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THE MINIMUM-UNBIASED-PERCENTAGE ERROR (MUPE) METHOD IN CER DEVELOPMENT

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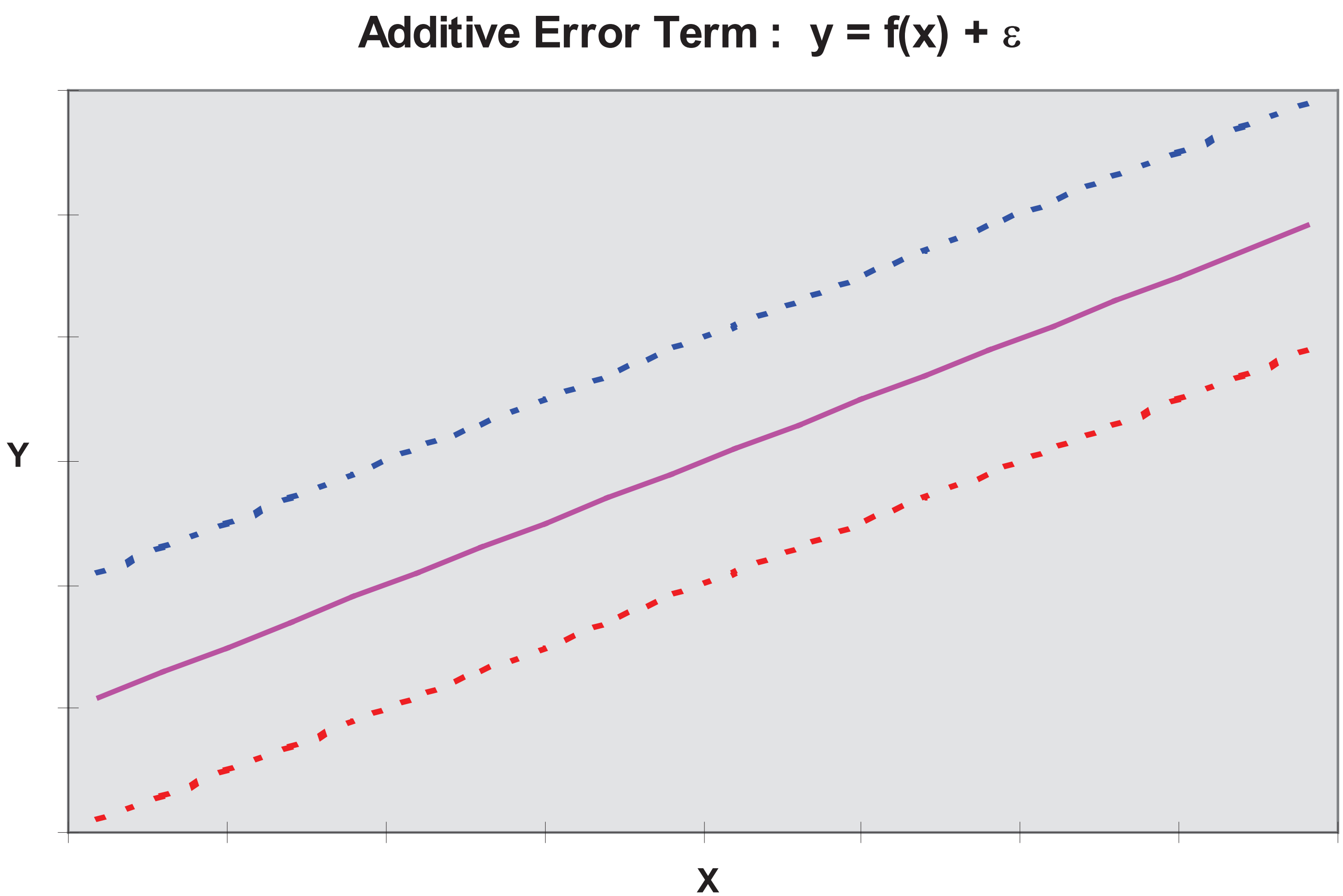
Objective

- Find a regression technique to model a multiplicative error term without bias
- It requires no transformation and no correction factor

