln\left[\frac{(\$53{,}192)^2 + (\$2{,}823)^2}{(\$53{,}192)^2}\right]} = 0.053$$

$$x_{0.8} = e^{0.8416\sigma_{YT} + \mu_{YT}} = e^{(0.8416)(0.053)+(10.880)} = \$55{,}542$$

Example

- So, the 80th percentile of the cost of the portfolio is \$55,542.
- However, the sum of the 80th percentiles of the individual programs is \$59,083.
- This is a difference of 6%.
- Moreover, the sum of the 80th percentiles corresponds to the 98th percentile of the cost of the portfolio, as shown below.

$$\Phi\left(\frac{\ln(\$59{,}083)-\mu_{YT}}{\sigma_{YT}}\right)=\Phi\left(\frac{\ln(\$59{,}083)-10.880}{0.053}\right)=\Phi(2.0071)=0.9776$$

(approximately the 98th percentile)

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Example

- So, at what percentile should we budget the individual programs in order that the portfolio is budgeted at the 80th percentile?
- As the table below shows, budgeting the ten individual programs at their 65th percentiles is equivalent to budgeting the portfolio at the 80th percentile.

In this example, the sum of the 65th percentiles is equivalent to the 80th percentile of the sum of the cost distributions.

Program	μ_X	σ_X	μ_Y	σ_Y	65.17th %ile
1	$ 6,457	$ 660	8.768	0.102	$ 6,684
2	$ 9,741	$ 1,766	9.168	0.180	$ 10,281
3	$ 4,359	$ 188	8.379	0.043	$ 4,429
4	$ 1,535	$ 181	7.329	0.118	$ 1,596
5	$ 9,579	$ 1,338	9.158	0.139	$ 10,015
6	$ 1,759	$ 283	7.460	0.160	$ 1,848
7	$ 2,978	$ 414	7.989	0.138	$ 3,113
8	$ 5,063	$ 819	8.517	0.161	$ 5,321
9	$ 8,486	$ 1,203	9.036	0.141	$ 8,877
10	$ 3,235	$ 432	8.073	0.133	$ 3,377
Total	**$ 53,192**				**$ 55,542**

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Example

- It turns out in this example, if we budget all individual programs at the 65th percentile, then the total cost budget will be equal to the 80th percentile.
  - 80th percentile of sum of distributions is $55,542.
  - Sum of 65th percentile of individual distributions is $55,542.
- Moreover, the sum of the 80th percentiles of the individual distributions is equal to the 98th percentile of the portfolio distribution.
- Thus, it is inefficient to budget each individual program at their 80th percentiles.
- The next chart displays the results we might expect for portfolios of different sizes.

How to Select the Appropriate Percentile
(When Costs are Normally Distributed)

- Assume individual program cost estimates have correlated, normal distributions.
- If you want the portfolio budget at the 80th percentile, then you must decide which percentile to budget for each individual program.
- This can be done analytically for normally distributed cost estimates.
- It is necessary to choose *p*, the desired probability of not exceeding the budget for each program, which satisfies the following equation:

$$x_{T,0.8} = x_{1,p} + x_{2,p} + \cdots + x_{N,p} = \sum_{i=1}^{N} x_{i,p}$$

where $x_{T,0.8}$ is the 80th percentile of the portfolio cost, and

$x_{i,p}$ is the p^{th} percentile of the i^{th} cost estimate.

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How to Select the Appropriate Percentile
(When Costs are Normally Distributed)

- Since the 80th percentile of a standard normal random variable is $z_{0.8}$ = 0.8416, the 80th percentile of the cost of the portfolio is:

$$x_{T,0.8} = \mu_T + 0.8416\sigma_T$$

likewise,

$$x_{1,p} + x_{2,p} + \cdots + x_{N,p} = (\mu_1 + z_p\sigma_1) + (\mu_2 + z_p\sigma_2) + \cdots + (\mu_N + z_p\sigma_N)$$

where z_p is the value of the standard normal distribution corresponding to the percentile to be chosen for the individual programs.

