Joint Agency
Cost Schedule Risk and Uncertainty Handbook

This Handbook defines processes and procedures for performing cost and schedule risk and uncertainty analysis in support of life cycle cost estimates for major acquisition programs.

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Preface

The Government cost analysis community recognizes the need to capture the inherent uncertainty of acquisition programs into realistic cost estimates to support milestone decision processes. Programmatic, cost, schedule, and technical uncertainties are present from the earliest concept exploration phase, through system development, acquisition, deployment, to operations and sustainment. Many estimating processes have focused on producing a single, discrete dollar value that in turn becomes the budget. Realistically, estimating processes develop a range of likely values, with objective and quantifiable analysis of uncertainty intrinsically embedded. The goal of this handbook is to introduce industry best practices for incorporating uncertainty into our cost estimates in order to provide decision makers with the information necessary to make sound, defendable investment decisions.

Cost estimates are required throughout the life-cycle of Government acquisition programs, starting from the early stages when requirements may be ill-defined and when potential solutions to provide the capabilities have yet to be developed. Ideally, analysts rely on objective data analysis to develop their estimates. Often, however, it is necessary to resort to subjective means. In both cases, there is uncertainty in the data, the process and the ultimate estimates. This handbook provides guidance on how to capture and quantify this uncertainty in a manner that supports the decision maker's needs.

This handbook emphasizes the need to shift away from estimates based solely on the "best guess" of system and programmatic parameters and encourages the cost analyst to build models that address technical, programmatic, cost and schedule uncertainties, and risks as inter-dependent – not separate – processes. This handbook is "tool-independent" and uses a single model to support examples that show the reader how to implement the guidance in a realistic setting. Illustrations are drawn from three industry- common cost risk and uncertainty analysis tools: @RISK, Crystal Ball, and ACEIT RI$K. Regardless of the tool used, adherence to the guidelines yields equivalent results.

The effective incorporation of risk uncertainty in cost and schedule estimates is a challenging task. This handbook is promulgated to help establish a systematic, structured, repeatable and defendable process for delivering comprehensive estimates to Government leadership to get the best possible capability with increasingly limited available resources.

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1.0 INTRODUCTION

1.1 PURPOSE

The purpose of the Cost Schedule Risk and Uncertainty Handbook (CSRUH) is to describe best practice techniques to model cost estimate uncertainty in order to calculate and report the cost risk. The handbook also provides guidance on how to capture schedule uncertainty and the impact of discrete events (without double counting) that may influence the cost risk assessment. The goal of the handbook is to define and clearly present simple, well-defined cost risk and uncertainty analysis processes that are repeatable, defendable and easily understood. While capturing schedule uncertainty is a theme throughout this document, the handbook content is applicable to the development of any uncertainty model. It also provides an introduction to building a fully integrated cost and schedule risk and uncertainty analysis. To aid in the distinction of these very different types of models, we will use the following definitions:

- **CISM**: The Cost Informed by Schedule Method is a cost uncertainty model that has some level of duration uncertainty built into it such that duration uncertainty influences the cost simulation results. Building a CISM model, illustrated in **Figure 1-1**, is the focus of this handbook. However, the guidance is also applicable to the development of cost uncertainty models that are not informed by schedule.
- **FICSM**: The Fully Integrated Cost/Schedule Method is typically a cost loaded schedule model with cost/schedule risks and uncertainty addressed. The elements of a FICSM model are introduced in **Appendix B**. The FICSM approach is currently gaining community interest and may be the way of the future. It is introduced in this handbook and will be more fully addressed at a later date.

Throughout this handbook, many technical terms are used that are derived from comprehensive and often conflicting cost/schedule uncertainty sources listed in **Section 7.0**. For a consolidated list of definitions, see **Appendix A** . It is expected that the reader has a good knowledge of cost estimating and statistics.

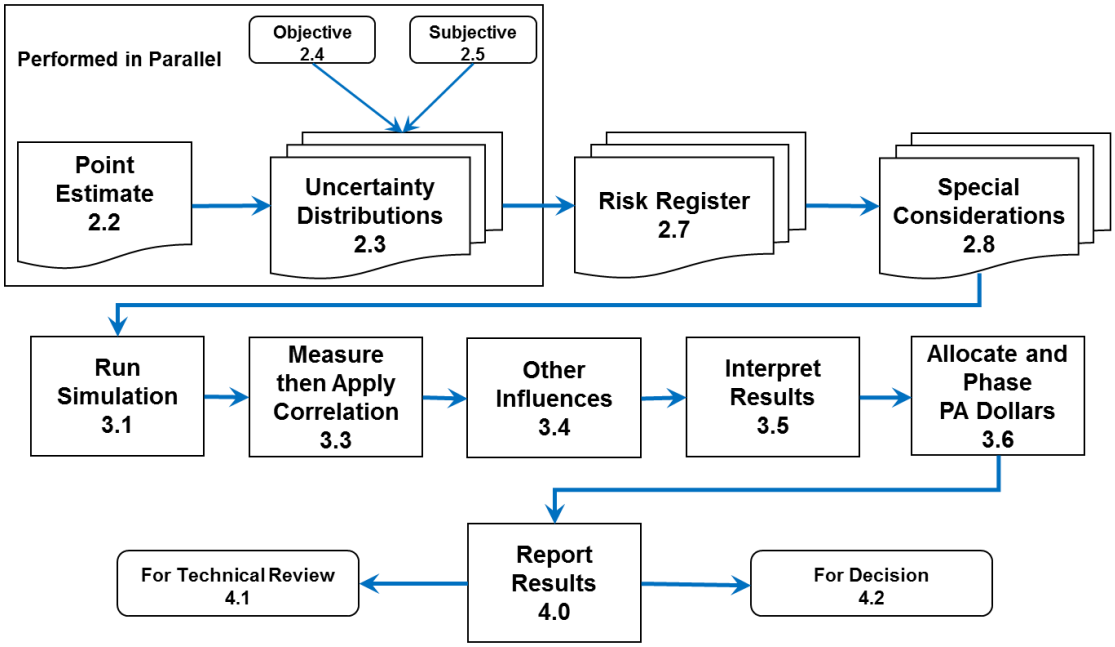


Figure 1-1 CISM Model Development Flow

Risk and uncertainty data should be collected and input into the model at the same time as the point estimate. Recommend incorporating schedule uncertainty via CISM or FICSM approach.

1.2 RISK AND UNCERTAINTY

1.2.1 The Requirement for Cost Risk and Uncertainty Analysis

Cost analysts do their utmost to develop the best cost estimate possible from the available information. This should include building the cost model to be consistent with the project's planned schedule. The fact remains that every assumption and variable driving the cost estimate represents only one point within a range of possible values. For this reason, a cost estimate of this type is called a "point estimate."

Ensuring the cost model point estimate is sensitive to schedule uncertainty is not easy. Typically, analysts collect completed project data as a basis for developing cost estimating relationships (CERs). Depending on the nature of the data, the CERs may already capture some degree of schedule uncertainty. A cost analyst must be able to defend the uncertainty built into the model and demonstrate there is no "double counting" of uncertainty.

The key steps in the cost estimating process are illustrated in **Figure 1-2**[1]. This handbook focuses on the mechanics of conducting the risk and uncertainty analysis.

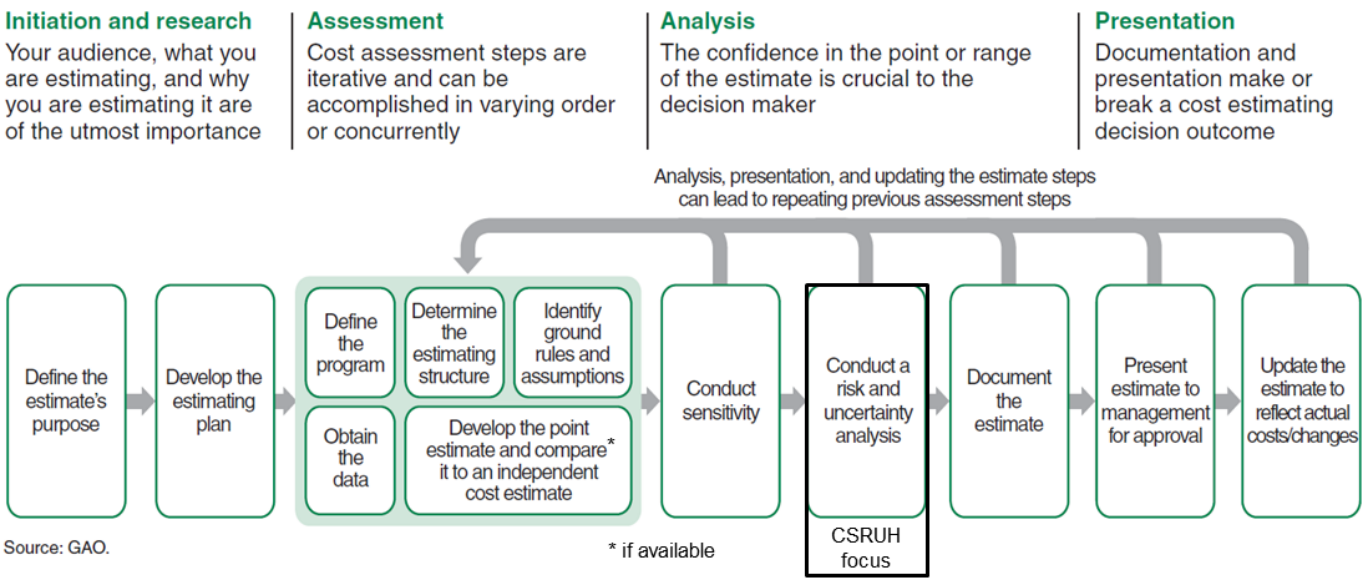


Figure 1-2 The Cost Estimating Process

1.2.2 The Difference Between Risk, Opportunity, and Uncertainty

There is an important distinction between the terms risk, opportunity, and uncertainty:

- **Risk** is the probability of a loss or injury
- **Opportunity** is a favorable event or outcome
- **Uncertainty** is the indefiniteness about the outcome of a situation

These definitions are supported by **References 24, 53** and **68**. In a situation that includes favorable and unfavorable events, uncertainty captures both opportunity and risk. Uncertainty is defined in cost models for the purpose of estimating risk. In the context of a cost model, risk is the probability that a specific funding level will be exceeded. The definition of risk, opportunity, and uncertainty in the cost estimating context is illustrated in **Figure 1-3**.

[1] This figure is replicated from the 2009 GAO Cost Estimating and Assessment Guide (**Reference 68**).

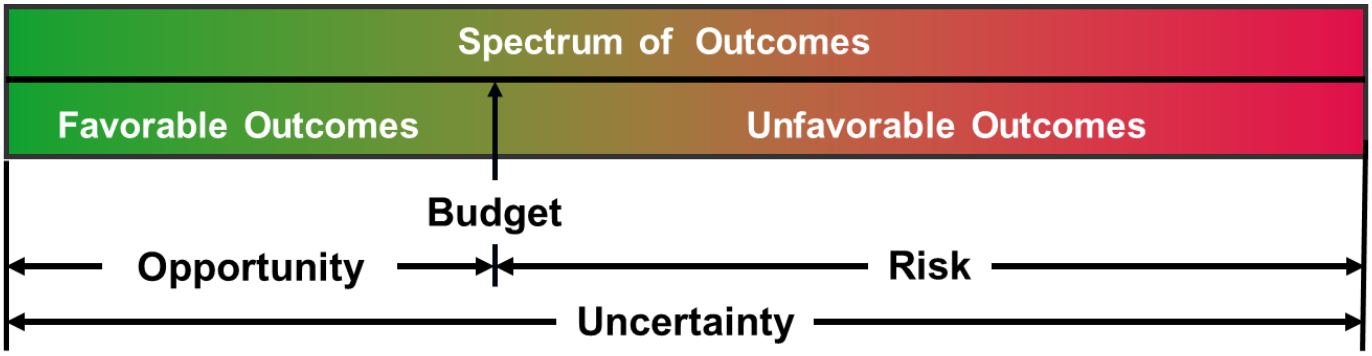


Figure 1-3 Risk, Opportunity and Uncertainty in the Cost Estimating Context

The goal of the simulation model is to combine all the sources of cost uncertainty in order to estimate the risk of exceeding a given budget.

1.3 SOURCES OF UNCERTAINTY

1.3.1 Uncertainty to be Captured

Every program has many sources of uncertainty. The goal is to model the combined effect of all sources of uncertainty in order to assess the risk of exceeding a given budget. At a minimum, the model needs to capture the uncertainty of:

- Parametric CERs including factors and cost improvement curve (CIC) equations
- CER inputs, complexity factors for analogies, engineering judgment
- Any other cost drivers (man-hours, head counts, rates, ratios, overhead, fee, etc.)
- The planned schedule (durations)
- Risk register events, both probability of occurrence and the consequence

This handbook will provide guidance on how to apply correlated uncertainty across these elements of a cost risk and uncertainty model.

1.3.2 Uncertainty Captured From the Risk Register

The CISM modeling process defined in this handbook begins by addressing the uncertainty (see **Section 1.4.1**) in the cost model which will include modeling the impact of the risk register. The goal is to model the combined effect of these sources of uncertainty in order to assess the risk of exceeding a given budget. But modeling the uncertainty in the cost estimate is not sufficient. The analyst needs to capture the risk register as well.

The Risk Management Guide for DoD Acquisition (**Reference 53**) contains baseline information and explanations for a well-structured risk management program. Risk management is a fundamental program management tool for effectively managing cost, schedule, and technical risks associated with system acquisition. The Risk Management Guide for DoD Acquisition provides guidance on how to identify and assess negative events and chose not to address opportunities. A direct quote is “Risk addresses the potential variation in the planned approach and its expected outcome. While such variation could include positive as well as negative effects, this guide will only address negative future effects since programs have typically

experienced difficulty in this area during the acquisition process." In the CSRUH, however, the risk register will capture not only the potential negative outcomes (risks) but positive outcomes (opportunities) as well.

Opportunities should not be confused with risk mitigation plans. To clarify the difference between risk, risk mitigation, and opportunity in the context of the risk register, consider a project that has an expensive test in the uncertainty model, but the 90% probability that the test will have to be repeated is not yet captured. There are several approaches to model that situation, including:

- Adding a risk event to the risk register that captures the 90% probability and cost impact of the second test
- Adding a risk event to the risk register but also adding risk mitigation effort (cost) to the model to reduce either the probability of the second test or its consequence or both. Risk mitigation effort in this context is not free. We are not talking about plans to "scrutinize more", "manage better" or other such euphemisms. Risk mitigation effort in this context is a deliberate decision to add scope to the project in order to reduce/eliminate an identified risk. Obviously the cost of the risk mitigation effort needs to be substantially less than the impact of event to be mitigated to make it worthwhile.
- Adding both tests to the uncertainty model. In this case the risk register should include the 10% opportunity (cost savings) that the second test is not required. The probability of this opportunity could be increased if risk mitigation effort is also added.

Since the risk register contains both risks and opportunities, it would seem appropriate to assign it a name other than: risk register. However, risk register is a very common term used to define potential events that could have either a negative (risk) or positive (opportunity) impact on the project. As an example, the Association for the Advancement of Cost Engineering (AACE) International recommended practice 57R-09, Integrated Cost and Schedule Risk Analysis (SRA), (**Reference 79**) defines risk events as events that may or may not happen, but if they do happen they will have a *negative* or *positive impact* on the cost or schedule or both. The risk register is a listing of all identified risk events that impact the cost model.

The primary source of the risk register will be from the program risk management team. However, the risk management team will not have a cost focus. They will be assessing a wide variety of risks to the program's objectives and characterizing their impact on cost, schedule or both. The role of the cost analyst will be to use this information to select those risks not already inherently captured in the cost uncertainty model and add them as discrete events with a probability of occurrence and the event's impact on cost and/or schedule. Capturing the risk register is discussed in **Section 2.7**.

The uncertainty model must capture uncertainty of cost methods, cost method inputs, and the risk register.
Where possible, the influence of duration uncertainty should also be captured.
Any additional uncertainty that can be modeled in a defensible way should also be captured.

1.3.3 Uncertainty That Could be Captured

There are many other potential sources of cost estimate uncertainty. If defendable methods are available, the analyst should consider capturing the uncertainty of the following:

- Inflation
- Acquisition strategies
- Requirements creep
- Significant change in the planned scope
- Different contracting options/strategies

- Congressional/Service actions (e.g. budget perturbations)
- Anything outside the project manager's control that will affect the project that can be modeled

No standard, approved modeling methods are provided in this handbook for these uncertainties. Inclusion of these uncertainties should be done with great care to reduce the likelihood of double counting or creating overly pessimistic assessments.

1.3.4 Uncertainty That Should Not Be Captured

Special consideration should be given to uncontrollable events that can impact the cost of a program. In most cases, these events should not be included in the uncertainty assessment. If at all, they could be the subject of a separate sensitivity analysis and discussion. Events such as natural disasters (hurricanes, earthquakes, etc.), industry collapses (bankruptcies, litigation, etc.), mission changing events (e.g. space shuttle disaster), and world events (e.g. September 11th) should generally be excluded from explicit uncertainty modeling.

1.3.5 Elements That Could Be Regarded as Certain

There are actually several elements in the model that should not be regarded as uncertain. The most obvious portions of the estimate that should be considered certain are sunk costs (see **Section 2.8.2**). Items that ***may*** be considered certain include:

- Elements with inconsequentially low cost need not receive uncertainty treatment unless there are many of these elements. Defining inconsequentially low cost is an analyst or agency judgment.
- Sunk costs
- A design fact (i.e. for each item *A* procured, three of item *B* will be procured)
- Unit of Measure conversion factors
- Production minimum or maximum sustainable rates
- Production quantity is often a source of uncertainty but generally not assigned an uncertainty distribution. Instead, various quantity scenarios are best treated as discrete what-if cases, each of which is subjected to the risk and uncertainty analysis.
- Some forms of government furnished equipment can be considered as certain if the analyst is only concerned about guarding against uncertainty for those items for which your agency or department has to pay

1.4 OVERVIEW OF COST RISK AND UNCERTAINTY METHODS

1.4.1 Simulation-Based Cost Risk and Uncertainty Analysis

With several powerful commercial and government simulation tools from which to choose, simulation has become the most popular method of modeling cost uncertainty. For details on simulation methods see **Appendix A.10**. The specific type of tools to be used is a function of the type of model to be created:

- **CISM models**: The simulation tools most commonly used in the DoD cost analysis community suitable for CISM models are Crystal Ball (CB), @RISK, and the Automated Cost Estimator (ACE). CB and @RISK are Excel-based general-purpose commercial risk and uncertainty modeling tools. ACE is a DoD-funded tool designed for the government cost analysis environment. This handbook does not recommend any specific tool, but does acknowledge that these three are in common use and widely accepted. Examples in this handbook will be modeled using all three of these tools and it will be shown that probability results are within a percent or less of each other. It is the responsibility of

the analyst to determine which tool to use in accordance with their organization's policy and tool availability.