Application for MUPE

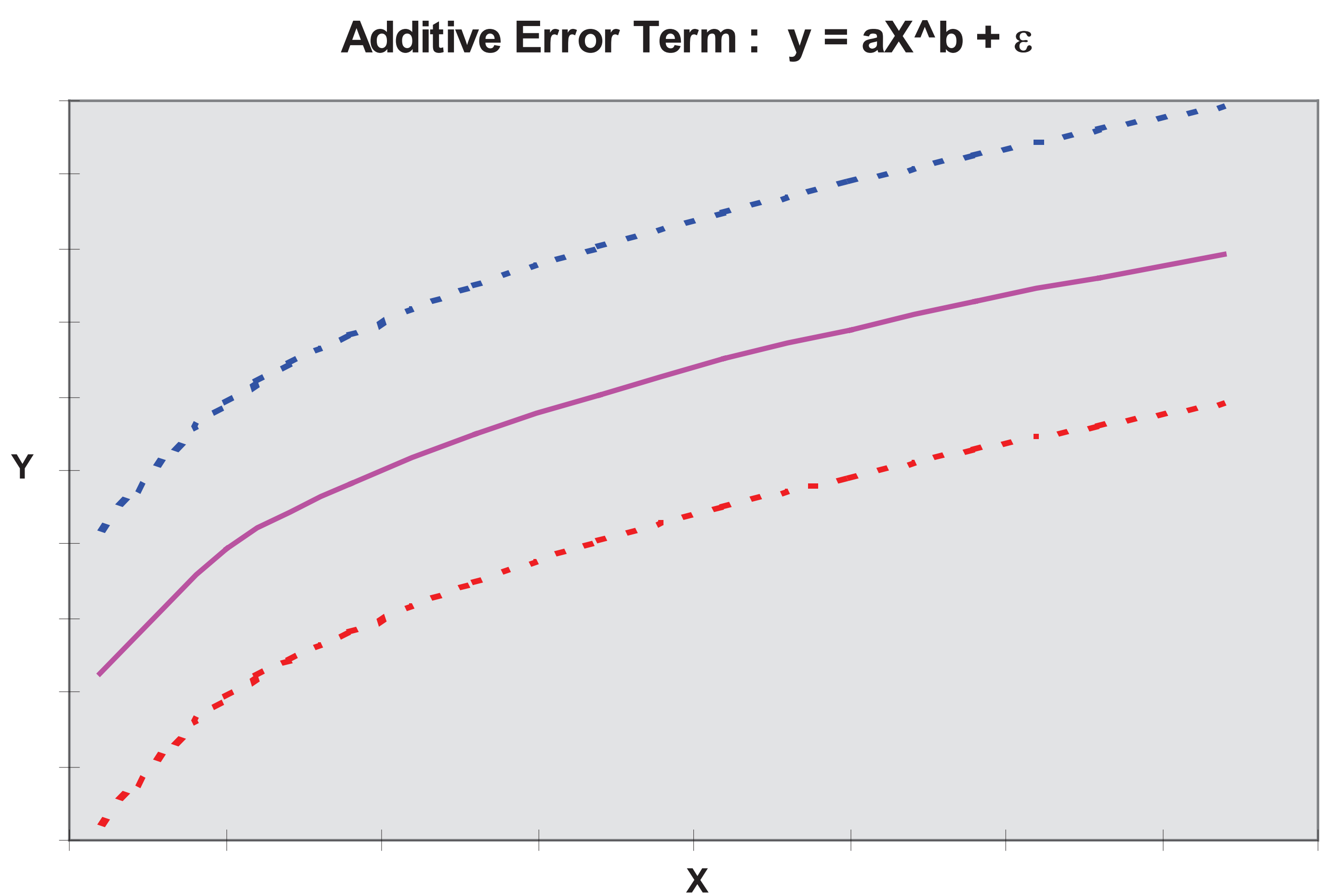
- Fan-shaped pattern exists in database, or
- Data pattern not noticeable (especially in small samples)
  - Data range of dependent variable over one order of magnitude
  - Management controls proportional errors

Additive Error Term



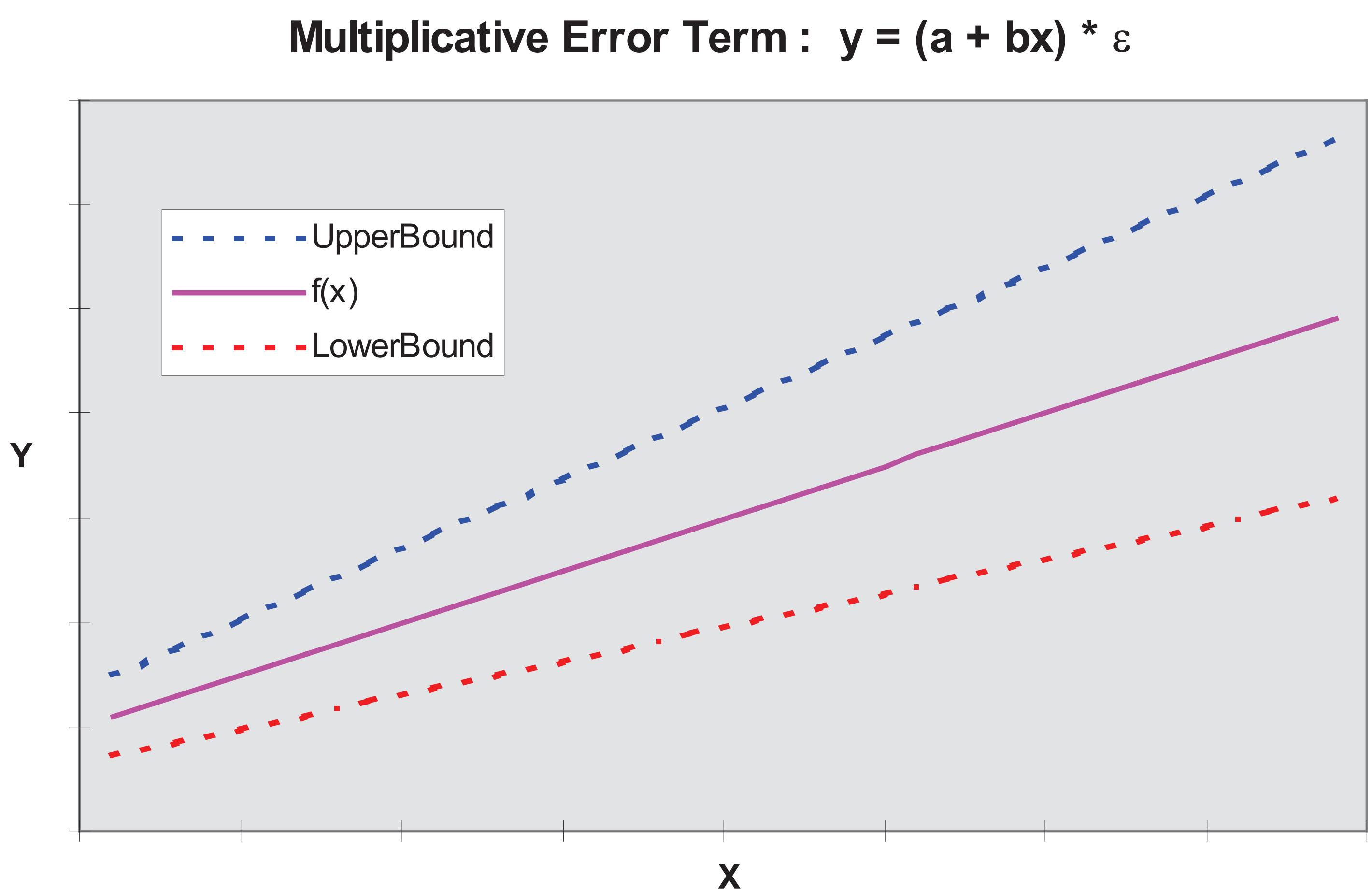
Note: Error distribution is independent of the scale of the project. (OLS)