How to Select the Appropriate Percentile

(When Costs are Normally Distributed)

- So, the equation to be solved is:

$$\mu_T + 0.8416\sigma_T = (\mu_1 + z_p\sigma_1) + (\mu_2 + z_p\sigma_2) + \cdots + (\mu_N + z_p\sigma_N) = \sum_{i=1}^{N}\mu_i + z_p\sum_{i=1}^{N}\sigma_i$$

- And, since $\mu_T = \sum_{i=1}^{N}\mu_i$ and $\sigma_T = \sqrt{\sum_{i=1}^{N}\sigma_i^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\rho_{ij}\sigma_i\sigma_j}$

  the resulting equation is:

$$\sum_{i=1}^{N}\mu_i + 0.8416\sqrt{\sum_{i=1}^{N}\sigma_i^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\rho_{ij}\sigma_i\sigma_j} = \sum_{i=1}^{N}\mu_i + z_p\sum_{i=1}^{N}\sigma_i$$

How to Select the Appropriate Percentile

(When Costs are Normally Distributed)

- Solving for z_p results in:

$$z_p = \frac{0.8416\sqrt{\sum_{i=1}^{N}\sigma_i^2 + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\rho_{ij}\sigma_i\sigma_j}}{\sum_{i=1}^{N}\sigma_i}$$

- Now, all that is left is to solve for z_p, then determine $\Phi(z_p)$ from the standard normal distribution.
- The result is the percentile at which we must budget each of the individual programs to ensure the 80th percentile of the cost of the portfolio is realized.

Lognormal Distributions

- Consider a portfolio of N individual programs, each with a lognormal cost distribution with common mean μ_X and common variance σ_X^2.
- Furthermore, suppose each program is correlated with every other program with common correlation ρ.
- Using classical statistical theory, the mean and variance of the cost of the portfolio are calculated as:

$$\mu_{XT} = \sum_{i=1}^{N} \mu_{Xi} = N\mu_X$$

$$\sigma_{XT}^2 = \sum_{i=1}^{N} \sigma_{Xi}^2 + 2\rho \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \sigma_{Xi}\sigma_{Xj} = N\sigma_X^2 + N(N-1)\rho\sigma_X^2$$
$$= N\sigma_X^2\left(1+(N-1)\rho\right)$$

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Calculating Percentiles

- Each individual program's p_i^{th} percentile is:

$$x_{pi} = e^{z_{pi}\sigma_Y + \mu_Y}$$

- And the p_T^{th} percentile of the portfolio is:

$$x_{pT} = e^{z_{pT}\sigma_{YT} + \mu_{YT}}$$

- So, if we want to choose p_i such that the sum of each individual's p_i^{th} percentile is equal to the portfolio's p_T^{th} percentile, then we have:

$$e^{z_{pT}\sigma_{YT} + \mu_{YT}} = e^{z_{pi}\sigma_Y + \mu_Y} + e^{z_{pi}\sigma_Y + \mu_Y} + \cdots + e^{z_{pi}\sigma_Y + \mu_Y}$$

$$\Rightarrow e^{z_{pT}\sigma_{YT} + \mu_{YT}} = Ne^{z_{pi}\sigma_Y + \mu_Y}$$

Calculating Percentiles

- Solving for z_{pT} in terms of z_{pi} and N is straightforward:

$$z_{pT} = \frac{\mu_Y + z_{pi}\sigma_Y - \mu_{YT} + \ln(N)}{\sigma_{YT}}$$

- Likewise, solving for z_{pi} in terms of z_{pT} and N is also simple:

$$z_{pi} = \frac{\mu_{YT} + z_{pT}\sigma_{YT} - \mu_Y - \ln(N)}{\sigma_Y}$$

- Once z is determined, the corresponding percentile is simply $\Phi(z)$ *(from a standard normal distribution).*

When Costs are Uncorrelated…

- The following two tables give, for different values of *N*, (1) the percentile that is necessary for each individual program in order that the portfolio is budgeted at the 80th percentile, and (2) the percentile of the portfolio budget that is realized when each individual program is budgeted at the 80th percentile.
- These tables assume individual program costs are uncorrelated.
- These tables also assume the individual programs' variances are similar to that of a recent space program.