- **FICSM Models**: There are even more tools to choose from and in use throughout the government to support FICSM model building and they tend to either be add-ins to specific scheduling tools, or stand-alone tools that will import the schedule model files. They include Primavera Risk Analysis (PRA), @RISK for Project, Risky Project, ACEIT (the Joint Analysis Costs and Schedule – JACS component), Acumen Fuse and Polaris to name a few. Unlike the spreadsheet-based tools, these schedule-based simulation tools are relatively new to the industry, many of them emerging in the last few years. Also unlike the spreadsheet-based tools, the differences between them make it challenging to have them agree completely with each other. The state-of-the-art for FICSM models is changing very fast and so are the tools. For that reason, the FICSM model example in **Appendix B** was only developed in one tool to demonstrate one possible way to generate a FICSM result.

1.4.2 Other Methods

There are acceptable methods other than simulation for performing an uncertainty analysis. These include:

- **Outputs-Based Uncertainty**: The outputs-based method applies uncertainty directly to the cost model results rather than to the model's inputs and is discussed further in **Section 5.4**
- **Enhanced Scenario Based Method (eSBM)**: eSBM is not a simulation-based approach. It is based on developing one or more potential scenarios and using historical information to help define the impact. eSBM also provides an output uncertainty distribution around scenario results based on historical cost growth data. eSBM is discussed further in **Section 5.2**.
- **Method of Moments**: Method of moments is an analytical approach to estimating total program uncertainty. It can be useful when there is a need to sum large numbers of uncertain elements. Method of moments principles are applied in **Section 3.3.3** to compare an analytical solution to a simulation result when summing correlated uncertainties. It is also discussed in **Section 5.3**.

2.0 COST INFORMED BY SCHEDULE METHOD MODEL

2.1 STRATEGIC APPROACH

A systematic process for developing a CISM model, regardless of the tool used, is summarized in **Figure 2-1**. In this section of the handbook we will provide guidance on each step of the process. The important messages of **Figure 2-1** are:

- The point estimate is the anchor of the risk and uncertainty modeling process. It must be complete before the risk and uncertainty modeling can be completed.
- Cost methods and their drivers must be assessed for uncertainty. Use of schedule duration sensitive cost estimating methods and uncertainty around schedule duration is strongly encouraged.
- Risk register items must address risk and opportunity and they may influence methods or drivers
- Wherever feasible, define uncertainty parameters as a percent of the point estimate to avoid problems when performing what-if drills as uncertainty will scale with the point estimate
- Measure the correlation present in the model to aid in determining if additional correlation is required
- Measure, do not guess, the number of simulation trials required to obtain a stable, converged result
- Compare results with those shown in **Section 3.5.3** to help with understanding the results

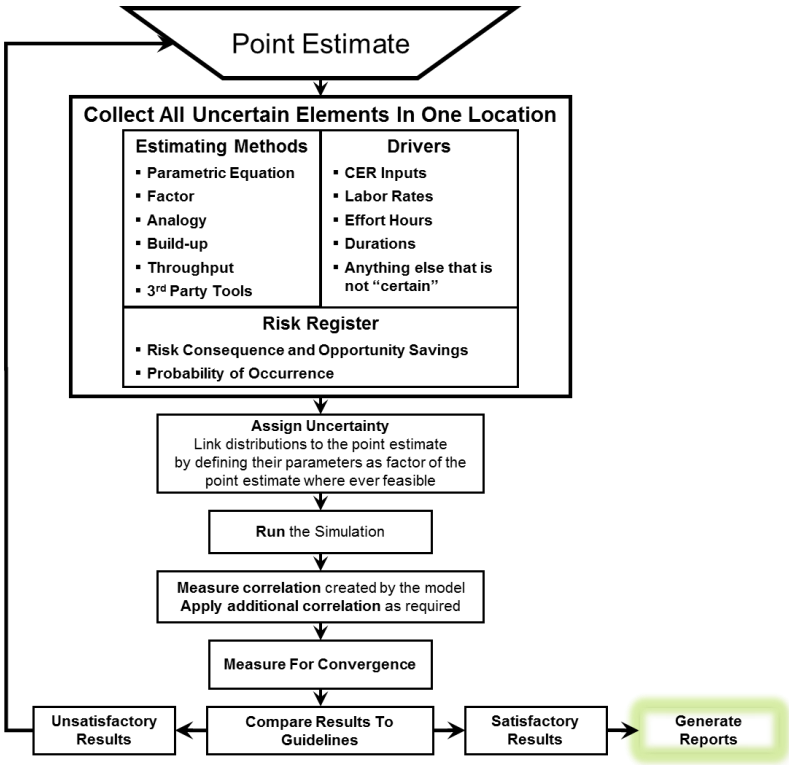


Figure 2-1 Overview of the Simulation Method

The process is iterative as indicated by the feedback loop passing though the "unsatisfactory results" block. Pausing to review for consistency in how the risk and uncertainty assumptions have been applied and to compare the statistical results with metrics suggested in **Section 3.5** will improve the quality of the overall result. In the CISM modeling approach, uncertainty is applied to every element significantly influencing the cost estimate, including task durations. Analysts are encouraged to find ways to at least introduce schedule uncertainty into the cost model. The example model will illustrate one way to introduce schedule uncertainty into a cost model as inspiration for the analyst to introduce schedule into his/her models.

2.2 THE POINT ESTIMATE

2.2.1 Point Estimate Definition

The starting point for any uncertainty analysis method is a point estimate. The point estimate's cost and schedule must be based upon a realistic, documented definition of the program. Depending on the objective of the estimate, the point estimate can be based upon:

- **Program of Record**: defined in the program documents
- **Assessed Technical Baseline**: alternative that reflects a technical assessment (see **Reference 95**)
- **What-If Case**: for a specific sensitivity analysis

When incorporating schedule uncertainty into the cost model, analysts should be cognizant of the fact that cost estimates are structured very differently than schedule estimates:

- **The cost point estimate (PE)** will be derived from an approved work breakdown structure (WBS). MIL-STD-881C (**Reference 81**) presents direction for effectively preparing, understanding, and presenting a work breakdown structure. It provides the framework for Department of Defense (DoD) Program Managers to define their program's WBS and also for defense contractors in their application and extension of the contract's WBS. The model's O&S phase framework may follow

the structure in the OSD CAIG's O&S Cost Estimating Guide (**Reference 60**). Cost estimating methods are driven by uncertain technical, schedule, and programmatic inputs. This is consistent with the idea that the point estimate inputs should reflect the best assessment consistent with the cost estimate objective. **Appendix A.2** contains further details on the nature of a cost point estimate.

- **The schedule point estimate (PE_S)** is an integrated network of activities containing all the detailed discrete work packages and planning packages (or lower-level tasks of activities) necessary to support the events, accomplishments, and criteria of the project plan. Please see **Appendix A.1.10** for a discussion of an Integrated Master Plan (IMP) and Integrated Master Schedule (IMS). In this handbook and the example model (both CISM and FICSM), we assume that neither the IMP nor the IMS are available. Consequently, the schedule developed from program documents should follow the concepts of an IMS, but at a much reduced level of detail. Ideally, it will follow the same WBS structure as used to develop the point estimate (**Reference 68**).

The cost and schedule point estimates must be consistent with each other and the estimate objective.

2.2.2 Point Estimate Total Does Not Capture Risk or Uncertainty

The total PE is the sum of methodologies applied at the lowest level in the WBS. The PE at any level in the WBS does not change with the application of risk and uncertainty. But with the application of risk and uncertainty, other values (e.g., mean, median, mode or some other percentile) are available to be selected as a point estimate. The cost PE should not include any extra dollars inserted into individual cost elements to address some measure of estimate uncertainty. Nor should elements like an Engineering Change Order (ECO) be used to capture uncertainty of other cost elements. WBS elements like ECO should be estimated in the same way as other WBS elements: based on methods derived from historical data or expert opinion, but without additive “pads” or “margins” to address suspected shortfalls in the PE of other WBS elements. Planned engineering can be captured explicitly. Unplanned engineering changes should be captured through uncertainty modeling.

The degree to which the selected cost estimating method may underestimate or overestimate cost should be addressed in the uncertainty analysis, not by understating or overstating the results of the cost estimating method. Cost drivers such as weight, code count, volume, power, rates, etc. should reflect a documented value and not some lower or upper bound. The potential for a cost driver to be something other than its documented value should be handled in the uncertainty analysis process.

This handbook provides guidance on how to apply risk and uncertainty to the PE and PE_S. No matter how much effort is applied to estimating lower-level WBS elements, the parent (or total) levels in the PE can only be the most-likely value if every child estimate is a most-likely with perfectly symmetrical uncertainty. In practice, this is never the case for cost estimates. The objective of the risk and uncertainty analysis is to estimate the combined uncertainty of every element based upon cost, schedule, technical, and risk register influences (avoiding double counting risks) to provide a basis for estimating the probability (risk) of exceeding a specific budget.

Parent level totals throughout the point estimate WBS will not reflect most likely results.

2.2.3 Point Estimate Construction

When performing cost risk and uncertainty analysis, the analyst should ensure that the point estimate is as complete as possible. No amount of agonizing over distribution shapes, bounds or correlation will make up for the exclusion of elements required in the estimate or using estimating relationships that are completely inappropriate for the system in question. Yes, collect the necessary data and build uncertainty as the model is developed, but ensure that the point estimate is as complete and robust as possible.

This handbook uses a simplified missile WBS (based on MIL-STD 881C) as shown in **Figure 2-2**. **This example will be used throughout the handbook** to illustrate cost risk and uncertainty analysis methods, processes, and effective reporting. The models are available with this handbook in @RISK, Crystal Ball and the Automated Cost Estimator (ACE). For details, see **Section 7.2.2**.

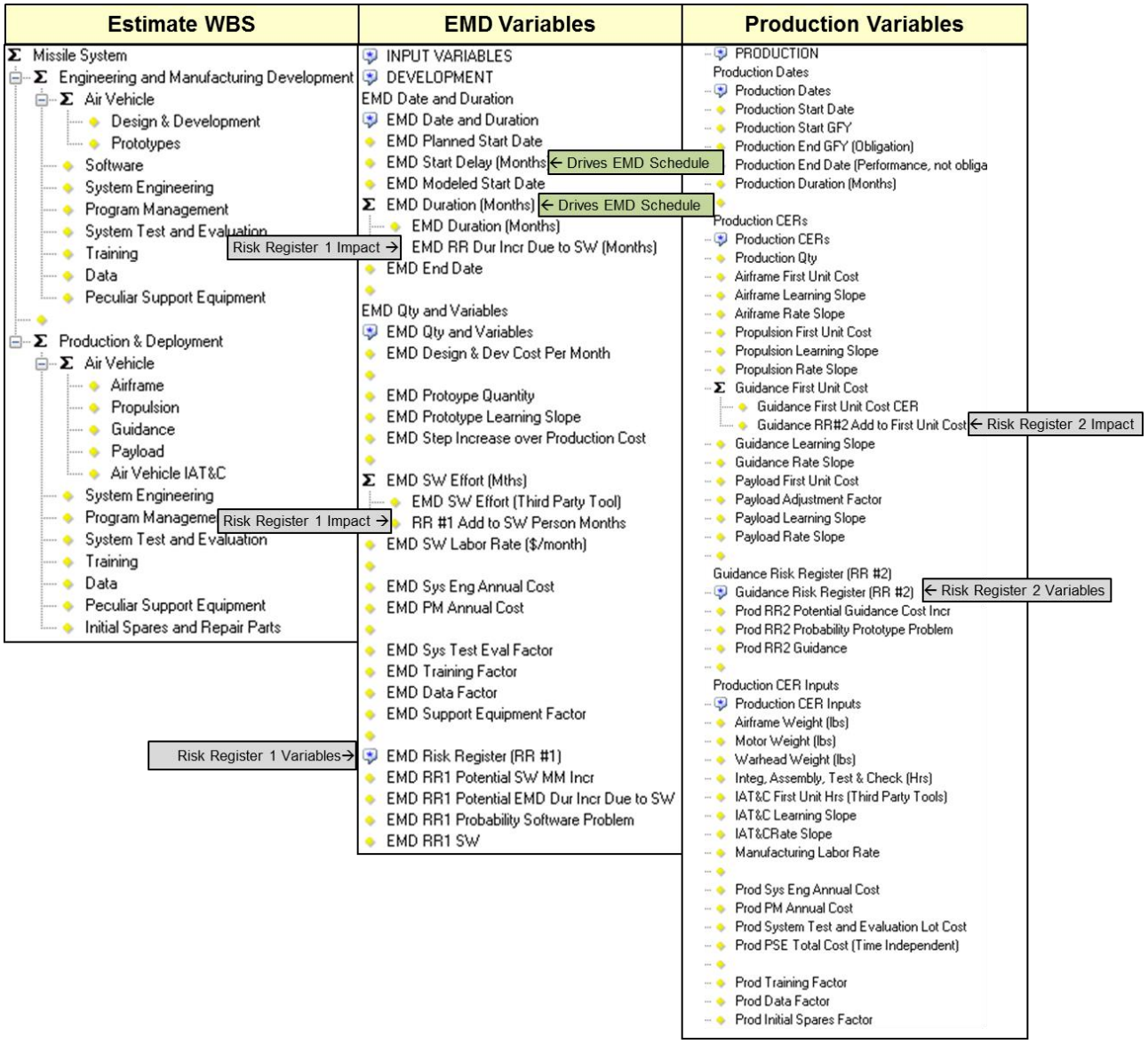


Figure 2-2 Example WBS and Input Variables

As shown in **Figure 2-2**, only a few duration variables are in use and it will be shown this has a significant impact on the cost uncertainty result. Additionally, two risk register events are included with a total of one duration and two cost impacts. Note how the risk register is integrated into the model. This approach makes it easier to build the model, but there remains the requirement to expeditiously identify and cross check risk register items with the source.

While O&S is not explicitly addressed in either the handbook or the example, the guidance is intended to be generic enough to be applicable to any type of estimate, as most cost estimating methods are addressed.

2.2.4 Modeling Approaches

Once the analyst has identified the scope and schedule and has defined the program WBS (see WBS example in **Figure 2-2**), there is the task of populating the lowest level of the WBS with cost estimating methods.

While the modeling approach to each element arises from the four standard estimating methods (i.e., analogy, parametric, engineering, and extrapolation), the choice of how the methods are implemented will determine how uncertainty is to be assigned. This handbook will provide specific guidance on the following:

- **Cost as a function of technical and/or schedule parameters**: Uncertainty is assigned to the equation itself and to its inputs
- **Cost as a factor of another cost**: The factor is one form of a parametric equation, however it tends to be used to estimate cost as a factor of another cost. The factor is uncertain and additional uncertainty is inherited through the cost variable.
- **Level of Effort**: Quantity times the cost per unit, burn rate times a duration, and other engineering calculations. In these instances the equation or build-up method itself is not uncertain. However, each variable in the method needs to be assessed for uncertainty.
- **Throughputs**: Analogies, quotes, and subject matter expert (SME) opinions are almost always uncertain. While the source value may be a known, its applicability to the new program may be uncertain.
- **Third Party tool results**: When moving results from another model or tool into another, it is not enough to import the point estimate; the uncertainty needs to be imported as well

2.2.5 Duration Sensitive Cost Estimating Methods

There are many ways to cause the cost model to be sensitive to duration. Duration should be assigned at the lowest possible level in the cost model (rather than assigning the same top level duration to all lower level tasks). Applying uncertainty directly to dates is discouraged. The date 01 October 2013 is interpreted by Excel as 41458. Plus or minus a small percentage results in a very long duration relative to most projects. It is more intuitive and easier to control when uncertainty is applied to durations in terms of months or days. The example model includes several such methods that should not be construed as required or even recommended practices. They are included here only to illustrate and inspire. Each of the following are built into the example @RISK, Crystal Ball and ACE example models:

- **EMD Start**: A planned start date of 01Oct2013 is defined. A start delay of two months is defined and used to calculate a modeled start date using Excel and ACE date functions[2]. This structure makes it easy to cause the modeled start date to be uncertain. This may have no effect on a constant year estimate, but it will impact the TY estimate.
- **EMD Duration**: A most-likely EMD duration is defined in months and used to calculate the EMD end date. The EMD end date is also influenced by a risk register event.
- **Link EMD End to Production Start:** Production start is modeled as EMD end. There are several alternative variations that could be considered.
  - **Independent**: Production start could be modeled similar to EMD start, and independent of EMD end. In our model, the Production schedule is fixed and unlinked to EMD by default.
  - **Lead/Lag**: The PE_S could include an uncertain lead/lag providing for production start to either have a gap or overlap EMD end. It is not uncommon, for instance, for production to begin before EMD has concluded.

[2] Note that Excel YearFrac(FromDate, ToDate, [Basis]) either omitting or using a value of 0 for the optional basis variable forces Excel to assume 30 day months and 360 day years. Many financial calculations are made under these assumptions. Use "1" for the optional basis value to force YearFrac to use actual dates, and to match the ACE DateYearDiff function.

- **Production Duration**: For the example model, the baseline assumption is that 600 missiles will be obligated at a rate of 150 per year. The first and last year quantities are influenced by the production start date. The model adjusts the duration of production for the assumed production rate.