Additive Error Term



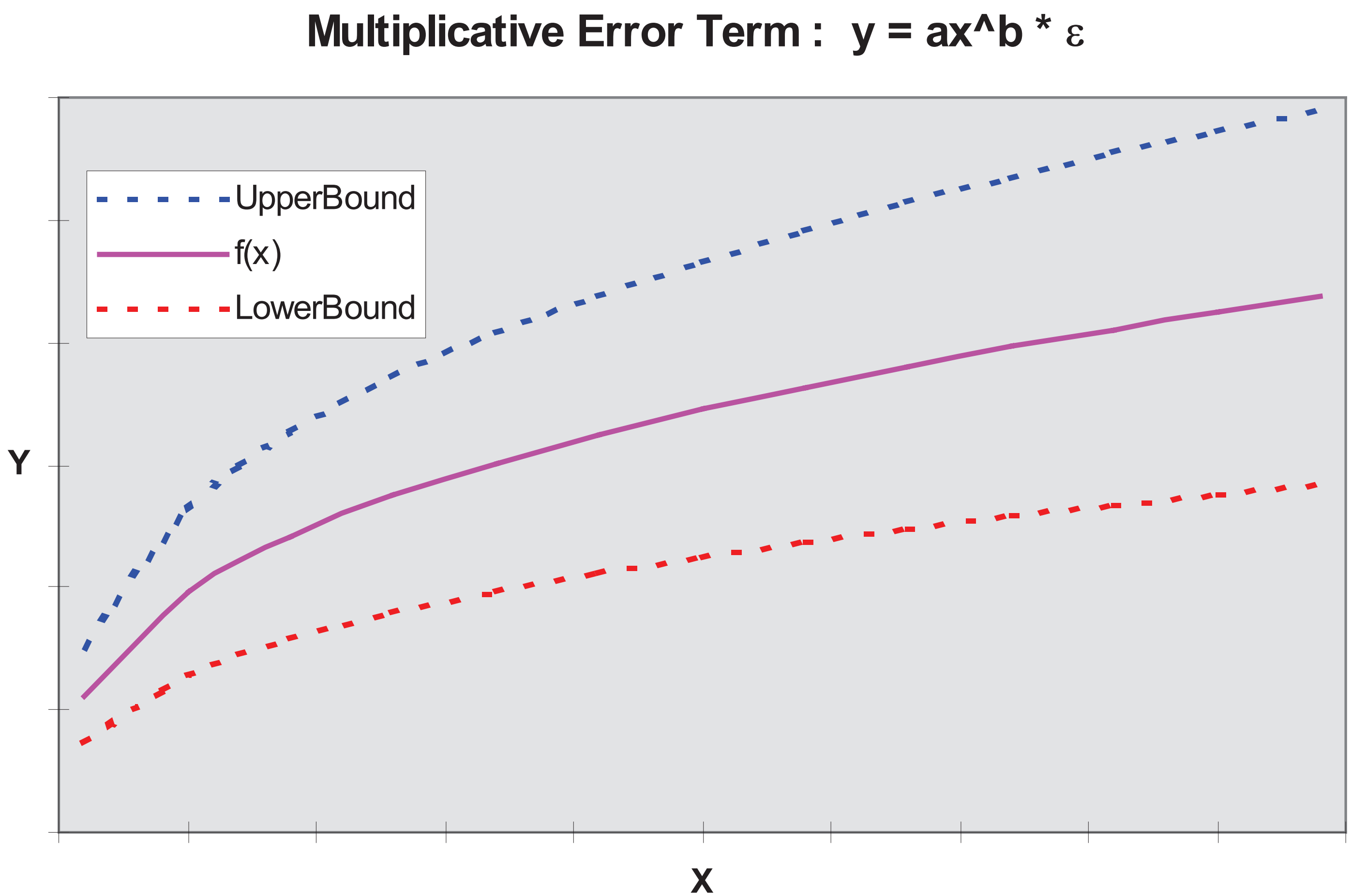
Note: This requires non-linear regression.

Multiplicative Error Term



Note: This requires non-linear regression.

Multiplicative Error Term



Note: This equation is linear in log space.

Error Term Assumptions - Background

	ADDITIVE ERROR	MULTIPLICATIVE ERROR
Distrn Assumptions	$N(0, S^2)$, independent	$LN(0, S^2)$, independent
Typical Model Form	Linear – $y = a + b\,x$	Exponential – $y = a\,x^b$
Historical Rationale	(Mathematical Convenience)	
Legitimate Reasons	Absolute Errors	Proportional Errors
What should be cost errors?	Cost variation is independent of the scale of the project	Cost variation is proportional to the scale of the project