N	(1) Individual percentile equivalent to 80th percentile total
1	80.0%
2	73.6%
3	70.3%
4	68.2%
5	66.7%
6	65.6%
7	64.7%
8	64.0%
9	63.4%
10	62.9%
20	60.0%
30	58.7%
40	58.0%
50	57.4%
100	56.1%
1000	53.8%
10000	53.1%

N	(2) Percentile of Total equivalent to sum of 80th percentiles
1	80.0%
2	87.3%
3	91.5%
4	94.2%
5	96.0%
6	97.2%
7	98.0%
8	98.6%
9	99.0%
10	99.3%
20	100.0%
30	100.0%
40	100.0%
50	100.0%
100	100.0%
1000	100.0%
10000	100.0%

When Correlation is $\rho = 0.2$…

- The following two tables give, for different values of *N*, (1) the percentile that is necessary for each individual program in order that the portfolio is budgeted at the 80th percentile, and (2) the percentile of the portfolio budget that is realized when each individual program is budgeted at the 80th percentile.
- These tables assume individual program costs are correlated with $\rho = 0.2$.
- These tables also assume the individual programs' variances are similar to that of a recent space program.

N	(1) Individual percentile equivalent to 80th percentile total
1	80.0%
2	75.2%
3	73.0%
4	71.7%
5	70.9%
6	70.3%
7	69.9%
8	69.5%
9	69.2%
10	69.0%
20	67.9%
30	67.6%
40	67.4%
50	67.2%
100	67.0%
1000	66.8%
10000	66.7%

N	(2) Percentile of Total equivalent to sum of 80th percentiles
1	80.0%
2	85.3%
3	88.0%
4	89.7%
5	90.8%
6	91.5%
7	92.1%
8	92.6%
9	92.9%
10	93.2%
20	94.6%
30	95.1%
40	95.3%
50	95.4%
100	95.7%
1000	96.0%
10000	96.0%

When Correlation is ρ = 0.4…

- The following two tables give, for different values of *N*, (1) the percentile that is necessary for each individual program in order that the portfolio is budgeted at the 80th percentile, and (2) the percentile of the portfolio budget that is realized when each individual program is budgeted at the 80th percentile.
- These tables assume individual program costs are correlated with ρ = 0.4.
- These tables also assume the individual programs' variances are similar to that of a recent space program.

N	(1) Individual percentile equivalent to 80th percentile total
1	80.0%
2	76.6%
3	75.2%
4	74.4%
5	73.9%
6	73.6%
7	73.3%
8	73.2%
9	73.0%
10	72.9%
20	72.3%
30	72.1%
40	72.0%
50	72.0%
100	71.9%
1000	71.7%
10000	71.7%

N	(2) Percentile of Total equivalent to sum of 80th percentiles
1	80.0%
2	83.6%
3	85.3%
4	86.2%
5	86.8%
6	87.3%
7	87.6%
8	87.8%
9	88.0%
10	88.2%
20	88.9%
30	89.1%
40	89.3%
50	89.3%
100	89.5%
1000	89.6%
10000	89.7%

When Correlation is ρ = 0.6…

- The following two tables give, for different values of *N*, (1) the percentile that is necessary for each individual program in order that the portfolio is budgeted at the 80th percentile, and (2) the percentile of the portfolio budget that is realized when each individual program is budgeted at the 80th percentile.
- These tables assume individual program costs are correlated with ρ = 0.6.
- These tables also assume the individual programs' variances are similar to that of a recent space program.

N	(1) Individual percentile equivalent to 80th percentile total
1	80.0%
2	77.8%
3	77.0%
4	76.6%
5	76.3%
6	76.1%
7	76.0%
8	75.9%
9	75.8%
10	75.8%
20	75.5%
30	75.4%
40	75.3%
50	75.3%
100	75.2%
1000	75.2%
10000	75.2%

N	(2) Percentile of Total equivalent to sum of 80th percentiles
1	80.0%
2	82.2%
3	83.2%
4	83.6%
5	84.0%
6	84.2%
7	84.3%
8	84.4%
9	84.5%
10	84.6%
20	84.9%
30	85.1%
40	85.1%
50	85.2%
100	85.2%
1000	85.3%
10000	85.3%

When Correlation is ρ = 0.8…

- The following two tables give, for different values of *N*, (1) the percentile that is necessary for each individual program in order that the portfolio is budgeted at the 80th percentile, and (2) the percentile of the portfolio budget that is realized when each individual program is budgeted at the 80th percentile.
- These tables assume individual program costs are correlated with ρ = 0.8.
- These tables also assume the individual programs' variances are similar to that of a recent space program.