In order to cause the model cost to be sensitive to duration, the following modeling choices were made:

- **Directly influenced by duration**: System engineering, program management, and test and evaluation costs are often directly related to duration since it is necessary to retain staff when durations expand and staff can be reassigned when projects end early. Estimating methods tend to estimate total cost. It is important to also estimate the nominal duration associated with the total cost. This provides a basis for estimating a monthly or annual burn rate ($/month or $/year) and thus create an estimating method that is directly influenced by duration (burn rate * duration).
- **Cost Improvement Curve influenced by duration**: Production duration is influenced by total quantity and production rate. In this situation, a unit or cumulative average CIC (learning curve) result is not influenced by duration, but a rate-affected CIC result will be influenced by duration. The example model employs a rate-affected CIC. If the rate per year is changed, duration and total cost will change. Additional CIC guidance is in **Section 2.8.5**.
- **Indirectly influenced by duration**: Many elements such as training, data, peculiar support equipment, parts obsolescence, and initial spares are often estimated as factors of the prime mission product (in our case, Missile Production). Thus, as the production cost is influenced by duration, this impact is inherited by elements that are estimated based on a factor times the production cost.

Table 2-1 identifies the cost estimating methods in the example cost model and how they are influenced by duration. This handbook will provide focused guidance on how to apply uncertainty to the above methods.

Table 2-1 Cost Modeling Approaches in the Example Model

WBS Description	Estimate Method	Duration Sensitive
Missile System		
Engineering and Manufacturing Development		
Air Vehicle		
Design & Development	**[DurationBased]** EMD_DesignDevPerMth*EMD_Duration	Direct
Prototypes	**[Factor for T1]** EMD_Prod * ProdToEMDStepUpFact * Learning	Time Independent
Software	**[Analogy]** ThirdPartyToolSWManMonths * SWLaborRate$	Time Independent
System Engineering	**[Build-up]** EMD_SEFTE * EMD_SELabRate$ * EMD_Duration	Direct
Program Management	**[Build-up]** EMD_PMFTE * EMD_PMLabRate$ * EMD_Duration	Direct
System Test and Evaluation	**[Factor]** EMD_STE_Fac * EMD_Proto$	Time Independent
Training	**[Factor]** EMD_Trng_Fac * EMD_AV$	Indirect
Data	**[Factor]** EMD_Data_Fac * EMD_AV$	Indirect
Peculiar Support Equipment	**[Factor]** EMD_SptEquip_Fac * EMD_Proto$	Time Independent
Production & Deployment		
Air Vehicle		
Airframe*	**[Parametric CER: TRIAD]** 25.62 + 2.101 * AirFrameWt ^ 0.5541	CIC Rate
Propulsion*	**[Parametric CER: OLS Loglinear]** 1.618 * MotorWt ^ 0.6848	CIC Rate
Guidance*	**[Throughput]** 100	CIC Rate
Payload*	**[Parametric CER: OLS Linear] (**30.15 + 1.049 * WarheadWt) * AdjustFactor	CIC Rate
Air Vehicle IAT&C*	**[Third Party Tool]** IACO_HsPerUnit * MfgLaborRate$	CIC Rate
System Engineering	**[Build-up]** Prod_SEFTE * Prod_SELabRate$ * Prod_Duration	Direct
Program Management	**[Build-up]** Prod_PMFTE * Prod_PMLabRate$ * Prod_Duration	Direct
System Test and Evaluation	**[Throughput]** $1,250 per year	Direct
Training	**[Factor]** Trng_Fac * AV_Prod$	Indirect
Data	**[Factor]** Data_Fac * AV_Prod$	Indirect
Peculiar Support Equipment	**[Throughput]** $7,634.27	Time Independent
Initial Spares and Repair Parts	**[Factor]** InitSpares_Fac * AV_Prod$	Indirect

* = CER to estimate the first unit cost for a rate affected unit CIC

2.2.6 Role of Sensitivity Analysis on the Point Estimate

A sensitivity analysis is a systematic approach used to identify the impact of potential changes to one or more of an estimate's major input parameters on the total cost. The objective is to vary input parameters over a range of probable values and recalculate the estimate to determine how sensitive the outcomes are to changes in the selected parameters.

A sensitivity analysis chart is shown in **Figure 2-3**. The example illustrates the cumulative effects of changes to EMD and Production cost drivers (see **Section 5.2.3** for details). Sensitivity analyses, while useful for finding cost drivers, are not always sufficient to quantify the program cost risk. For example, each result shown in **Figure 2-3** is merely a point estimate that does not convey the risk and uncertainty associated with changes to cost drivers. (In contrast, eSBM is a type of a sensitivity analysis that does capture risk: see **Section 5.2**)

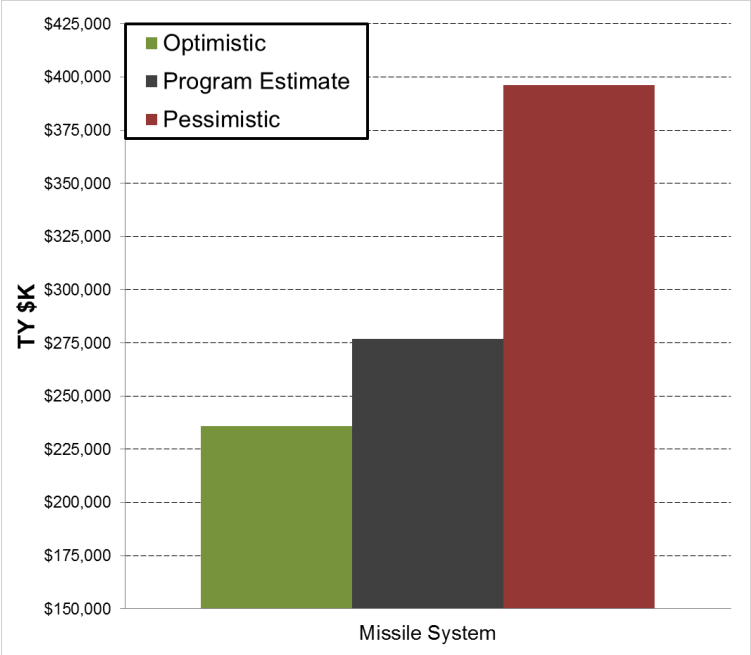


Figure 2-3 Example of a Sensitivity Analysis Chart

2.2.7 Role of Uncertainty Analysis on the Point Estimate

The point estimate represents one possible estimate based upon a given set of program characteristics, cost method selection, and input variable selections. The PE serves as the reference point on which the cost uncertainty analysis is anchored. For any given element in the estimate, uncertainty may be determined via one of three approaches:

- Objective (statistical analysis of relevant historical data)
- Subjective (expert opinion)
- Third Party Tools (separate models)

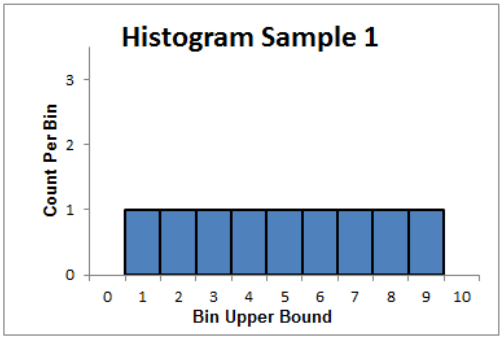
2.3 UNCERTAINTY DISTRIBUTIONS

2.3.1 Probability Distributions, Histograms and S-curves

A probability distribution is used to define uncertainty in the model as it assigns probabilities to the possible outcomes of a random event. A first step in assessing the behavior of collected data is to create a histogram.

A histogram is a bar chart where the *y*-axis is a count of the number of data points occurring within the width of each bar. The bars of a histogram are drawn so that they touch each other to indicate that the variable is continuous. **Figure 2-4** illustrates the concept with two sample sets. Each sample set contains nine observations. In Sample 1, there are nine unique and equally spaced points giving rise to one point per interval. In Sample 2, more than one point is binned into several of the intervals and a shape takes form.

Obs	Sample 1
1	6.0
2	7.0
3	3.0
4	8.0
5	1.0
6	4.0
7	9.0
8	2.0
9	5.0



Obs	Sample 2
1	3.8
2	1.9
3	1.3
4	2.3
5	4.9
6	2.8
7	3.1
8	1.0
9	2.6

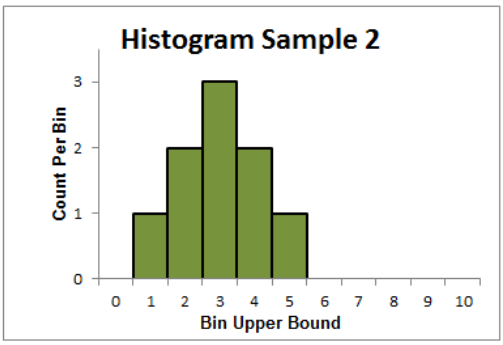


Figure 2-4 Histogram Examples

The frequency count for each bin can be transformed into percentages by dividing by the total number of sample points. In this case, the *y*-axis becomes the probability of a value falling within each bin, with the aggregate probability of all bins equaling 1. This type of histogram is often called the probability density function (PDF) plot. Graphing the cumulative probability is called the cumulative distribution function (CDF) plot, but is more commonly known as the "S-curve". A convention to help distinguish between the two is that the PDF *y*-axis is labeled with values between 0 and 1 while the CDF is labeled with percentage values. **Figure 2-5** illustrates Sample 2 in PDF and CDF form.

Obs	PDF	CDF
1	0.00	0.0
2	0.11	11%
3	0.22	33%
4	0.33	67%
5	0.22	89%
6	0.11	100%
7	0.00	
8	0.00	
9	0.00	

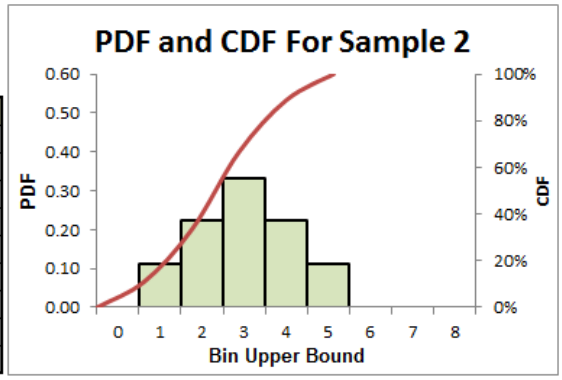


Figure 2-5 PDF and CDF Example

While random numbers can be graphed into a histogram to determine if there is any pattern (distribution), a known distribution can be used to generate random numbers. This is the core of the uncertainty simulation process. Understanding uncertainty simulation begins by understanding histograms.

A critical part of the construction of any histogram is determining how many intervals (bins) the data should be grouped into. There is no standard approach, but there are several well-known options. **Table A-14** lists five popular options. The Mann-Wald method (discussed in detail in **Reference 8**) is used by Crystal Ball and CO$TAT to determine the number of bins for the chi-squared goodness of fit test (see **Section 2.4.3.5** and **Appendix A.9.5**). Mann-Wald-divided-by-two (**Equation 2-1**) is recommended for use as a first approximation of bin count for histograms and as the basis for the chi-squared goodness-of-fit test for samples less than thirty data points. Some tools "roundup" and/or do not allow the number of bins to be less than a specific value like 6.

Equation 2-1 Mann-Wald / 2 Bin Count Equations

$$2\left(\frac{2n^2}{\left(\Phi^{-1}(\alpha)\right)^2}\right)^{0.2}$$

Excel: Max(6, ROUNDUP(2*(2*ObsCount^2/NORMSINV(ChiSigLvl)^2)^0.2,0))

Where, ***n*** is the sample size

Φ^{-1} is the inverse standard normal distribution
α is the level of confidence for the chi-squared test

Equation 2-1 is the general form of the Mann-Wald / 2. If the equation is evaluated at the 5% significance level, it reduces to $1.88n^{0.4}$ which is suitable for most cases.

Mann-Wald/2 is recommended for use as a first approximation of bin count.

2.3.2 Choosing and Defining Cost Model Probability Distributions

There are a large number of possible distribution shapes defined in the literature and available through a variety of tools. In an effort to promote consistency across program estimates, analysts are encouraged to limit their selection of distributions to those defined in **Table 2-2**[3]. See **Appendix A.6** for mathematical details on these distributions.

Table 2-2 Recommended Uncertainty Distributions

DISTRIBUTION	TYPICAL APPLICATION	KNOWLEDGE OF MODE	NUMBER OF PARAMETERS REQUIRED	RECOMMENDED PARAMETERS
Lognormal	**Default** when no better info. Probability skewed right. Replicate another model result. Power OLS CER uncertainty.	Mean or median known better than the mode	2	**Median, high** (some tools have a 3rd parameter : "Location". By default, it is zero. Used to "shift" the lognormal left or right (even into negative region)
Log-t	Log-t when < 30 data points		3	**Add Degrees of Freedom**
Triangular	Expert opinion. Finite min/max. Probability reduces towards endpoints. Skew possible. Labor rates, labor rate adjustments, factor methods	Good idea	3	**Low, mode, and high**
BetaPert	Like triangular, but mode is 4 times more important than min or max.	Very good idea	3	**Low, mode, and high**
Beta	Like triangular, but min/max region known better than mode.	Not sure	4	**Min, low, high, and max**
Normal	Equal chance low/high. Unbounded in either direction Linear OLS CER uncertainty.	Good idea, but unbounded in either direction	2	**Mean/Median/Mode and high value**
Student's-t	t when < 30 data points		3	**Add Degrees of Freedom**
Uniform	Equal chance over uncertainty range. Finite min/max.	No idea	2	**Low and High** (some tools require min and max)
Empirical Fit	Unable to fit a distribution to the data	Not required	N/A	**Enter source data and estimated probability for each data point**

Note: *Low/high* are defined with an associated percentile
Min/Max are the absolute lower/upper bound (also known as the 0/100)

The order of the distributions in **Table 2-2** is not arbitrary. **Figure 2-6** illustrates the frequency of each distribution found across 1400 fits of various cost data, factors, and CER residuals in **Reference 80.** The

[3] See http://pages.stern.nyu.edu/~%20adamodar/New_Home_Page/StatFile/statdistns.htm for an alternative discussion on distribution selection, including a flow chart at the end of the article.

finding that lognormal dominates is consistent with the AFCAA CRUH recommendation that lognormal should be the default distribution. Note that uniform was never found to be and normal was rarely found to be the best fit. Also, because beta and triangular were so close to each other, triangular was placed ahead of beta due to its simplicity and popularity.

In the absence of better information, choose lognormal as the distribution shape.

In the absence of better information and the distribution is known to be left skewed, use betaPERT.

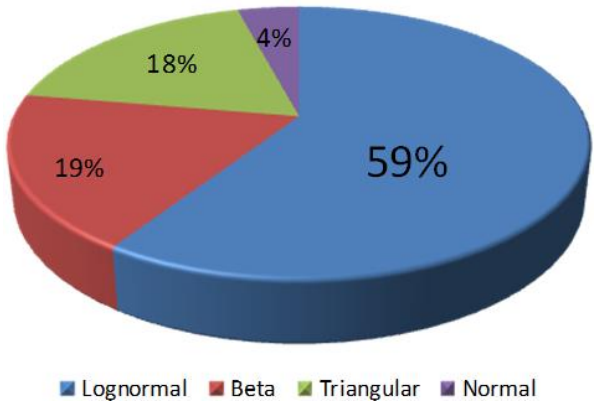


Figure 2-6 Relative Frequencies of Distribution Shapes

Lognormal distributions have a defined lower bound that is never less than zero. They have an upper bound of infinity, thus providing at least some probability of a large cost overrun. The skew of a lognormal[4] is pre-defined. This handbook recommends that in the absence of better information, choose lognormal as the shape of the uncertainty distribution. If it is known that the distribution is left skewed, betaPERT is recommended when there is no evidence to do otherwise.

The empirical fit distribution in **Table 2-2** is a good choice when no other distribution seems to fit the data particularly well. This can often be the case when attempting to replicate distributions that combine the uncertainty from a variety of cost, schedule, and risk register elements or when trying to replicate third party tool results in a spreadsheet or ACE model. Most tools refer to empirical fits as "custom" distributions. Details of the empirical fit approach can be found in **Section 2.4.3.6**.

2.3.3 Operating and Support Probability Distributions

Some agencies include failure analysis mathematics in their operations and support cost models to estimate the number of spares and/or maintenance actions. A few of these distributions are introduced here. For details on their specification and uses please see **Reference 45** and **63**. These distributions include:

- **Poisson distribution** can be used to define the number of failures in a specified time when the average number of failures is small. It is also a useful distribution to estimate testing, inventory levels, and computing reliability. The Poisson distribution is a discrete distribution that requires only a single parameter, the mean, to define the distribution. A common use of the Poisson is to simulate the number of failures per year by using the inverse of the mean time between failures as the parameter.
- **Exponential distribution** is a continuous distribution that can be used to estimate the time between failures. The parameter in this case is the mean time between failures.

[4] All three tools (CB, @RISK, and ACE) use different conventions to describe a lognormal. However, with care, the identical lognormal distribution can be successfully replicated in all three.

- **Weibull distribution** is a continuous distribution often used to estimate time between failures. A common approach is to assume a high failure rate at the beginning of life due to manufacturing errors (infant mortality), reducing to a constant (the design failure rate), and then increasing at the end of life (wear out). Infant mortality and wear out phases are often modeled with the Weibull distribution.

2.3.4 Uncertainty Distribution Descriptive Statistics

The descriptive statistics for any distribution are described in **Appendix A.4**. Also, see **Appendix A.5** for the definition of probability (or percentile). Most distributions used in cost uncertainty analysis will have a mean and a median. But not all distributions will have a mode, finite minimum, or finite maximum. **Figure 2-7** illustrates several descriptive statistics and two subjective points (low and high) that could be used to define the distribution. **Section 2.5.3** provides further guidance on the source and meaning of a low and high point in the distribution. The probability (percentile) of these particular low and high values is shown. For instance, there is an eight percent probability that a random draw from **Figure 2-7** will return a number between 55.8 and 80. It is important to distinguish between min/max, the absolute bounds of a distribution, and the low and high which will always have a percentile associated with them to uniquely define their location in the distribution.