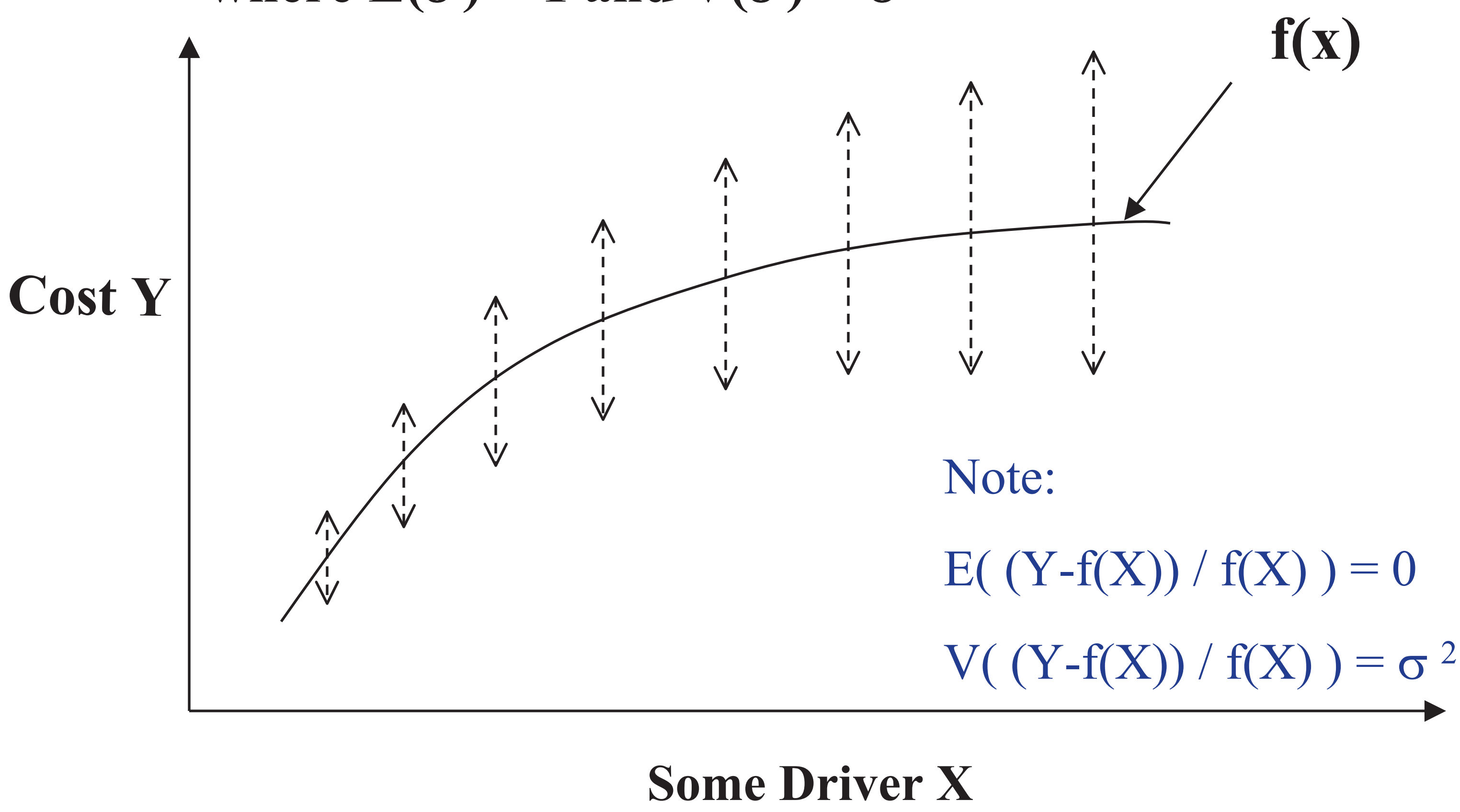
- **Model form should not drive error term assumption**
- **Error term should not drive model form**

Multiplicative Error Model - MUPE

Definition for cost variation:

$$Y = f(X)^*\varepsilon$$

where $E(\varepsilon) = 1$ and $V(\varepsilon) = \sigma^2$



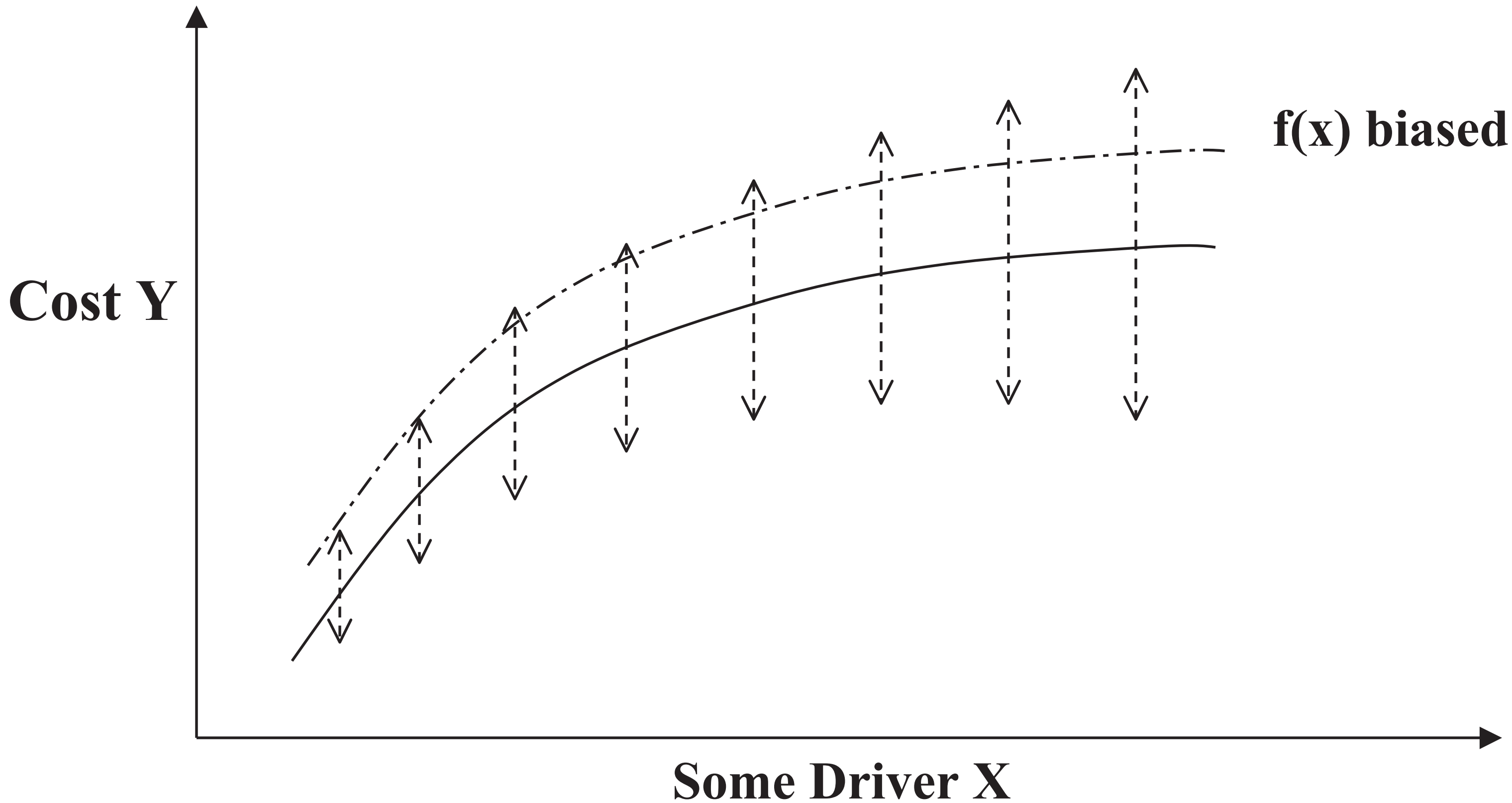
Note:

$E((Y\text{-}f(X)) / f(X)) = 0$

$V((Y\text{-}f(X)) / f(X)) = \sigma^2$

Bias in a Regression Equation

a biased high regression equation:



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Candidate Error (Residual) Forms

- **Log-Error** ($\varepsilon \sim LN(0, \sigma^2)$) ⇨ **Least squares in log space**
  - **Residuals of the cost function transformed into log-space**

    Error = Log (y_i) - Log f(x_i)

- **Weighted Residuals** ⇨ **Least squares in unit space**
  - **by the reciprocal of the observation**

$$Error = \frac{y_i - f(x_i)}{y_i}$$

  - **by the reciprocal of the predicted *value***

$$Error = \frac{y_i - f(x_i)}{f(x_i)}$$

Regression Methods to Implement Multiplicative Errors

- Least squares in log space
  - Log-Error Model

- Least squares in unit space with weighted residuals
  - Weighted by observation - WLS
  - Weighted by predicted value
    - MPE (Minimum-Percentage Error) Method
    - MUPE (Minimum-Unbiased-Percentage Error) Method
    - ZPB/MPE Method (Constrained MPE Method)

Methodologies

- Analyses of real examples using four different methods
  - MPE (Minimum-Percentage Error) Method
  - MUPE (Minimum-Unbiased-Percentage Error) Method
  - Log-Error Model
  - ZPB/MPE Method (Constrained MPE Method)

MPE and MUPE Methods

- **Two possible ways to perform the optimization for the weighted least squares using the <u>predicted</u> values in USCM7**

  - **MPE** □ **high bias due to simultaneous minimization**

$$\textit{Minimize} \quad \sum_{i=1}^{n} \left(\frac{y_i - f(x_i)}{f(x_i)} \right)^2$$

  - **MUPE** □ **bias eliminated**

$$\textit{Minimize} \quad \sum_{i=1}^{n} \left(\frac{y_i - f(x_i)}{f_{\boldsymbol{k-1}}(x_i)} \right)^2$$

where k is the iteration number

Constrained MPE Method (ZPB/MPE)

- **Alternative method to remove the high bias in MPE equations for the general level of the function**
  - ➢ **Constrained Excel Solver solution**

$$Minimize \quad \sum_{i=1}^{n} \left(\frac{y_i - f(x_i)}{f(x_i)} \right)^2$$

$$Subject \quad to \quad \sum_{i=1}^{n} \left(\frac{y_i - f(x_i)}{f(x_i)} \right) = 0$$

Comparison between ZPB/MPE and MPE

- For most equations (i.e.,$Y = a X^{b} Z^{c}$, $Y = a + bX + cZ$, etc.)
  - Sensitivity coefficients (associated with the driver variables) are the same between MPE & ZPB/MPE equations
  - Only leading term or level of function adjusted
  - Findings also proven by mathematical derivations
- For triad equations (i.e., $Y = a + b X^{c} Z^{d}$)
  - All coefficients changed

Bias Characteristics of Different Methods

- **Log-error equations will be biased low**
  - **Bias can be adjusted using a correction factor, i.e.,**

$$\exp\left(\frac{\sigma^2}{2}\left(1 - \frac{p}{n}\right)\right)$$

p = number of estimated coefficients

- **Residuals weighted by the observation will be biased low**
  - **Bias cannot be easily justified**
- **MPE equations will be biased high**
  - **Bias in equation can be adjusted in simple cases by a correction factor**

$$\frac{1}{1+\sigma^2_{mupe}} \quad (\cong 1 - \sigma^2_{mpe})$$

- **MUPE (noted as IRLS) will be asymptotically unbiased**
  - **MUPE has zero proportional error for points in the database, it is also proven to produce consistent parameter estimates.**

Heuristic Goodness-of-fit Measures

- **Multiplicative error (standard error of model):**

$$\sqrt{\sum_{i=1}^{n}\left(\frac{y_i - \hat{y}_i}{\hat{y}_i}\right)^2 \Big/ (n-p)}$$

- **Average percentage error (APE)*:**

$$\left(\sum_{i=1}^{n}\frac{y_i - \hat{y}_i}{\hat{y}_i}\right)\Big/ n$$

Note:

$$SEE_{OLS} = \sqrt{\frac{1}{n-p}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$$

APE is termed as **average bias in USCM7.*

MPE Method is biased

- Biased high for the level of the function
  - Correction for bias by a correction factor:

    $CF = 1 - \sigma^2_{MPE}$
- Biased low for
  - variance / standard error
    - Correction for bias in estimate of variance by:

      $CF = 1 / (1 - \sigma^2_{MPE})$
  - average percentage error (APE)
    - Correction for bias in APE by:

      $CF = 1 / (1 - APE_{MPE})$

Note: True APE = $APE_{MPE} / (1 - APE_{MPE})$

Heuristic Goodness-of-fit Measures

- **Pearson's Correlation Squared (r^2):**

$$r^2 = \frac{\left(Cov(Y, \hat{Y})\right)^2}{Var(Y) * Var(\hat{Y})}$$

  - **$r^2 = R^2$ in OLS**
  - Pearson's correlation coefficient measures the linear association between y (actual cost) and y-hat (predicted cost), it cannot explain the actual deviation between y and y-hat if the model is not OLS

Heuristic Goodness-of-fit Measures

- **R-Squared:**

$$R^2 = 1 - \frac{SSE}{SST} = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2} \quad (= \frac{\sum (\hat{y}_i - \bar{y})^2}{\sum (y_i - \bar{y})^2} = \frac{SSR}{SST} \; in \; OLS \;)$$

- **Adjusted R-Squared:**

$$Adj\,.R^2 = 1 - \frac{SSE\;/(n-p)}{SST\;/(n-1)} = 1 - \frac{\sum (y_i - \hat{y}_i)^2 /(n-p)}{\sum (y_i - \bar{y})^2 /(n-1)}$$

- **Note: R-Squared is a measure of the amount of variation about the mean explained by the fitted equation in OLS**

Analysis of Examples

- **Comparisons of examples of complex equations drawn from Unmanned Spacecraft Cost Model, 7th Edition (USCM7) and other sources**
  - ➢ **6 examples from USCM7**
    - – **COMM/TT&C Digital Electronics**
    - – **TT&C Digital Electronics**
    - – **TT&C RF Distribution**
    - – **EPS Generation**
  - ➢ **2 examples from other sources**
- **Analyze and compare results using 4 different methods**

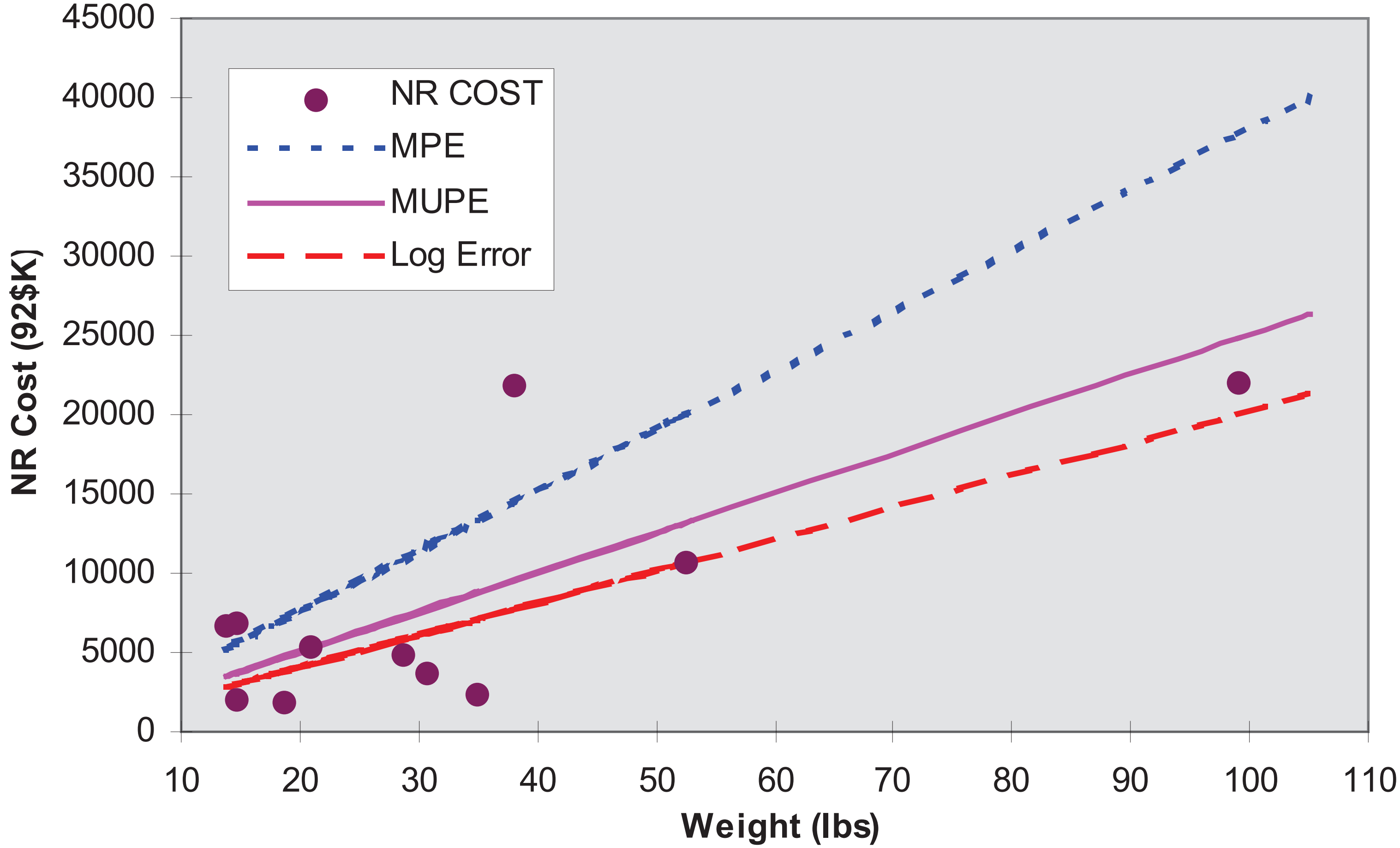
COMM/TTC Digital Electronics NR CER (n = 11)