N	(1) Individual percentile equivalent to 80th percentile total
1	80.0%
2	79.0%
3	78.6%
4	78.4%
5	78.3%
6	78.2%
7	78.2%
8	78.1%
9	78.1%
10	78.1%
20	78.0%
30	77.9%
40	77.9%
50	77.9%
100	77.9%
1000	77.9%
10000	77.8%

N	(2) Percentile of Total equivalent to sum of 80th percentiles
1	80.0%
2	81.0%
3	81.4%
4	81.6%
5	81.7%
6	81.8%
7	81.9%
8	81.9%
9	82.0%
10	82.0%
20	82.1%
30	82.2%
40	82.2%
50	82.2%
100	82.2%
1000	82.2%
10000	82.2%

When Correlation is ρ = 1.0…

- The following two tables give, for different values of *N*, (1) the percentile that is necessary for each individual program in order that the portfolio is budgeted at the 80th percentile, and (2) the percentile of the portfolio budget that is realized when each individual program is budgeted at the 80th percentile.
- These tables assume individual program costs are perfectly correlated.
- These tables also assume the individual programs' variances are similar to that of a recent space program.

N	(1) Individual percentile equivalent to 80th percentile total
1	80.0%
2	80.0%
3	80.0%
4	80.0%
5	80.0%
6	80.0%
7	80.0%
8	80.0%
9	80.0%
10	80.0%
20	80.0%
30	80.0%
40	80.0%
50	80.0%
100	80.0%
1000	80.0%
10000	80.0%

N	(2) Percentile of Total equivalent to sum of 80th percentiles
1	80.0%
2	80.0%
3	80.0%
4	80.0%
5	80.0%
6	80.0%
7	80.0%
8	80.0%
9	80.0%
10	80.0%
20	80.0%
30	80.0%
40	80.0%
50	80.0%
100	80.0%
1000	80.0%
10000	80.0%

How to Select the Appropriate Percentile
(When Costs are Lognormally Distributed)

- Assume individual program cost estimates have known correlated, *lognormal* distributions.
- If you want the portfolio budget at the 80th percentile, then you must decide which percentile to budget for each individual program.
- This can be done analytically for lognormally distributed cost estimates.
- It is necessary to choose *p*, the desired probability of not exceeding the budget for each program, which satisfies the following equation:

$$x_{T,0.8} = x_{1,p} + x_{2,p} + \cdots + x_{N,p} = \sum_{i=1}^{N} x_{i,p}$$

where $x_{T,0.8}$ is the 80th percentile of the portfolio cost, and

$x_{i,p}$ is the p^{th} percentile of the i^{th} cost estimate.

How to Select the Appropriate Percentile

(When Costs are Lognormally Distributed)

- Since the 80th percentile of a standard normal random variable is $z_{0.8}$ = 0.8416, the 80th percentile of the cost of the portfolio with lognormal distributions is:

$$x_{T,0.8} = e^{\left(\mu_{Y,T} + 0.8416\sigma_{Y,T}\right)}$$

likewise,

$$x_{1,p} + x_{2,p} + \cdots + x_{N,p} = e^{(\mu_{Y,1} + z_p\sigma_{Y,1})} + e^{(\mu_{Y,2} + z_p\sigma_{Y,2})} + \cdots + e^{(\mu_{Y,N} + z_p\sigma_{Y,N})}$$

where z_p is the value of the standard normal distribution corresponding to the percentile to be chosen for the individual programs.

How to Select the Appropriate Percentile
(When Costs are Lognormally Distributed)

- So, the equation to be solved is:

$$e^{\left(\mu_{YT}+0.8416\sigma_{YT}\right)} = e^{\left(\mu_{Y,1}+z_p\sigma_{Y,1}\right)} + e^{\left(\mu_{Y,2}+z_p\sigma_{Y,2}\right)} + \cdots + e^{\left(\mu_{Y,N}+z_p\sigma_{Y,N}\right)} = \sum_{i=1}^{N} e^{\left(\mu_{Y,i}+z_p\sigma_{Y,i}\right)}$$

- Unfortunately, this equation does not have an elegant solution (that this author is aware of), but can easily be solved for z_P using numerical methods, such as a non-linear solver.
- Then, after solving for z_P, it is necessary to find $\Phi(z_P)$ from the standard normal distribution which is then used to calculate the percentile at which we must budget each of the individual programs.