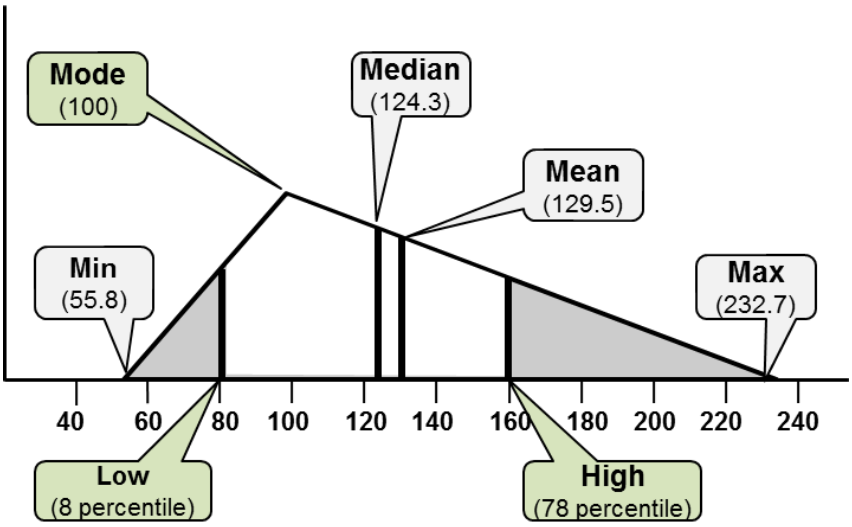


Figure 2-7 Distribution Parameters of a Notional Triangle

2.3.5 Uncertainty Distribution Skew

Since cost tends to overrun more than it under runs, it is not common for an element in a cost model to have a symmetric distribution, meaning an equal chance of an underrun or an overrun. **Appendix A.4.5** provides the mathematical definition of skewness, the formal way to define distribution asymmetry. It also describes a more simplified definition, skew, that we use in this handbook. The area under the curve of any probability distribution is one (1.0). We define skew as the measure of the area to the left of the mode. For example, if the area to the left of the mode is 0.25 then the distribution is right skewed (most of the area is to the right of the mode). If the area to the left is 0.75 then the distribution is left skewed. Skew follows the longest tail of the data. This definition, skew, is easier to interpret and directly relatable to the cost analyst. A skew of 0.25 tells the analyst that 75% of the data is higher than the mode. The skewness equivalent is 0.42, it is harder to calculate and not as easy to interpret. See **Appendix A.4.5** for details.

Figure 2-8 presents several illustrations of left skewed, not skewed and right skewed distributions. In each illustration the mode is labeled as the point estimate with the exception of the uniform distribution. Since the uniform distribution has no mode, the point estimate shown on each uniform illustration is used as a reference point in order to illustrate left skewed, not skewed or right skewed relative to the point estimate.

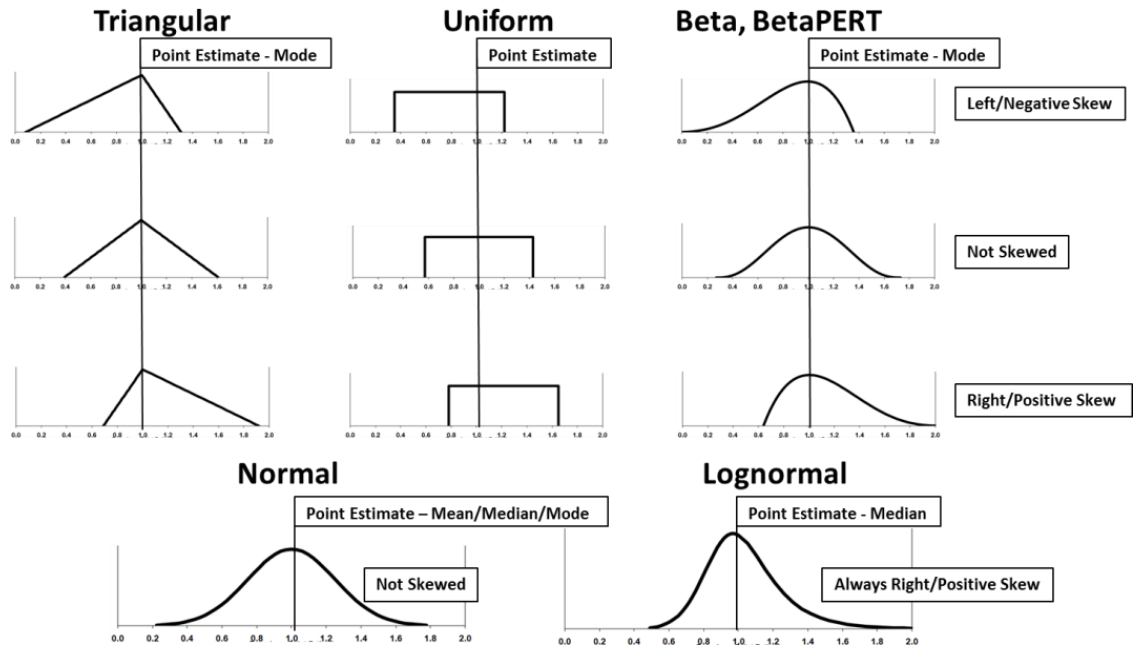


Figure 2-8 Illustration of Distribution Skew

All uncertainties should be ***data-driven*** rather than subjective, whenever possible.
All distributions should be selected based on fitted data rather than subjective, whenever possible.

2.4 OBJECTIVE UNCERTAINTY

2.4.1 Overview

Objective uncertainty is derived from the application of well-defined statistical processes on known outcomes[5]. In the context of cost analysis, this begins with the collection and normalization of relevant historical data. This handbook will address two objective methods for estimating uncertainty:

- Developing parametric equations through regression analysis
- Fitting distributions to normalized, historical data or CER residuals

2.4.2 Uncertainty of Parametric CERs

2.4.2.1 Types of Parametric CERs

Parametric CERs are derived from regression analysis. Regression analysis is a statistical technique to determine which, if any, independent variables (e.g., item characteristics) explain the variation of a dependent variable (e.g., item's cost across a variety of projects). How well parametric equations derived in this manner predict the source data provides a statistical, objective assessment of the uncertainty of the CER result. The CER error is modeled either as an additive or multiplicative factor as illustrated in **Figure 2-9** (from **Reference 13**) where y is the cost of interest, $f(x)$ is the CER functional form and ε is the error term. The error is measured by comparing the actual cost to the cost predicted by the CER for each data point used to create the CER. Regression analysis seeks to minimize the CER error term. For cost estimating, the multiplicative error term is preferred because cost error tends to scale with the estimate. For a more detailed discussion please see **Appendix A.8.4.**

[5] The known outcomes themselves can be uncertain since there is no guarantee that the dataset has been normalized properly or is fully applicable to the estimate. However, for the purposes of developing parametric CERs, this data is often treated as known.

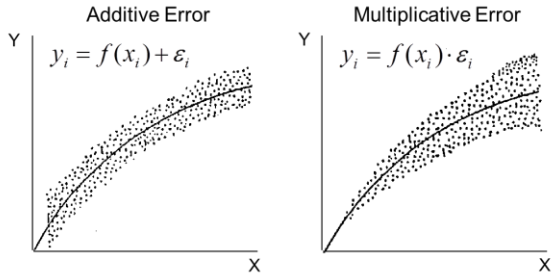


Figure 2-9 Additive and Multiplicative Functional Forms

The most common regression methods are described in detail in **Appendix A.7** and summarized here:

- **Ordinary Least Squares (OLS), Unit Space**: OLS solves for linear relationships and minimizes an additive error term. An example of an OLS derived CER is: Cost = 30.15+1.049*WarheadWt.
- **Ordinary Least Squares, Log Space**: OLS solves for linear relationships and minimizes an additive error term in log space (regression performed on the log of the data). The error term is multiplicative in unit space. An example of a Log Linear OLS derived CER is: Cost=1.618*MotorWt^0.6848.
- **Minimum Unbiased Percentage Error (MUPE)**: Derives CERs with multiplicative error terms directly in unit space. An example of a CER that cannot be derived using OLS but can be derived with MUPE is: Cost= 25.62 + 2.101 * AirFrameWt ^ 0.5541 (known as a triad form).
- **Zero Bias Minimum Percent Error (ZMPE)**: ZMPE is another method that can derive CERs with multiplicative error terms directly in unit space. Like MUPE, it can be used to derive any CER functional form, including those that OLS cannot derive.

2.4.2.2 Choosing the Parametric CER Distribution Shape and Point Estimate Location

Regardless of the CER method employed, even if the CER inputs (independent variables) are known precisely, the CER will return a result that is not certain. Depending on how the CER is developed, the error is either additive or multiplicative (a factor of the CER result). In cost estimating, we expect the potential error of the CER to scale with the CER result thus making multiplicative error terms the preferred approach for modeling the CER uncertainty (see **Appendix A.8.4**).

Two critical decisions when applying uncertainty to CERs are: selecting the uncertainty shape and defining where the point estimate falls within the distribution. Both of these decisions should be based upon an understanding of the regression method used to develop the CER. It is also possible to fit a distribution to the residuals (see **Section 2.4.3.8**). Point estimate location considerations for the most common regression methods are:

- **OLS Linear**: A premise of the OLS method is that the errors will be normally distributed in fit space. The OLS linear CER result is the center (mean, median, mode) of the normal distribution.
- **OLS Log Space**: Applying OLS in log space yields a multiplicative lognormal uncertainty in unit space. The point estimate result is the **median**, not the mean of the lognormal uncertainty.
- **MUPE**: The MUPE CER delivers the mean; it has zero proportional error for all points in the CER. Goodness-of-fit measures can be derived to judge the quality of the model if the CER error is assumed to be normal (a common assumption).
- **ZMPE**: The ZMPE method also delivers the mean and zero proportional error for all the data points in the CER. Distribution shape is arbitrary; however, some analysts prefer using lognormal.

Two critical decisions: Select the uncertainty shape and define where the point estimate falls.

2.4.2.3 Calculating the Prediction Interval

CER error terms provide an average error for the equation. This average error needs to be adjusted to account for the number of sample points, degrees of freedom, and where the estimate is located in the data set. Most statistical software packages will do this calculation for OLS CERs and some will address MUPE. For details on how to calculate simple OLS CER prediction intervals manually, see **Appendix A.8.5** and **A.8.8.** If the CER was generated using a statistical tool, or if the detailed statistical results are available, this information should be used to calculate the prediction interval.

> The prediction interval is the preferred source of distribution bounds.

2.4.2.4 Estimating the Prediction Interval

When obtaining a prediction interval is not possible, the CER's standard error (SE, also called standard error of the estimate SEE, see **Appendix A.1.9**) may be used to obtain objective uncertainty. This may arise in the absence of the data and the time to create the CER and its prediction interval, or the lack of detailed statistical results from a published CER. Also known as the CER standard deviation, the SE alone is generally not sufficient to define CER uncertainty. An adjustment is required to account for the location of the point estimate within the dataset used to generate the CER. As the estimate moves away from the center of the CER dataset, the spread of the prediction interval will increase.

Table 2-3 contains the SE multipliers based on the sample size and the independent variable value's position relative to the mean of those used to create the CER. Each multiplier in the table was computed using **Equation A-54** contained in **Appendix A.8.3** and published in **Reference 15**. To use the table, determine:

- **the size of the sample** used to generate the CER. If there is only a qualitative assessment, use: small = 5, medium = 15, large = 25.
- **the distance from CER center**: this is the distance the CER input is from the mean of the independent variable as a ratio to its standard deviation. If it is not possible to make this calculation, the ratio can be estimated by judging how similar the project is to the source data for the CER.

Table 2-3 Factors to Adjust CER SE Sample Size and Sample Relevance

SE Multiplier		Number of Data Points in Sample						
		5	10	15	20	25	30	
Distance / StdDev for Independent Variable	**0.00**	1.095	1.049	1.033	1.025	1.020	1.017	Very Similar
	0.25	1.101	1.052	1.035	1.026	1.021	1.018	
	0.50	1.118	1.061	1.041	1.031	1.025	1.021	
	0.75	1.146	1.075	1.051	1.038	1.031	1.026	Similar
	1.00	1.183	1.095	1.065	1.049	1.039	1.033	
	1.25	1.230	1.121	1.082	1.062	1.050	1.042	
	1.50	1.285	1.151	1.103	1.078	1.063	1.053	Dissimilar
	1.75	1.346	1.186	1.127	1.097	1.078	1.066	
	2.00	1.414	1.225	1.155	1.118	1.095	1.080	
	2.25	1.487	1.267	1.185	1.142	1.115	1.096	Different
	2.50	1.565	1.313	1.218	1.167	1.136	1.114	
	2.75	1.647	1.362	1.253	1.195	1.159	1.134	
	3.00	1.732	1.414	1.291	1.225	1.183	1.155	Very Different

Should there be no basis for determining sample size or distance from the data center, **the recommended default multiplier is 1.2** as this is the average of the multipliers in the first three columns of **Table 2-3**.

> If the only parameter to define the distribution is the standard deviation or standard error, multiply it by 1.2 to estimate the prediction interval.

Simulation tools will allow a normal or lognormal distribution to be defined using different parameters. Typically, the mean and standard deviation are sufficient but in situations where the CER result is not the mean or when the item being estimated is different to those in the CER's dataset, an adjustment must be made. A CER result may not be the mean (could be the median, mode or something else). The CER error expressed as SEE is a measure of the CER average error. If the prediction interval is not available, but the CER SEE is available, the steps illustrated in **Table 2-4** can be used to derive the OLS CER lognormal error distribution parameters (mean and standard deviation). The CER SEE (measure of average error) needs to be adjusted for the specific point estimate. The calculation steps depend on how the SEE is reported (log or unit space). The SEE adjustment factor used in the example calculation is drawn from **Table 2-3** based on the CER being constructed from 10 data points and an assessment that the program being estimated is "different" from the programs in the CER source data.

Table 2-4 Template to Derive an Estimate's Lognormal Distribution Parameters

	Range Name	Value	Equation
Given the Following Information:			
Given: Parametric CER Result	CERResult	$60.91	UC1 = 1.618 * 200 ^ 0.6848; where MotorWt = 200
Mean Log Space	MeanLogSpace	4.1095	LN(**CERResult**)
Interpretation of CER Result		50.0%	CER result is the median of the log normal distribution
Determine SEE Multiplier using Table 2-3 (if data is available, use formulas in A-8.3):			
SEE Multiplier	SEEAdjust	1.267	10 data points, Project is "different" from CER source data
CER error (SEE) given in LOG space, calculate the Mean and SEE in unit space to model a lognormal distribution:			
Given: SEE Log Space	SEELSpace	0.1413	
Calculate:			
Adjusted SEE Log Space	SEELSpaceAdj	0.1790	**SEELSpace** (0.1413) multiplied by **SEEAdjust** (1.267)
Mean Unit Space	MeanUnitSpace	$61.90	EXP(**MeanLogSpace**+**SEELSpaceAdj**^2/2)
Adjusted SEE Unit Space		$11.17	SQRT((EXP(**SEELSpaceAdj**^2)-1)*__MeanUnitSpace__^2)
CER error (SEE) given in UNIT space, calculate Mean and SEE in unit space to model the lognormal distribution:			
Given: SEE Unit Space	SEEUSpace	$8.737	
Calculate:			
SEE Log Space	SEELSpace	0.1413	SQRT(LN((1+SQRT(1+4*(**SEEUSpace**/**CERResult**)^2))/2))
Adjusted SEE Log Space	SEELSpaceAdj	0.1790	**SEELSpace** (0.1417) multiplied by **SEEAdjust** (1.267)
Mean Unit Space	MeanUnitSpace	$61.90	**CERResult***EXP((**SEELSpaceAdj**^2)/2)
Adjusted SEE Unit Space		$11.17	SQRT((EXP(**SEELSpaceAdj**^2)-1)*__MeanUnitSpace__^2)

2.4.2.5 Summary of How to Define Parametric Objective Uncertainty

Overall, the order of preference for determining CER uncertainty is:

- **Statistical Tool**: Use the statistical tool that is used to perform the regression analysis to generate a prediction interval for the estimate
- **Calculate the prediction interval**: If, in addition to the CER equation, detail statistical results of the regression are available, use the mathematics of **Appendix A.8.5** or **A.8.8** to calculate a prediction interval
- **Estimate the prediction interval**: If only the CER equation and its standard error are available, use the guidance of **Appendix A.8.3** to estimate a standard error adjusted for the particular estimate

2.4.3 Uncertainty Based on Fitted Distributions

2.4.3.1 Overview

Fitting, rather than assuming, a distribution to data that defines the uncertainty of an element in the model is attractive as it is an objective, rather than a subjective, analysis of the element in question. However, the

process of performing a best-fit analysis has many subjective steps that highly influence the outcome. The following sections of the handbook outline the key steps and recommend an approach that brings some level of consistency to the process. Two utilities are available with the CSRUH, one of which is wholly contained within Excel. See **Section 7.1.1** for details.

There are two specific targets for distribution fitting:

- **CER Residuals**: While the predictive statistics obtained from common regression methods yield objective measures of dispersion, the distribution shape is an assumption rather than a fact. An objective method without assumptions is to determine the best-fitting distribution to the CER's residuals.
- **CER Inputs**: There is a need to model the uncertainty of the independent variables driving CERs. A common approach is to collect representative values, use their mean as the point estimate and assume a normal or t-distribution and the sample standard deviation to define the uncertainty. A better approach is to fit a distribution to the data.

2.4.3.2 Defining the Cumulative Probability of the Input Data

In order to compare a fitted distribution to the source data, it is necessary to first assign a cumulative probability to each data point. There are many differing opinions in the literature on the best way to perform this calculation. When the number of data points is large, there is little difference between the methods. But when the number of data points is small, as is generally the case in cost estimating, this calculation will highly influence distribution fitting results. If the intent is to compare results across distribution fitting packages, one must first understand how they assign probability to the source data as this produces the reference against which the fit is performed.

Table 2-5 lists several common methods for estimating the probability of each data point in the data set. The cumulative probability formulas use *i*= the rank order of the data point and *n*= the number of observations.