- **MPE:** $Y = 211.24 * Wt^{.787} * NumLinks^{.853}$
  ($s = .221$, $r^2 = .94$)
- **MUPE:** $Y = 194.12 * Wt^{.804} * NumLinks^{.825}$
  ($s = .221$, $r^2 = .94$)
- **Log-Linear:** $Y = 186.61 * Wt^{.811} * NumLinks^{.815}$
  ($s = .231$, $r^2 = .94$)
- **ZPB/MPE:** $Y = 204.02 * Wt^{.787} * NumLinks^{.853}$
  ($s = .221$, $r^2 = .94$)

COMM/TTC Digital Electronics NR CER (n = 11)

- **MPE:** $Y = 359.88 * \text{Weight}$ ($s = .58$, $r^2 = .54$, $\overline{R}^2 = -.02$)
  (+43%)
- **MUPE:** $Y = 251.08 * \text{Weight}$ ($s = .69$, $r^2 = .54$, $\overline{R}^2 = .52$)
- **Log-Error:** $Y = 202.48 * \text{Weight}$ ($s = .69$, $r^2 = .54$, $\overline{R}^2 = .50$)
  (-19%)
- **ZPB/MPE:** $Y = 251.08 * \text{Weight}$ ($s = .69$, $r^2 = .54$, $\overline{R}^2 = .52$)

Comparison Chart for Weight based CER



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COMM/TTC Digital Electronics NR CER in USCM7

- **MPE:** $Y = 345.78 * Weight$ ($s = .53$, $r^2 = .65$, $\overline{R}^2 = .27$)
  (+35%) average bias = 26% in USCM7

- **MUPE:** $Y = 256.88 * Weight$ ($s = .62$, $r^2 = .65$, $\overline{R}^2 = .64$)

- **Log-Error:** $Y = 218.95 * Weight$ ($s = .58$, $r^2 = .65$, $\overline{R}^2 = .63$)
  (-15%) $Y * CF = 255.34 * Weight$

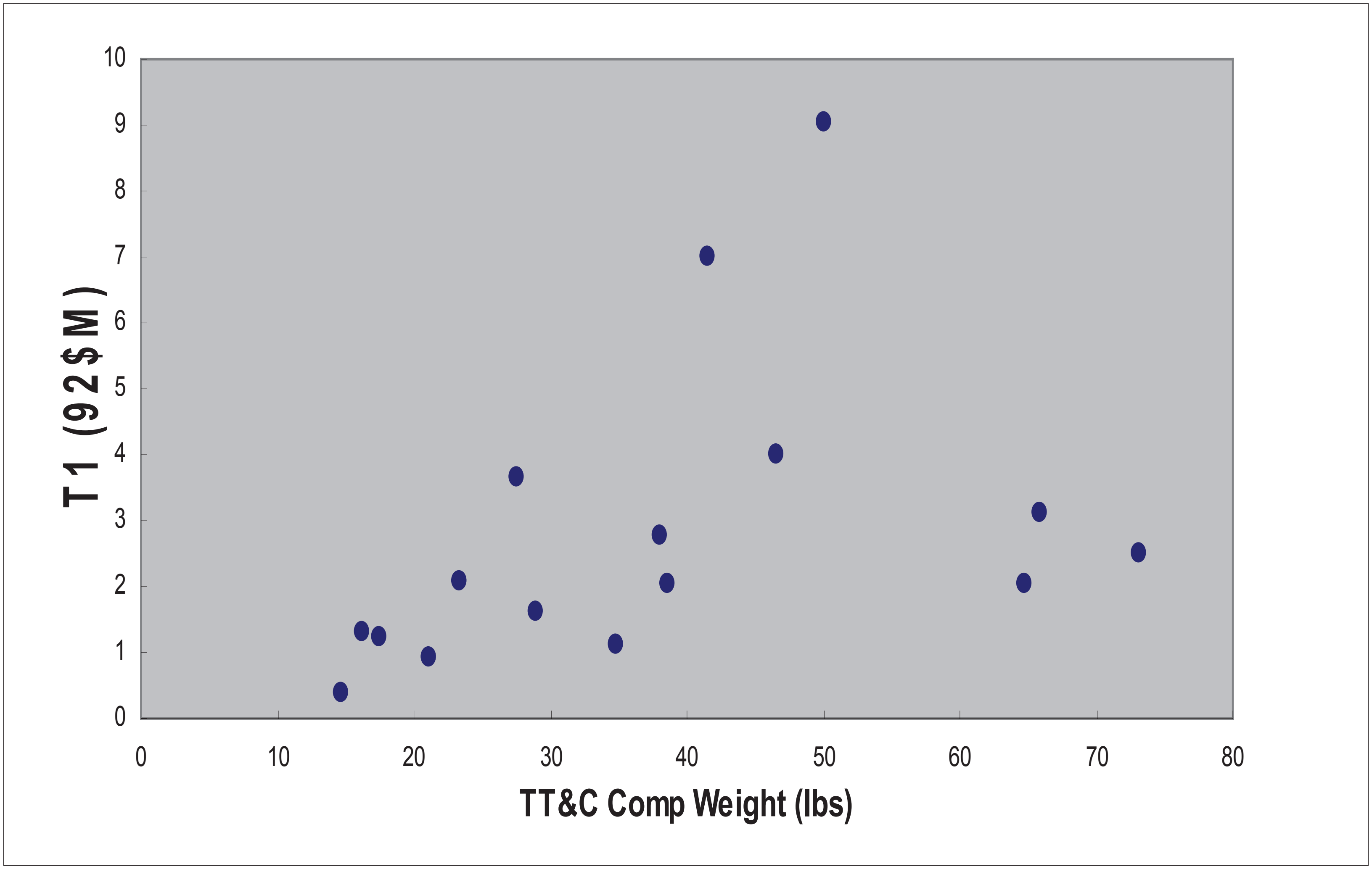
- **ZPB/MPE:** $Y = 256.88 * Weight$ ($s = .62$, $r^2 = .65$, $\overline{R}^2 = .64$)

- **Note:** The average bias measures listed in USCM7 are biased low.