Table 2-5 Methods to Estimate Cumulative Probability

Approach	Cumulative Probability	Extreme Data Treatment
Method 1	i/n	The maximum observed value is considered an accurate extreme, but something more extreme than the minimum observed value is considered possible.
Method 2	(i-1)/(n-1)	The minimum and maximum observed values in a data set are both considered accurate extremes.
Method 3	(i-.5)/n	Something more extreme than both the minimum and maximum observed values is considered possible.
Method 3 with Correction for Continuity	(i-1 + 0.5*Freq)/ n	Same as method 3, however duplicate values take on the probabilty halfway between the point below and above the duplicate.
National Institute for Science and Technology	i/(n+1)	Similar to method 3, however greater probability of values beyond the data extremes.
Method 4	(i-1)/n	The minimum observed value is considered an accurate extreme, but something more extreme than the maximum observed value is considered possible.

Each method is applied to the example model Motor and Airframe weight data in **Table 2-6**. Selecting a method depends on the analyst's judgment about the certainty of the observed extremes.

Table 2-6 Application of Cumulative Probability Methods Examples

Observation	Motor Wgt	Method 1 i/n	Method 2 (i-1)/(n-1)	Method 3 (i-.5)/n	Method 3 with CoC (i-1+ 0.5*Freq))/ n	NIST i/(n+1)	Method 4 (i-1)/n
System #1	90	10%	0%	5%	5%	9%	0%
System #2	112	20%	11%	15%	15%	18%	10%
System #3	130	30%	22%	25%	25%	27%	20%
System #4	170	40%	33%	35%	35%	36%	30%
System #5	195	50%	44%	45%	45%	45%	40%
System #6	210	60%	56%	55%	55%	55%	50%
System #7	225	70%	67%	65%	65%	64%	60%
System #8	290	80%	78%	75%	75%	73%	70%
System #9	320	90%	89%	85%	85%	82%	80%
System #10	340	100%	100%	95%	95%	91%	90%

Observation	Airframe Wgt	Method 1 i/n	Method 2 (i-1)/(n-1)	Method 3 (i-.5)/n	Method 3 with CoC (i-1+ 0.5*Freq))/ n	NIST i/(n+1)	Method 4 (i-1)/n
System #1	230	10%	0%	5%	5%	9%	0%
System #2	300	20%	11%	15%	15%	18%	10%
System #3	330	30%	22%	25%	25%	27%	20%
System #4	440	40%	33%	35%	35%	36%	30%
System #5	480	50%	44%	45%	50%	45%	40%
System #6	480	50%	44%	45%	50%	45%	40%
System #7	620	70%	67%	65%	65%	64%	60%
System #8	720	80%	78%	75%	75%	73%	70%
System #9	790	90%	89%	85%	85%	82%	80%
System #10	800	100%	100%	95%	95%	91%	90%

Method 3 and the correction for continuity (CoC) method (**Reference 12**) split the difference between the two extreme approaches. However, as can be seen in the shaded cells of **Table 2-6** for Airframe Weight, the CoC method returns a probability for 480 halfway between the points below and above as 480 was repeated twice in this dataset. The other methods apply the cumulative probability associated with the first occurrence of the value.

As can be seen in **Figure 2-10**, Method 1 and 4 bound the possibilities and fitting the distribution for both could yield different results. Choosing the approach to calculate cumulative probabilities is ultimately based on whether or not the observed minimum and maximum are considered to be sufficient extremes. If there is no basis to make a determination, Method 3 with COC is the recommended default approach.

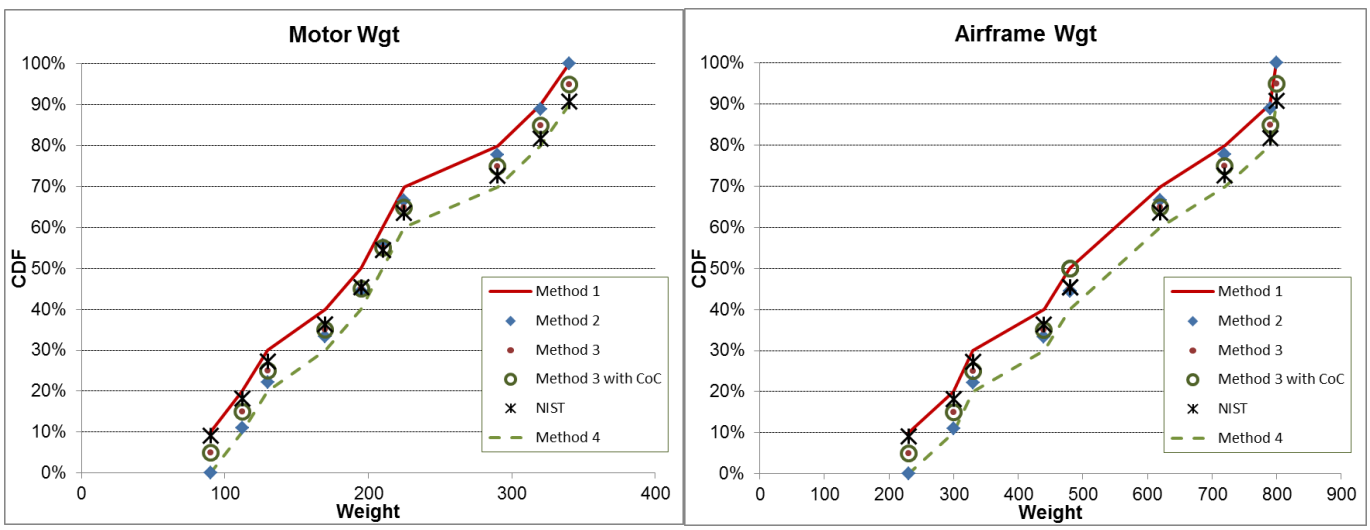


Figure 2-10 Comparing Methods for Estimating Probability

2.4.3.3 Best Fit

There are many commercial tools available that will find the distribution that best fits a given sample of observations. Each tool uses a different approach and offers different options. These tools will generate slightly different results given the same dataset and results may also change as newer versions are released. Tools such as ACEIT and @RISK will perform fits on as few as six and five data points respectively. Other tools such as Crystal Ball require fifteen or more data points. The actual number of data points required is a function of the number of parameters that need to be estimated. A normal distribution, for instance, requires two. A beta distribution requires four. In both cases, a degree of freedom is lost in the curve fitting process. Therefore, at least six data points are required to perform an unconstrained fit using the beta distribution while retaining one degree of freedom. This is necessary to develop a goodness-of-fit statistic.

2.4.3.4 Fit Constraints

It is not unusual for the best-fitting distribution to not capture the entire sample. For instance, the lower or upper bound of a triangular or beta distribution fit may be higher or lower than one or more sample points. The fit, however, could be constrained to ensure the distribution captures or "surrounds" the sample data.

Even when none of the sample points are below zero, it is not unusual for a best-fit distribution to have a tail that stretches significantly into the negative region. Mathematically, this can be absolutely sound and correct. Pragmatically, especially in the cost analysis domain, such distributions can introduce undesirable effects in a simulation. Again, the fit can be constrained to ensure the lower or upper bound does not exceed a specific value (either high or low).

The several thousand fits developed for the Air Force Cost Risk and Uncertainty Metrics Manual (CRUAMM), **Reference 80** and **Attachment 1,** were performed with the surround and a minimum lower bound of zero constraints in place. To support research that required thousands of curve fits, the Air Force developed a simple distribution fitting utility in Excel to fit lognormal, normal, triangular, and beta distributions to selected data. The utility offered two objective functions to minimize: Sum of Squared Error (SSE) and Sum of Squared Percent Error (SSPE). Details of this tool can be found in **Reference 72.** This utility is used to support curve fitting for examples in this handbook. Note the fits developed for the examples in this handbook were unconstrained because none of the aforementioned bounds were present.

Run the fit process unconstrained first and then assess the need to run a constrained fit.

2.4.3.5 Goodness-of-Fit

Finding the best-fitting distribution through a minimum error optimization does not necessarily mean the best fit is statistically significant. There are several goodness-of-fit metrics to measure the statistical significance of the fit. Three of the most popular ones are (see **Appendix A.9** for details):

- **Kolmogorov-Smirnov (K-S)**: The sample CDF is compared to the fitted CDF and the maximum vertical distance between them is found. This generally happens in the middle, making K-S a preferred test when interested in accuracy at the center of the distributions.
- **Anderson-Darling (A-D)**: Measures the total area between the sample and fit CDF and with weightings that can focus on the fit in the tails. A-D is a preferred test when accuracy in the tails is needed.
- **Chi-Squared (also Chi^2)**: Compares the sample frequency to the fitted frequency by bin (columns in a histogram). This is the most common test because it is the easiest to calculate, is fast, and can calculate significance for any fitted continuous or discrete distribution. It does have a weakness. chi-squared is sensitive to the number of segments (bins) used to stratify the data. There are no clear guidelines for selecting the number and location of the bins. However, the number of sample points needs to be greater than the number of bins. In **Section 2.3.1**, Mann-Wald/2 is recommended as the first approximation.

Both K-S and A-D are limited in the number of distributions for which the significance of the fit can be calculated (i.e., cannot calculate the significance of a beta and triangular fit). For this manual, the chi-squared was chosen because it is the only available test that would produce a set of comparable *p*-values across all fitted distributions. However, it is recommended that all goodness-of-fits statistics be considered.

ACE performs the chi-squared test, Crystal Ball performs all three and @RISK 6.0 provides all three plus two other goodness-of-fit measures:

- **An Information Criterion (AIC)**: is used to measure the relative goodness-of-fit for a statistical model. AIC is founded in information theory, offering a relative measure of the information lost when a given model is used to describe reality. This criterion was developed by Hirotsugu Akaike under the name of "an information criterion" (AIC) and first published in 1974 (**Reference 4**).
- **Bayesian Information Criterion (BIC)**: was developed by Gideon E. Schwarz in 1978 (**Reference 6**). The BIC is also used for model selection among a finite set of models and is closely related to the AIC. Just like the AIC, the BIC statistic is also derived from the log-likelihood function. Both statistics take into account the number of estimated parameters of the fitted distribution. However, the BIC penalizes more strongly than AIC for the number of estimated parameters.

It is important to note, however, that AIC and BIC statistics do not provide a measure of the statistical significance of a particular fit. That is, the actual values of the AIC and BIC statistic do not have meaning, except in relative terms, when comparing one proposed distribution type to another. This is also the case for the K-S and A-D test for many distributions such as triangular and beta. chi-squared is the only test that will provide a measure of statistical significance for any distribution, but its results are heavily influenced by the number of bins selected to perform the test.

2.4.3.6 Empirical Fit

If no fitted distribution is satisfactory, an empirical distribution that simply represents observed data is an acceptable alternative. This is a cumulative distribution function with incremental steps based on individual data points instead of a theoretical probability distribution with parameters estimated from the data. The general approach is assumption-free except for the treatment of the extreme boundaries. To create an empirical fit, rather than fit distributions to the data, simply enter each data point and its estimated probability into the simulation tool. Keep in mind that working with the empirical distribution does not involve a statistical procedure so no statement about goodness-of-fit is being made.

2.4.3.7 Suggested Fit Process

As described in the previous sections, there are many considerations when performing curve fit analysis:

- **Minimization**: Some tools offer more than one objective function for minimization, for instance choosing between SSE and SSPE
- **Goodness-of-fit test**: One or some combination of the available tests could be used. But the test and its criteria should remain constant through all comparisons. For instance, using chi-squared with a significance level of five percent (5%).
    - For chi-squared, simply changing the number of bins used to perform the test could yield an acceptable result
- **Matching parameters**: The best fit with acceptable goodness-of-fit may still result in a distribution with a mean, standard deviation, or visual (graphic) comparison that is undesirable. While not sufficient to reject the fit, such issues may be motivation to look at the second or third choice.
- **Outliers**: It may be necessary to consider removing outliers, a course of action that should be discouraged unless there is a compelling reason to do so. Improving the fit, on its own, is not a reason to remove an outlier.
- **Favor basic distributions**: Just because a distribution is ranked best does not mean it is the only acceptable choice. Examination of the parameters and the visual (CDF or PDF) may demonstrate that several distributions are all nearly as good. In such cases, it is recommended that if lognormal, triangular, normal, beta, betaPERT, or uniform is nearly as good, then select one of them. They are

easier to work with and easier to explain. However, if another distribution is a clear winner that is not one of them (a basic distribution), it should be given serious consideration.

- **Empirical Fit**: If the data follows a distribution with more than one mode or simply does not match any common distribution shape, consider creating an empirical fit directly in the simulation. In Crystal Ball this is the custom distribution; In @RISK there is the RiskCumul function; And in ACE it is called the Custom CDF.

In order to bring consistency to the process, a flow chart was developed in **Reference 80.** A variation is presented in **Figure 2-11** as a place to start for agencies to develop their own standard.

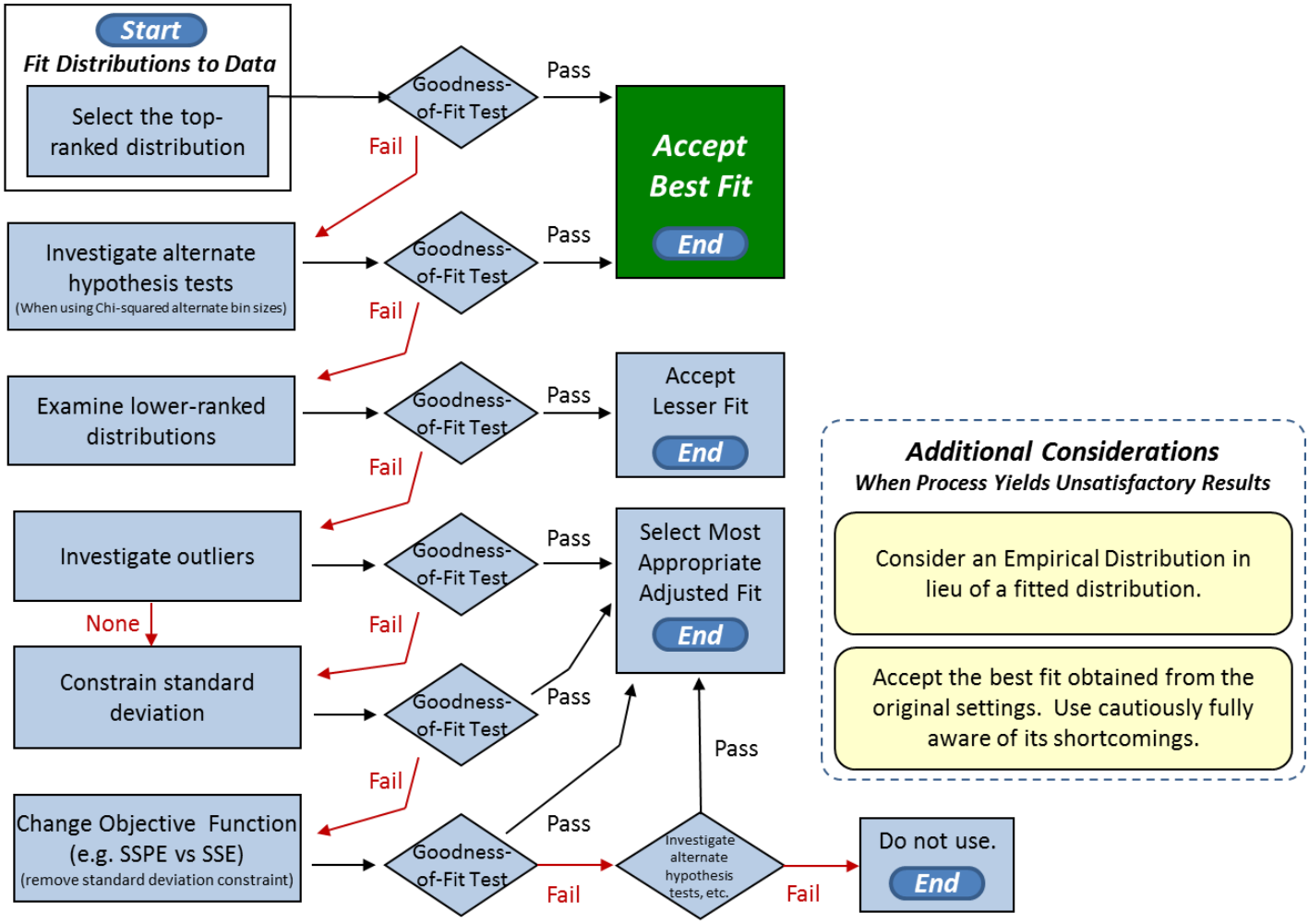


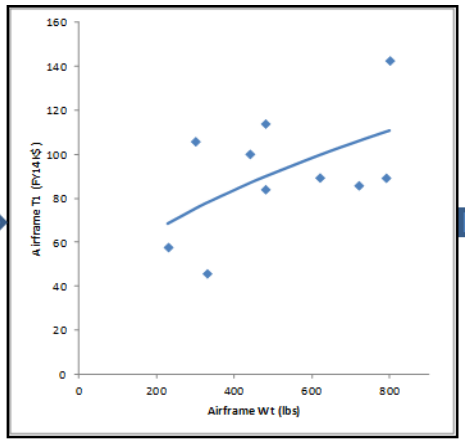
Figure 2-11 Distribution Fit Process

2.4.3.8 Fitting Distribution to Regression Residuals

As stated earlier, the regression method also includes an assumption for the distribution of the CER uncertainty. However, it is possible to fit (rather than assume) the shape and bounds for the regression equation from its residuals. The first step is to divide the actual data point value by the predicted data point value. For the example model, this was performed on the Airframe CER results as shown in **Figure 2-12**. The upper-left panel of the figure presents the data and the upper-center panel presents the fitted equation and scatter plot. The upper-right panel shows the residuals sorted in ascending order along with the computed cumulative probability using Method 3 with the CoC formula from **Table 2-5**. The bottom panel shows the results of the distribution fitting process and the resulting S-curve for each candidate distribution. The fits were ranked based upon the fit SEE. In this case triangular, ranked number 1, passed the goodness-

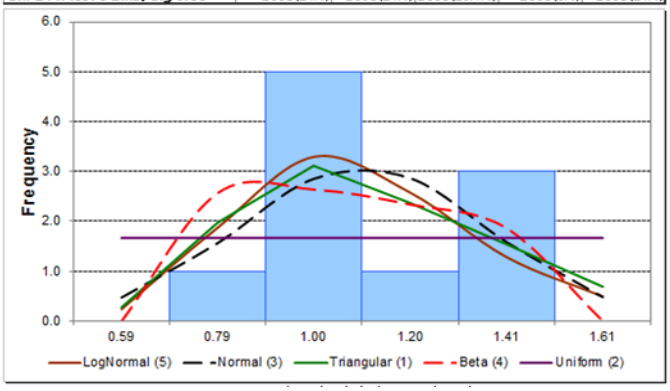
of-fit test, and yielded a mean and standard deviation very close to the sample. In order to pass the chi-squared goodness-of-fit test, the number of bins had to change from Mann-Wald/2 to Mann-Wald (6 bins or greater was sufficient to pass). The triangular distribution was used to model the Airframe CER uncertainty.

	Airframe Wt (lbs)	First Unit Cost $K2014
System #7	230	57.8
System #10	300	105.9
System #6	330	45.8
System #4	440	100.1
System #3	480	113.9
System #8	480	84.1
System #2	620	89.4
System #1	720	85.9
System #9	790	89.3
System #5	800	142.7



	Residual	CDF
System #6	0.5880	5%
System #9	0.8091	15%
System #1	0.8093	25%
System #7	0.8446	35%
System #2	0.8969	45%
System #8	0.9353	55%
System #4	1.1526	65%
System #3	1.2674	75%
System #5	1.2860	85%
System #10	1.4089	95%

	Sample	LogNormal	Normal	Triangular	Beta	Uniform
Mean	0.9998	1.0045	0.9998	1.0004	1.0003	0.9998
StdDev	0.2636	0.2599	0.2588	0.2510	0.2464	0.2446
CV	0.2636	0.2587	0.2589	0.2509	0.2463	0.2446
Min	0.5880			0.4760	0.6039	0.5762
Mode		0.9115	0.9998	0.8484	0.6733	
Max	1.4089			1.6770	1.5152	1.4234
Alpha					1.0276	
Beta					1.3347	
Data Count	10	% < 0 =	0.01%	None	None	None
Standard Error of Estimate		0.0699	0.0671	0.0642	0.0698	0.0643
Rank		5	3	1	4	2
SEE / Fit Mean		6.95%	6.71%	6.42%	6.98%	6.43%
Chi^2 Fit test 9 Bins, Sig 0.05		Good (24%)	Good (24%)	Good (28.7%)	Good (9%)	Good (24%)



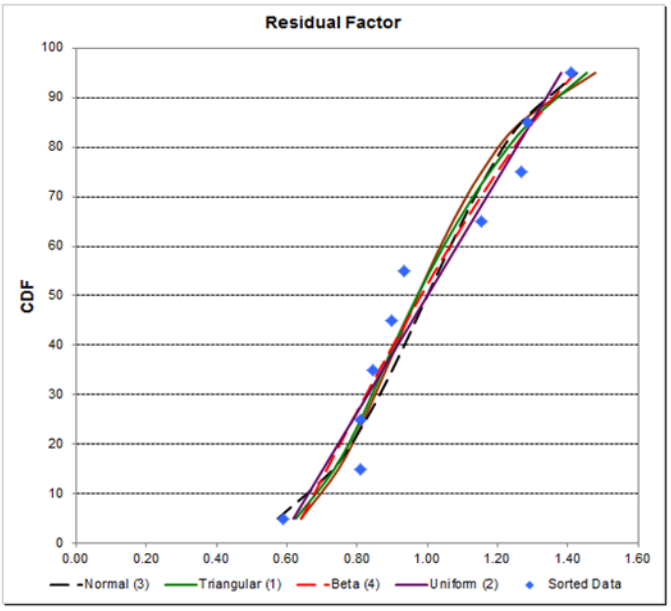
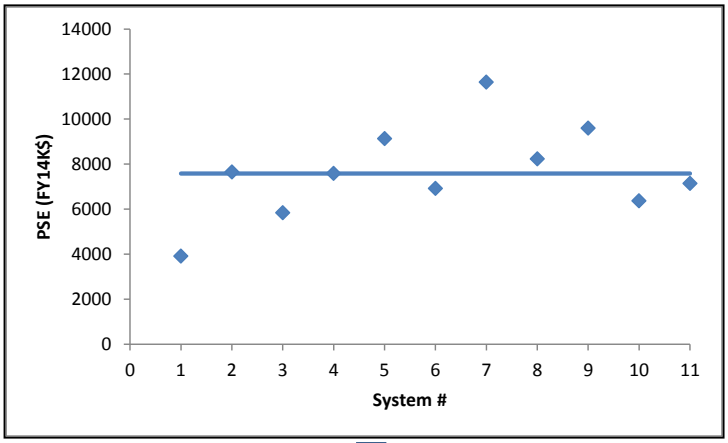


Figure 2-12 Fitting a Distribution to Example CER Residuals

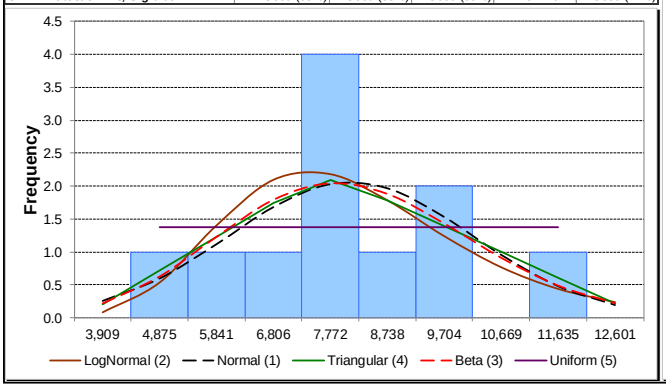
2.4.3.9 Fitting Distribution to a Throughput or CER Input

The process is also applicable to measuring the spread in a single variable of a dataset for which there are no available explanatory variables. In these cases it is common to use the median (or mean) value as a throughput estimate for the value in the cost model. Again, the shape and bounds of the throughput method can be determined by fitting a distribution to the values. For the example model this process was used to estimate Production Peculiar Support Equipment (PSE) cost. The normal curve was the best fit for PSE as shown in **Figure 2-13**. **Figure 2-12** and **Figure 2-13** have similar types of tables and plots. However, the CDF in **Figure 2-13** is computed directly from the data values (PSE $), while the CDF in **Figure 2-12** is computed from residuals. To model the PSE cost uncertainty as a multiplier to the median point estimate of 7,580, divide the parameters by 7,580.

	PSE $	CDF
System #1	3909	5%
System #3	5832	14%
System #10	6369	23%
System #6	6917	32%
System #11	7142	41%
System #4	7580	50%
System #2	7641	59%
System #8	8228	68%
System #5	9128	77%
System #9	9596	86%
System #7	11635	95%
Median	7580	



	Sample	LogNormal	Normal	Triangular	Beta	Uniform
Mean	7,634.2727	7,668.7976	7,634.2727	7,635.7767	7,642.8685	7,634.2727
StdDev	2,047.5470	2,057.5933	2,040.9037	1,969.0556	2,031.4244	1,878.8132
CV	0.2682	0.2683	0.2673	0.2579	0.2658	0.2461
Min	3,909.0000			3,047.7122	-1,206.7498	4,380.0727
Mode		6,909.4256	7,634.2727	7,195.7043	7,375.0091	
Max	11,635.0000			12,663.9137	26,548.9502	10,888.4727
Alpha					12.6081	
Beta					26.9357	
Data Count	11	% < 0 =	0.01%	None	0.00%	None
Standard Error of Estimate		371.6543	354.4826	407.5106	396.8587	616.0862
Rank		2	1	4	3	5
SEE / Fit Mean		4.85%	4.64%	5.34%	5.19%	8.07%
Chi^2 Fit test 5 Bins, Sig 0.05		Good (83%)	Good (83%)	Good (55%)	Unknown	Good (21%)



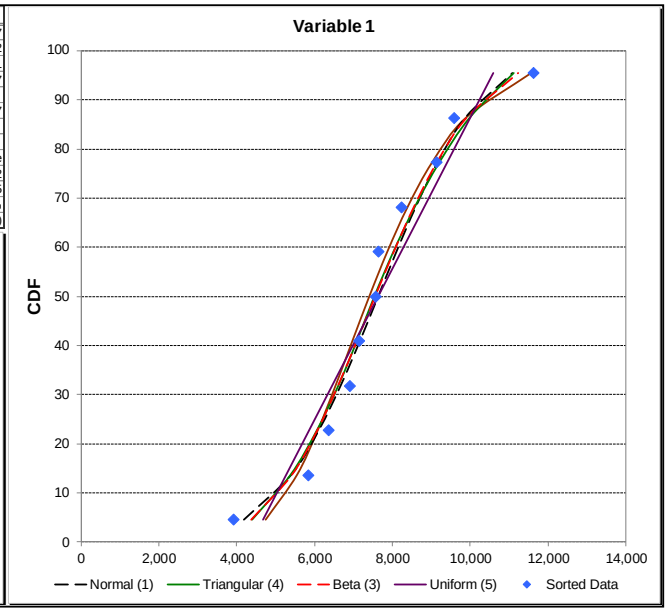


Figure 2-13 Fitting a Distribution to a Throughput Example

2.4.4 Inheriting Uncertainty from One Model to Another

Often when an overall program estimate is assembled, the estimated cost for some elements may be obtained via separate and distinct cost models. These may be third-party tools whose inner workings are closed to the analyst, such as SEER or True Planning. These may also be transparent models configured separately due to organizational needs or modeling expediency or a portfolio of programs. For example, a program's sensor subsystem may be estimated via a stand-alone radar cost model. In such situations it is recommended that the uncertainty distribution inherent in those models be carried forward into the aggregate model. Simply multiplying a third-party tool's point estimate result by a "management reserve factor" is discouraged. The third-party cost result should be treated as a random variable with uncertainty just like all the other estimated values in the model.

When a given cost element is modeled using a separate model, apply uncertainty to each of the elements in that model consistent with the instructions of that model's user manual and the guidance discussed in this handbook. Popular parametric third party tools provide sufficient simulation results to allow the analyst to replicate the uncertainty in the aggregate model.

There are at least three methods available:

- **Custom Based on Extracted Simulation Data**: This is the most accurate approach. Simply extract the simulation trial data from the third party tool and use it to create a custom distribution. Some tools limit the number of trials they can import (ACE, for instance, is limited to 10,000).
- **Custom Based on Extracted Percentile Results**: As demonstrated in **Reference 67,** this is a very easy way to accurately replicate a simulation result. Extract percentile results from the third party tool and use them to define a custom distribution in the aggregate model. The more results that are exported, the greater the accuracy. Twenty data points (every five percent) is normally sufficient.
- **Lognormal Based on Two Values**: All the simulation tools will generate a lognormal distribution from two curve values. In the absence of more detailed information, a reasonable fit can be achieved by extracting the median and the 85% value (**Reference 67**). An alternative is to extract the mean and standard deviation from the product's simulation results.

Once the third-party tool's resultant distribution is entered into the host cost model, assign appropriate correlations with other elements.

When using separate models, it is recommended that the uncertainty distribution inherent in those models be carried forward into the aggregate model.

2.5 SUBJECTIVE UNCERTAINTY

2.5.1 Overview

This section applies to every element in the cost estimate where objective uncertainty distributions are not available. Guidance for choosing the distribution shape is provided in **Section 2.3.2**. In addition to shape, distributions are characterized by parameters describing their dispersion and skewness. Subjective dispersion parameters are commonly the low and high bounds of the value in question. Eliciting subjective assessments of standard deviation, probability or some other measure of variation is challenging. This section, therefore, focuses on how to interpret the lower and upper bounds obtained from expert opinion.

2.5.2 Elicitation of Subjective Bounds from Subject Matter Experts (SMEs)

Ideally, distribution parameters are developed from an objective assessment of relevant historical data as described in the prior section. Often, however, it is necessary to rely on informed opinion. The analyst generally has to resort to expert judgment, such as that possessed by engineers, managers, and other knowledgeable people. This process is called elicitation. It can be difficult to do and subject to numerous biases. Over-optimism can be traced both to cognitive biases, which are errors in the way the mind processes information, and to organization (motivation) pressures. Some motivational and cognitive biases are listed in **Table 2-7**.

SME predictions based upon an assessment of their own capabilities, experiences, and expectations can be tempered with statistical analysis of relevant historical data. Such data provides the SME with a reality check that should have a positive influence on their intuitive view of the situation.

An "inside view" is one that a SME may gravitate to spontaneously. It is when the estimate is based on a tight focus of the project at hand: resources, obstacles, scenarios, and extrapolating current trends. Even the most conservative "inside view" estimates tend to be exceedingly optimistic. An "outside view" is based upon an examination of historical data from completed projects similar to the one under consideration. Using that information as the basis to build a distribution of possible outcomes, the SME can be asked to identify the position of the current project in the distribution. The concept of an "inside view" versus an "outside view" is postulated and discussed in detail in **Reference 34**.

Table 2-7 Motivational and Cognitive Bias

Motivational Bias	Cognitive Bias
Social pressure (face-to-face)	Representativeness (small sample)
Impression (not face-to-face)	Availability (most recent)
Group Think	Anchoring and Adjustment
Wishful thinking	Inconsistency (opinion changes over time)
Career goals	Relating to irrelevant analogies
Misunderstanding	Underestimation
Project Advocacy	
Competitive Pressures	

As an additional point of view, the Association for the Advancement of Cost Engineering International (AACEI), in **Reference 97**, lists the following biases, many of which are listed in **Table 2-7**.

- **Confirmation Bias**: Gather facts that support certain conclusions; disregard other facts
- **Premature Termination**: Accept the first alternative that looks like it might work
- **Inertia**: Keep thought patterns that we have used in the past despite new circumstances
- **Selective Perception**: Screen-out information that we do not think is important
- **Optimism Bias**: See things in an unjustifiably positive light
- **Recency**: Place more attention on more recent information
- **Repetition Bias**: Believe what we are told most often and by the most different sources
- **Anchoring**: Unduly influenced by initial information; shapes our view of later information
- **Groupthink**: Peer pressure to conform to the opinions held by the group
- **Escalating Commitment**: Increase support of a decision over time (over-value sunk costs)
- **Attribution Asymmetry**: Attribute success to our abilities; attribute failures to bad luck

Best practices for elicitation include:

- Have historical minimum, maximum, and averages on hand for the discussion
  - Do not initially share with the expert to avoid unintended anchoring
  - As the dialog progresses, this will provide further context to the discussion
  - Do not use it to bludgeon the expert, but do use it to challenge or support estimates
- Use multiple experts
- Ask for an upper and lower value. Encourage brainstorming for reasons why the range could be larger, especially in the upper direction.
- Encourage the dialog to identify the value at which the outcome has a one in five chance of being lower than, or the value that has one in five chance of being higher than the estimate. Such a dialog makes the participants determine not only the bounds but also their interpretation.
- Seek the most-likely value near the end of the discussion
- Select a distribution shape based on the skew and firmness of the bounds. Lognormal is suggested as the default choice in the absence of compelling arguments for another shape. Refer to **Table 2-2** for further guidance on selecting a distribution shape.

- In the absence of better information, treat bounds as the 70-percent interval. For symmetrical distributions use the 15/85 percent boundary interpretation (see **Section 2.5.3** for details). For skewed distributions, consider skewing the bound interpretations as shown in **Section 2.5.4**.
- Crosscheck and, when appropriate, challenge experts' inputs against historical experience. Characterize adjustments with meaningful project comparisons derived from relevant historical data (i.e., challenge a software uncertainty assessment of +/- 50% with several relevant, real life examples where 300% or more growth was experienced). Ideally, prepare for elicitation discussions by having on hand meaningful cross-checks based upon well known, real-life examples.
- Iterate the evolving conclusions with the experts as needed (see **Reference 98** and **100** for details)

2.5.3 Interpretation of Bounds Obtained Through Subjective Assessment

The low and high bounds for subjective uncertainties obtained from experts need to be interpreted. **Reference 5**, reprinted in **Reference 71** observed that experts rarely identify 60% or more of the possible uncertainty range and never did better than 70%. The impact of interpreting the lower/upper bound to be the 15%/85% ***without adjustment*** (i.e. 70% of the total range) on a triangular, normal, and lognormal distribution is illustrated in **Figure 2-14** and **Figure 2-15**. The narrower distribution illustrates the distribution shape if the expert bounds are taken as "absolute," which is rarely the case (for the normal and lognormal illustration, "absolute" is interpreted as 3 standard deviations, or 99.9% of the possible range). The broader shape is the one that could be implied by the bounds obtained from the expert, if there is no adjustment for skew. For the lognormal, since it is not symmetrical, only the upper or lower bound can be defined. In **Figure 2-15**, the lognormal is shown when the expert upper bound is assumed to be at the 85 percentile.