TT&C Digital Electronics Recur CER (n = 16)

- **MPE:** $Y = 23.41 * Wt^{.922} * Nbox^{.659} * NLinks^{1.091}$
  $(s = .26, r^2 = .93, \bar{R}^2 = .90)$
- **MUPE:** $Y = 19.08 * Wt^{.96} * Nbox^{.68} * NLinks^{1.09}$
  $(s = .26, r^2 = .92, \bar{R}^2 = .91)$
- **Log-Linear:** $Y = 17.60 * Wt^{.967} * Nbox^{.69} * NLinks^{1.09}$
  $(s = .26, r^2 = .93, \bar{R}^2 = .91)$
- **ZPB/MPE:** $Y = 22.20 * Wt^{.922} * Nbox^{.659} * NLinks^{1.091}$
  $(s = .26, r^2 = .92, \bar{R}^2 = .90)$

TT&C Digital Electronics – Scatter Plot



TTC Digital Electronics Recur CER (n = 16)

- **MPE:** $Y = 103.33 * Wt$ $(s = .54,\ r^2 = .15,\ \overline{R^2} = -.28)$
  (+37%)

- **MUPE:** $Y = 75.24 * Wt$ $(s = .63,\ r^2 = .15,\ \overline{R^2} = .11)$

- **Log-Error:** $Y = 63.52 * Wt$ $(s = .59,\ r^2 = .15,\ \overline{R^2} = .11)$
  (-16%)

- **ZPB/MPE:** $Y = 75.24 * Wt$ $(s = .63,\ r^2 = .15,\ \overline{R^2} = .11)$

TT&C RF Distribution Comp Rec CER (n = 13)

- **MPE:**

  $Y = -7.386 + 29.18 * Wt + 70.68 * Active$ $(s = .56,\ r^2 = .47,\ \bar{R}^2 = .24)$

  (-5%, +5%, +85%)

- **MUPE:**

  $Y = -7.043 + 27.899 * Wt + 38.202 * Active$ $(s = .67,\ r^2 = .46,\ \bar{R}^2 = .35)$

- **Log-Error:**

  $Y = -6.165 + 25.08 * Wt + 30.293 * Active$ $(s = .60,\ r^2 = .47,\ \bar{R}^2 = .29)$

  (+12%, -10%, -20%)

- **ZPB/MPE:**

  $Y = -5.616 + 22.19 * Wt + 53.74 * Active$ $(s = .64,\ r^2 = .47,\ \bar{R}^2 = .33)$

  (+20%, -20%, +41%)

Note: ZPB/MPE is 76% of MPE

EPS Generation Recur CER (n = 17)

- **MPE:** $Y = 582.62 + 0.0979 * CellNum$
  $(s = .39,\ r^2 = .80,\ \overline{R}^2 = .51)$

- **MUPE:** $Y = 488.54 + 0.0864 * CellNum$
  $(s = .41,\ r^2 = .80,\ \overline{R}^2 = .71)$

- **Log-Error:** $Y = 437.787 + 0.080 * CellNum$
  $(s = .47,\ r^2 = .80,\ \overline{R}^2 = .73)$

- **MPE/ZPB:** $Y = 506.366 + 0.085 * CellNum$
  (87% of MPE) $(s = .41,\ r^2 = .80,\ \overline{R}^2 = .71)$

SEPM Factors

- **MPE:**

  $Y = 22.54 * \text{Lot\#}^{-.718} * \text{Rate}^{-1.02}$ ($s = .167$, $r^2 = .84$)

- **MUPE:**

  $Y = 21.93 * \text{Lot\#}^{-.713} * \text{Rate}^{-0.992}$ ($s = .169$, $r^2 = .84$)

- **Log-Error:**

  $Y = 21.65 * \text{Lot\#}^{-.712} * \text{Rate}^{-0.983}$ ($s = .167$, $r^2 = .84$)

- **ZPB/MPE:**

  $Y = 21.98 * \text{Lot\#}^{-.718} * \text{Rate}^{-1.02}$ ($s = .169$, $r^2 = .84$)

  (97.5% of MPE)

Example from ISPA

- **MPE:** $Y = 377.337 - 279.211 * X^{-0.141}$ (s = .664, r^2 = .42)
- **MUPE:** $Y = -1.145 + 53.734 * X^{0.658}$ (s = .871, r^2 = .36)
- **Log-Error:** $Y = 3.512 + 30.17 * X^{0.990}$ (s =1 .06, r^2 = .32)
- **ZPB/MPE:** $Y = -67.649 + 126.666 * X^{0.247}$ (s = .829, r^2 = .40)

Note: The above equations are different in both magnitude and sign.

Conclusions

- **MUPE method does <u>not</u> require transformation or correction factor.**
- **Traditional goodness-of-fit measures may not be adequate.**
  - Traditional statistics hold for linear equations.
- **Review multiplicative error, adjusted R^2, residual plot, percentage error table, etc. for MUPE equations.**
- **Pearson's correlation is <u>not</u> sensitive to different fitting methods. Use it with caution.**
- **ZPB/MPE method does not change the sensitivities of MPE equation; it only lowers the level of MPE equation by a certain percentage*. This finding is also proven by mathematical derivations.**
- **MPE method requires correction factors for both standard error and average percentage error because they are biased low.**
- **Log-error equations with correction factors are very close to MUPE equations in most cases.**
- **MPE and MUPE do not always converge, especially in learning curve analysis.**
  - If no convergence, use log-error models instead

* **true for most equations**

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