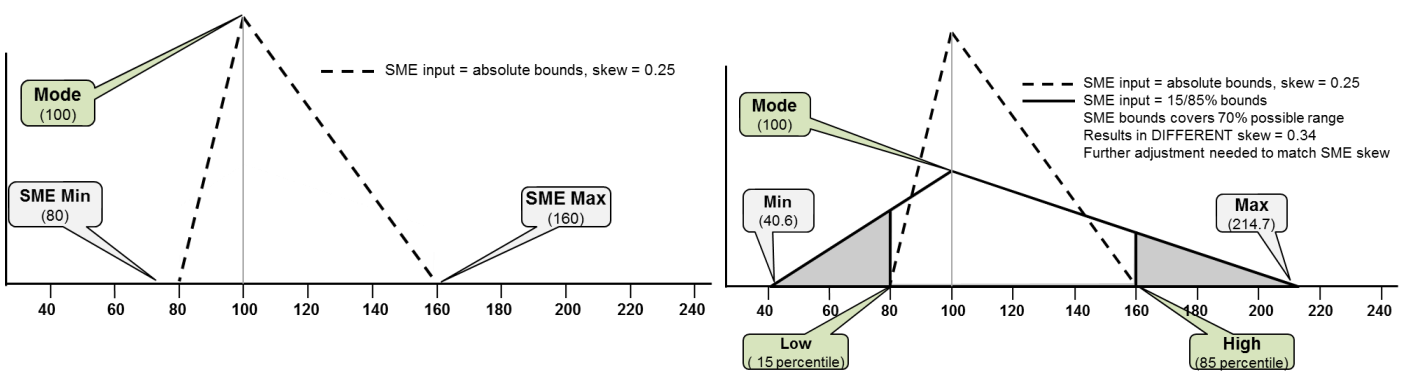


Figure 2-14 Impact of Unadjusted Bound Interpretation on a Triangular Distribution

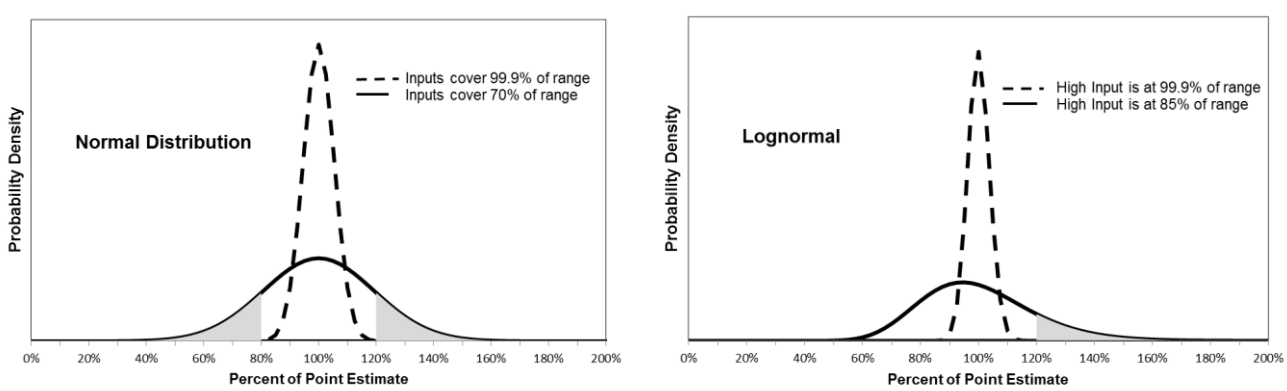


Figure 2-15 Impact of Bound Interpretation on a Normal and Lognormal Distribution

2.5.4 Adjusting Subjective Distribution Bounds for Skew

Reference 5 (reprinted at **Reference 71**) suggests that the 30% additional uncertainty should be applied symmetrically. Doing so, however, changes the skew of the final distribution from the one obtained from the expert. A recommended refinement (see **Reference 24**) is to adjust the low and high bound interpretation to retain the total probability captured by the expert and to also maintain the original skew. In this discussion, skew is defined as area to the left of the mode divided by the total area of the distribution. For a mathematical discussion of skew, see **Appendix A.4.5**.

Table 2-8 shows the sequence of steps to adjust the low and high probabilities to retain skew (see **Section 2.3.5** for a definition of skew). Triangular and uniform are solved using the same mathematics. There is also a closed form solution for betaPERT if the low/high bounds are symmetrical. Otherwise, Excel solver is used to solve for the betaPERT minimum to ensure the skew calculated from min/mode/max is the same as low/mode/high. The maximum is calculated as a function of the betaPERT minimum (note that the betaPERT low/high probability does not stray far from symmetrical). An Excel utility to perform this math is available with this handbook (see **Section 7.1.2**).

Table 2-8 Template to Calculate Adjustment for Skew

Template to Adjust A Triangular, Uniform or BetaPert Subjective Distribution Low and High Percentile for Skew					
Enter Case Title	Fig 2-16	EMD PM$	Motor Wgt	Warhead Wgt	**Comment or Formula**
Low	80	90	280	20	Input expert's low bound
Mode	100	100	290	25	Input expert's mode (most likely)
High	160	130	350	35	Input expert's high bound
Estimated uncertainty captured by expert	70%	70%	70%	70%	Default is 70%
Results for Triangular or Uniform Distributions					
Cumulative Probability Low	**8%**	**8%**	**4%**	**10%**	Round(Skew * (1 - UncertIncl),2)
Cumulative Probability High	**78%**	**78%**	**74%**	**80%**	UncertIncl + Adjusted Low Bound Interpretation
Min	**55.8**	**77.9**	**267.9**	**13.9**	(Mode-Skew * Max)/(1 - Skew)
Max	**232.7**	**166.3**	**422.7**	**47.1**	High+(High-Mode)*SQRT(UncertNotIncl)/(1-SQRT(UncertNotIncl))
Total uncertainty NOT captured by expert	0.30	0.30	0.30	0.30	1 - UncertIncl
Skew based upon inputs	0.25	0.25	0.14	0.33	(Mode-LowInp)/(HighInp-LowInp)
Revised Skew	0.25	0.25	0.14	0.33	(Mode-Min)/(Max-Min) i.e., CDF of Mode
Results for BetaPert					
Cumulative Probability Low	**17%**	**17%**	**16%**	**16%**	BETADIST(LowInp, Alpha, Beta, Beta_Min, Beta_Max)
Cumulative Probability High	**87%**	**87%**	**86%**	**86%**	BETADIST(HighInp, Alpha, Beta, Beta_Min, Beta_Max)
Min	**49.6**	**74.8**	**262.2**	**12.8**	Mode-1/((1-2*BETAINV(UncertNotIncl/2,Alpha,Beta)))*(Mode-LowInp)
Max	**251.2**	**175.6**	**456.8**	**49.3**	Beta_Min + Lambda * (Mode - Beta_Min) / (Alpha - 1)
Total uncertainty NOT captured by expert	0.30	0.30	0.30	0.30	1-(HighPercentBeta-LowPercentBeta) **Red cell indicates solver required**
Beta skew based upon inputs	0.37	0.37	0.28	0.42	BETADIST(Mode, Alpha, Beta, LowInp, HighInp)
Beta skew after adjustment	0.37	0.37	0.28	0.42	BETADIST(Mode, Alpha, Beta, Beta_Min, Beta_Max)

Figure 2-16 illustrates how the example triangular distribution from **Figure 2-14** is adjusted to maintain the skew.

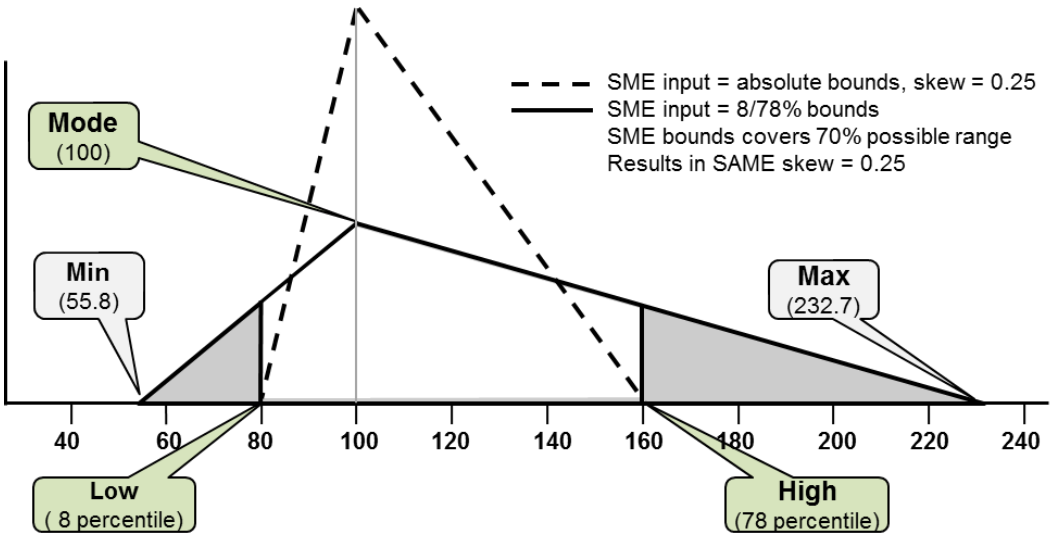


Figure 2-16 Triangular Distribution Adjusted to Maintain Original Skew

It is recommended that subjective bounds are adjusted for skew when using triangular, uniform or betaPERT. Additionally, the expert that provided the bounds should be consulted to verify the results are reasonable.

Unless there is evidence to do otherwise, *request* and *interpret* subjective bounds (expert opinion) as the 70% range and adjust for skew when using triangular, uniform or betaPERT.

2.5.5 Last Resort Subjective Uncertainty Guidance

When the analyst is unable to develop objective bounds from data, unable to obtain subjective bounds from Subject Matter Experts (SME) and unable to find a relevant document to reference, the analyst's own subjective designations of low, medium, and high uncertainty are a last resort. Prior to the release of the AFCAA CRUH (**Reference 55**), using +/- 5% of the point estimate was seen in many estimates. This is almost always unacceptably narrow in the cost analysis environment. Experts supporting the AFCAA CRUH reluctantly agreed to characterize low/medium/high distributions based on their observation that the CVs of regressed CERs tend to fall in the 0.15 to 0.35 range (good to not-so-good fits) for many commodities. For space systems, however, CVs of 0.45 and above are not uncommon. AFCAA has since performed a study, the Cost Risk and Uncertainty Metrics Manual (**Reference 80**). In general, that study found that CVs at lower levels in the WBS tend to be much wider. This finding is also supported by the NCCA SAR Cost Growth study (**Reference 88**). If these and other references are unavailable, **Table 2-9** is offered as a table of last resort. The defaults are based upon the following assumptions (note that the 15/85 bounds in this table do not need to be adjusted for skew):

- Lognormal: the standard deviation in log-space is 0.15 for low, 0.25 for medium, 0.35 for high and 0.45 for extreme high dispersion
- Weibull: the point estimate probability / factor of location (minimum) is 0.25 / 1.15 for low, 0.20 / 1.25 for medium, and 0.15 / 1.50 for high
- All other distributions: the standard deviation divided by the mean (i.e. the coefficient of variation – CV) is 0.15 for low, 0.25 for medium, 0.35 for high. 0.45 for extreme high and no skew
- Skew defined as (Mode-Low)/(High-Low). Left skew is 0.25, symmetrical is 0.50, right skew is 0.75

Table 2-9 Table of Last Resort Bounds for Subjective Distributions

Distribution	Point Estimate Position	Point Estimate and Probability	Mean*	CV*	15%	85%
Lognormal Low	Median	1.0 (50%)	1.0113	0.1509	0.8560	1.1682
Lognormal Med	Median	1.0 (50%)	1.0318	0.2541	0.7718	1.2958
Lognormal High	Median	1.0 (50%)	1.0632	0.3613	0.6957	1.4373
Lognormal Ehigh**	Median	1.0 (50%)	1.1067	0.4743	0.6273	1.5943
Normal Low	Mean	1.0 (50%)	1.0000	0.1501	0.8445	1.1555
Normal Med	Mean	1.0 (50%)	1.0000	0.2501	0.7409	1.2591
Normal High	Mean	1.0 (50%)	1.0024	0.3458	0.6400	1.3632
Normal EHigh	Mean	1.0 (50%)	1.0154	0.4258	0.5547	1.4703
Weibull Low	Mode	1.0 (25%)	1.1581	0.1794	0.9564	1.3695
Weibull Med	Mode	1.0 (20%)	1.3932	0.3324	0.9563	1.8547
Weibull High	Mode	1.0 (15%)	2.1037	0.5723	1.0000	3.2766
Triangle Low Left	Mode	1.0 (75%)	0.8775	0.1779	0.6953	1.0414
Triangle Low	Mode	1.0 (50%)	1.0000	0.1500	0.8338	1.1662
Triangle Low Right	Mode	1.0 (25%)	1.1225	0.1391	0.9586	1.3046
Triangle Med Left	Mode	1.0 (75%)	0.7959	0.3270	0.4923	1.0690
Triangle Med	Mode	1.0 (50%)	1.0000	0.2500	0.7230	1.2769
Triangle Med Right	Mode	1.0 (25%)	1.2041	0.2161	0.9310	1.5078
Triangle High Left*	Mode	1.0 (75%)	0.7454	0.4479	0.3467	1.1028
Triangle High	Mode	1.0 (50%)	1.0000	0.3501	0.6122	1.3878
Triangle High Right	Mode	1.0 (25%)	1.2858	0.2834	0.9034	1.7109
Triangle EHigh Left*	Mode	1.0 (75%)	0.7454	0.4960	0.3004	1.1501
Triangle EHigh	Mode	1.0 (50%)	1.0045	0.4439	0.5088	1.4998
Triangle EHigh Right	Mode	1.0 (25%)	1.3674	0.3426	0.8758	1.9140

Distribution	Point Estimate Position	Point Estimate and Probability	Mean*	CV*	15%	85%
Uniform Low Left	Mode	1.0 (75%)	0.8701	0.1724	0.6882	1.0520
Uniform Low	Mode	1.0 (50%)	1.0000	0.1500	0.8181	1.1819
Uniform Low Right	Mode	1.0 (25%)	1.1299	0.1328	0.9480	1.3118
Uniform Med Left	Mode	1.0 (75%)	0.7835	0.3191	0.4804	1.0866
Uniform Med	Mode	1.0 (50%)	1.0000	0.2500	0.6969	1.3031
Uniform Med Right	Mode	1.0 (25%)	1.2165	0.2055	0.9134	1.5196
Uniform High Left	Mode	1.0 (75%)	0.6969	0.5023	0.2726	1.1213
Uniform High	Mode	1.0 (50%)	1.0000	0.3500	0.5757	1.4243
Uniform High Right	Mode	1.0 (25%)	1.3031	0.2686	0.8788	1.7275
Uniform EHigh Left	Mode	1.0 (75%)	0.6949	0.5774	0.2085	1.1813
Uniform EHigh	Mode	1.0 (50%)	1.0000	0.4500	0.4544	1.5456
Uniform EHigh Righ	Mode	1.0 (25%)	1.3897	0.3238	0.8441	1.9353
Beta Low Left	Mode	1.0 (61%)	0.9393	0.1600	0.7750	1.0986
Beta Low	Mode	1.0 (50%)	1.0000	0.1502	0.8375	1.1625
Beta Low Right	Mode	1.0 (39%)	1.0607	0.1417	0.9014	1.2249
Beta Med Left	Mode	1.0 (63%)	0.8833	0.2827	0.6046	1.1517
Beta Med	Mode	1.0 (50%)	1.0000	0.2502	0.7255	1.2745
Beta Med Right	Mode	1.0 (37%)	1.1170	0.2240	0.8483	1.3957
Beta High Left	Mode	1.0 (66%)	0.8085	0.4191	0.4117	1.1862
Beta High	Mode	1.0 (50%)	1.0000	0.3501	0.6046	1.3955
Beta High Right	Mode	1.0 (33%)	1.2021	0.2912	0.8157	1.6061

** EHigh = Extreme High * To match these paramaters, tools must be set to truncate the distribution at zero.

Note the CVs in this table are intended for lower-level WBS elements and CER inputs. These are not to be confused with the CV benchmarks discussed in **Section 3.5.3**.

There is sufficient information in the table to model the distributions in any of the available tools. See **Appendix A.6.4** for the mathematics of converting the available set of lognormal distribution parameters to those that may be more convenient to use in the selected risk and uncertainty tool.

2.6 DOCUMENT COST METHOD AND COST DRIVER UNCERTAINTY

Uncertainty must be considered at several steps in the cost estimating process and will be addressed through a combination of objective and subjective uncertainty. **Table 2-10** illustrates one way to build the estimate and make it easy to review key pieces of information including:

- The distribution shape and position of the point estimate in the distribution
- Distribution parameters as a percent of the point estimate when uncertainty should scale with what-if cases and in the case of low/high values, their percentiles should be shown
- Distribution as values when the uncertainty range should not change with what-if cases
- The source of each uncertainty

The precise layout varies by organization and personal work style, but tables such as this one are a best practice for documentation and for use in technical reviews.

Table 2-10 Combining CER and Input Uncertainty

WBS Elements	Point Estimate	Cost Estimating Relationship	Form	PE Position	Low	Low Intrp	High	High Intrp	Comment
Airframe T1	$77.85	25.62 + 2.101 * AirFrameWt ^ 0.5541	Triangular	Mode = PE*85%	47.6%	0	167.7%	100	Fit Residual data
Propulsion T1	$78.56	1.618 * MotorWt ^ 0.6848	Log-t	Median			20.7%	90	Regression Result
Guidance and Control T1	$100.00	100	Triangular	Mode	85.0%	8	140.0%	78	Expert Opinion
Payload T1	$62.01	30.15 + 1.049 * WarheadWt	Student's-t	Mean			113.8%	90	Regression Result
Airframe Weight (lbs)	330.0	330	Uniform	Undefined	182.11	0	855.89	100	Fit to Data
Motor Weight (lbs)	290.0	290	Triangular	Mode	280.00	4	350.00	74	Expert Opinion
Warhead Weight (lbs)	25.0	25	Triangular	Mode	20.00	10	35.00	80	Expert Opinion

Define distributions in a single location within the model with key parameters clearly visible in an organized format. This will simplify performing what-ifs, supporting technical reviews and applying correlation.

2.7 CAPTURING THE RISK REGISTER IN THE COST MODEL

The risk register is a record of all events identified by the project team that may have either a positive or negative impact on the cost estimate. This record includes information such as:

- A description of the event
- The probability the event will occur
- The impact of the event should it occur. It is important to note that the impact of the event could be a risk (unfavorable result) or an opportunity (favorable result).
- Risk mitigation strategies
- The risk owner

The risk register needs to be captured by the CISM or FICSM model. This section provides guidance on how to capture the risk register in the cost model where "risk" references the risk register and "uncertainty" references the uncertainty captured in the model not addressed by the risk register.

2.7.1 Risk Register Events

The Risk Reporting Matrix illustrated in **Figure 2-17** is typically used to report risks identified within a program. The level of risk for each event is mapped to a low (green), moderate (yellow), or high (red) risk based upon its probability (likelihood) and cost (consequence). Each square is populated with the number of events that fall into the relevant square.

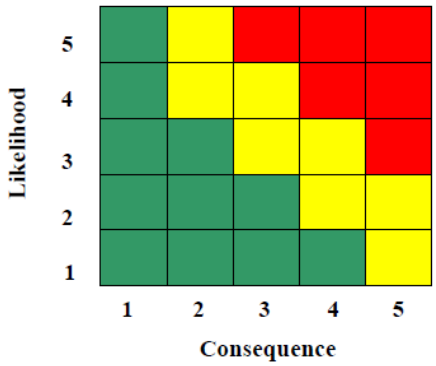


Figure 2-17 Risk Reporting Matrix

The DoD Risk Management Guide (**Reference 53**) provides the probability of occurrence for each level as shown in **Table 2-11**. For example, if the event is cited as Level 3 likelihood it has a fifty percent probability of occurring. Individual program offices may tailor these probabilities, but the following process remains the

same. Additionally, the program needs to define the cost impact and schedule impact values for each consequence level in **Table 2-11**. Correlation of risk consequences should also be addressed in the model.

Table 2-11 DoD Risk Management Guide Suggestions for Likelihood and Impact

Likelihood			Consequence			
Level	**Likelihood**	**Probability of Occurrence**	**Level**	**Technical Performance**	**Schedule**	**Cost**
1	Not Likely	~10%	1	Minimal or no consequence to technical performance	Minimal or no impact	Minimal or no impact
2	Low Likelihood	~30%	2	Minor reduction in technical performance or supportability, can be tolerated with little or no impact on program	Able to meet key dates. Slip < * month(s) Sub-system slip > * month(s) plus available float.	Budget increase or unit production cost increases. < ** (1% of Budget)
3	Likely	~50%	3	Moderate reduction in technical performance or supportability with limited impact on program objectives	Minor schedule slip. Able to meet key milestones with no schedule float. Slip < * month(s)	Budget increase or unit production cost increases. < ** (5% of Budget)
4	Highly Likely	~70%	4	Significant degradation in technical performance or major shortfall in supportability; may jeopardize program success	Program critical path affected Slip < * months	Budget increase or unit production cost increase < ** (10% of Budget)
5	Near Certainty	~90%	5	Severe degradation in technical performance; Cannot meet KPP or key technical/supportability threshold; will jeopardize program success	Cannot meet key program milestones. Slip > * months	Exceeds Acquisition Program Baseline threshold > ** (10% of Budget)

The cost analyst needs to obtain the risk register (introduced in **Section 1.3.2**) generated by the project risk management team and determine which of the risk events have not been captured by the cost model and are significant enough to add to the cost model. However, the sources of discrete risk items to be modeled are not limited to the risk register. Additionally, opportunities to reduce cost should also be captured. But it is essential that risk register items are not added if they are already addressed in the uncertainty captured in the current model.

Each item in the program risk register needs to be carefully assessed to properly augment the cost uncertainty model. Take care to properly interpret the cost and schedule consequences as often the risk management team may be providing impacts only to the program's current or pending contract. The analyst may need to extrapolate the consequences to the remainder of the life cycle.

2.7.2 Modeling Risk Register Events

When it is determined that risk register items are not already captured in the uncertainty model, there are two recommended approaches to model them:

- **Few Risk Register Items:** If there are only a few discrete uncertainties, their cost impact on the point estimate should be included in the form of supplemental "what-if" cases, regardless of their probability of occurrence. Funding at the "expected value" (probability of occurrence times the cost impact) is not recommended. Doing so does not capture sufficient funds to pay for the event should it occur, yet allocates unnecessary funds to the project if the event does not occur.
- **Many Risk Register Items**: If there are many discrete uncertainties, then they should be listed and uncertainty assigned using the "yes/no" criteria, which is modeled using the Discrete distribution in @RISK, the "Yes/No" distribution in Crystal Ball or the Probability of Occurrence column in ACE. The point estimate value for each risk register item should be zero. Their impact will be captured by the simulation.

Uncertainty should be applied to the cost impact as well as the probability of occurrence **(Reference 64)**. In the example, risk register item 2 (RR2) (see **Figure 2-2**) is for the case when there is some likelihood the baseline single-mode seeker fails to meet requirements and, as a consequence, an alternate multimode seeker would be necessary for the Guidance Section. This has a 30% chance of occurring. The median impact of this event is an additional $50K on the first unit cost. Both the likelihood (30%) and the consequence ($50K) are themselves point estimates and must receive uncertainty treatment. The 30%'s chance of occurrence uncertainty is modeled as a betaPERT distribution as shown in the left-hand panel of **Figure 2-18**. A Yes/No distribution is defined with the probability of yes linked to the 30%'s uncertainty forecast as shown in the center panel. The $50K's impact uncertainty is modeled as a lognormal as shown in the right-hand panel. The complete guidance system first unit cost (cell F112) that flows through the remainder of the estimate is modeled as the sum of the baseline (cell F113 which is also uncertain) point estimate and the result of the product of the Yes/No cell and the $50K's uncertainty cell (cell F114 = F123 * AE125).

Uncertainty should be applied to both the risk register cost impact and the probability of occurrence.

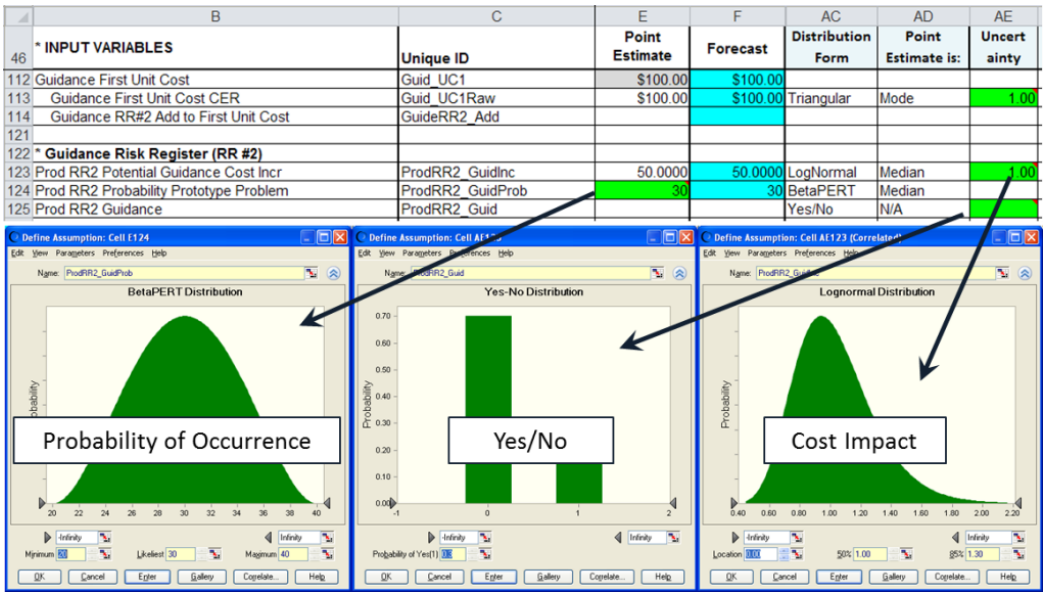


Figure 2-18 Discrete Uncertainty Example

Some considerations:

- Model the risk register events to directly impact the relevant elements as illustrated in **Figure 2-2**
- If the risk register event is highly probable, consider including its full impact in the point estimate and model the lower likelihood of it not happening as the risk register event
- When modeling uncertainty on a negative, the meaning of low and high can become confusing and for some distributions (like lognormal) impossible to define with the appropriate skew. In this case, consider modeling the value and its uncertainty in positive terms, but then subtract the results from the applicable element within the model.
- Keep in mind risk register items may have been encountered by the programs in the database and may already be captured by the uncertainty of the CER
- Model the impacts of risk register events to duration variables where applicable
- No risk register is comprehensive. Keep in mind there may be other sources of discrete risk in the program that should be modeled.

2.7.3 Modeling Risk Register Event Mitigation

The inclusion of risk mitigation plans often introduces additional cost elements into the cost estimate. This has the effect of "adding to" the initial cost estimate. However, the presence of these risk mitigation plans should have an impact on the uncertainties or risk register events assigned to the WBS elements that they address. For example, spending money to mitigate the cost impact of the guidance first unit failure risk may cause the event impact to be reduced, eliminated and/or the probability of occurrence to be reduced.

Risk register items are modeled as discrete events.
Exercise care to not double count risk.
Items to be modeled as discrete events are not limited to those obtained from the risk register.

2.8 SPECIAL CONSIDERATIONS

2.8.1 Truncating Objective or Subjective Uncertainty

In cost estimating, it is not unusual to obtain objective or subjective bounds that cause the distribution shape to stretch into the negative region (below zero) despite never observing a value in that region. As discussed in the fit distribution section, analysts may choose to force a lower bound when performing the fit. Alternatively, an unconstrained fit that stretches into the negative region can be truncated when used in the simulation tool. Employing distributions that include values less than zero is discouraged unless there is compelling evidence that negative values are a reasonably expected outcome and the model will perform properly. All tools provide the ability to truncate distributions at either a low point, high point or both.

Establishing the lower limit of the distribution to be zero will avoid nonsensical situations of negative dollars in cost elements or negative weight, power, volume, etc. in CERs. There are two significant impacts to be considered when truncating at zero. First, the distribution variance will be reduced and, second, the mean will shift to the right. Care should be taken to determine the impact and if it is acceptable. However, in situations when negative tails would represent nonsensical outcomes, the analyst is encouraged to either truncate or select a distribution, such as lognormal, that does not require truncation to simplify the explanation of the model.

Select distributions that will not give rise to nonsensical simulation draws. Truncate at zero when necessary.

2.8.2 Sunk Costs

Costs that have been incurred and cannot be recovered are called sunk costs. Further, for many acquisition decisions, funds that have been authorized and obligated in prior years are often deemed sunk though they have not been totally expended. Sunk costs are often part of a life cycle cost model because current and prior years are part of a system's total cost. Prior years' costs (and often current year's costs) should ***not have uncertainty distributions*** associated with them. In any event, it is essential to report both sunk and cost to-go when reporting cost in order to facilitate comparisons to project estimate at completion and previous estimates.

Updating an estimate to include sunk costs requires a firm grasp on the difference and the appropriate source for cost associated with government obligations and expenditures. When our cost estimate (like the example model) is built to support the budgeting process, it will be in terms of obligations. Sunk costs added to an obligation cost estimating model must reflect obligation dollars, not government expended nor contractor as expended. The best source for government obligation or expenditure data comes from the government financial execution branch. If utilizing expended dollars, the expenditures need to be traced back to the obligation year and applied as sunk costs for the obligation year in the cost estimate. The sunk costs should be entered as TY$ in the year of obligation. When using actual cost of worked performed from contractor

reports (CPRs or CSDRs) as a source for sunk costs, be aware that these sources represent the accruals on the contract and may not include the information necessary to trace the cost back to the obligation year. If using these data sources, the analyst has the burden of determining how to make adjustments so that the accruals can be properly entered as a sunk cost into an obligation estimate.

When sunk costs are in play, addressing uncertainty can be complicated. There are many ways to approach the modeling task. The uncertainty for the initial estimate is generally based on an analysis of total costs, not on costs to-go from some point in the project. Subtracting the sunk costs from the total cost estimate to arrive at cost to-go may make sense. But defining how much of the uncertainty remains in the cost to-go portion is more difficult to assess.

The following process is recommended for addressing sunk costs assigned to a specific element in the risk and uncertainty model:

- Every effort should be made to estimate the cost to-go based upon the best assessment of current circumstances. Noting that the total point estimate changes (sunk + cost to-go may not equal the original total point estimate cost) should not discourage using this approach.
- Divide the original uncertainty parameters by the point estimate cost to convert parameters to a percent of the original total point estimate
- Use these results to calculate the uncertainty parameters for the uncertainty on the cost to-go. Use a distribution of the same shape, but scaled to the cost to-go.
- If possible, obtain evidence to determine if the sunk costs are consistent with progress to-date. Ideally, this assessment should be data-driven, for instance through analysis of EVM data. However, it may be necessary to resort to expert opinion. If the evidence is compelling, consider multiplying the scaled uncertainty by a CER adjustment factor as described in **Section 2.8.6.**
- Determine if any risk register events associated with the element have been retired or need to be included in the assessment of the cost to-go

To illustrate this process on one element, Design and Development, two years of sunk cost are assumed. In the example model, the estimating structure is modified so that Design and Development is the sum of sunk and to-go costs. The sunk costs are throughput ($2M in 2014 and $3.6M in 2015) and the to-go cells compute as before. In the inputs section, two uncertain items have been retired. First, the EMD start date is now known with certainty (fixed as May 1, 2014). Second, the risk register item #2 concerning an alternate guidance section did not happen and has been retired. Since duration is an input parameter, it is now comprised of sunk months and to-go months. Similar to the modified cost estimating structure, the input parameter structure is modified so that EMD duration is the sum of sunk months and to-go months. The point estimate duration in this example is the original duration minus the sunk months. The bounds on the duration uncertainty distribution were converted to percentages. The changes to the model are shown in **Figure 2-19** and the resulting S-curves are shown in **Figure 2-20**. Note the total CV is reduced with the inclusion of sunk costs.

Sunk costs should not have uncertainty distributions associated with them.

B	C	E	F	G	H	I	J	K	L
DETAIL ESTIMATE based on PEs	Unique ID	Point Estimate	Forecast BY 2014	2014	2015	2016	2017	2018	2019
Design & Development Total	DesignDev_EMD_Total	$14,194	$14,194	$2,000	$3,600	$2,400	$2,400	$2,400	$1,394
Design & Development Sunk	DesignDev_EMD_Sunk	$5,600	$5,600	$2,000	$3,600				
Design & Development To Go	DesignDev_EMD_ToGo	$8,594	[illegible]			$2,400	$2,400	$2,400	$1,394

Change Element Structure to Sum of Sunk and To-go

Sunk Costs Throughput

To-go Cost Cell Formulas Unchanged

Sunk Duration

Uncertain EMD Start Date Retired

	B	C	E	F	AC	AD	AE	AG	AH	AJ	AK
46	* INPUT VARIABLES	Unique ID	Point Estimate	[illegible]ast	Distribution Form	Point Estimate is:	Uncertainty	Low	High	Percentile	Percentile
51	EMD Modeled Start Date	EMD_StartDate	01 May 2014	01 May 2014							
52	EMD Modeled Start Month	EMD_StartMth	5	5							
53	EMD Modeled Start GFY	EMD_StartGFY	2014	2014							
54	EMD Fraction of First Year	EMD_FirstYearFraction	0.4192	0.4192							
55											
56	EMD Duration (Months)	EMD_Mths	60	60.000							
57	EMD Duration Sunk (Months)		17	17.000							
58	EMD Duration (Months) PE Duration	EMD_MthsRaw	43	43	Triangular	Mode	1	0.90	1.20	10	80
59	EMD RR Dur Incr Due to SW (Months)										

Risk Register Item Retired

Point Estimate of Remaining Months

Remaining Months Scaled Uncertainty Bounds

Figure 2-19 Sunk Cost Example

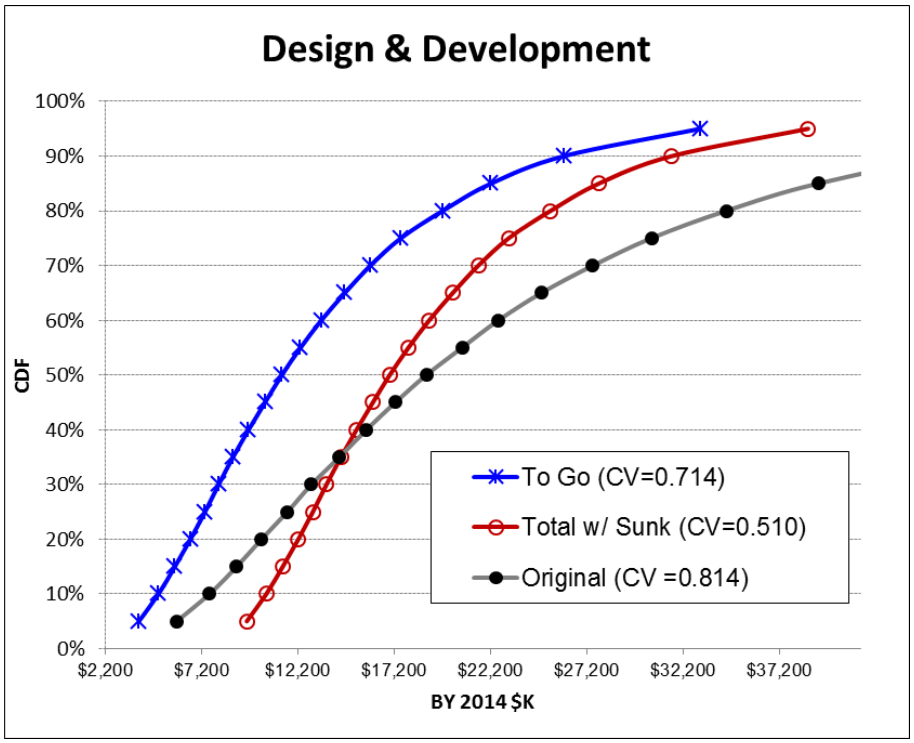


Figure 2-20 S-curves for Sunk Cost Example

2.8.3 Engineering Changes

It is extremely rare for a project to proceed through the acquisition cycles without experiencing engineering changes. Even so, cost elements for engineering changes such as Engineering Change Orders (ECO) are not meant to be the catchall for potential system cost growth or a place to allocate probability adjustment dollars (see **Section 3.6**). It is, therefore, not acceptable to use the ECO cost element to increase management reserve or as a substitute for uncertainty analysis. It is, however, appropriate to assign uncertainty to the method by which engineering change costs are estimated. When engineering changes are explicitly modeled as an element in the point estimate WBS, its uncertainty should be treated in a manner consistent with the method by which the element is estimated. Furthermore, care should be taken not to double count if discrete risk register items have already addressed possible new scope.

2.8.4 Inflation

Inflation is the overall increase in prices, or a general rise in the price of goods and services. Escalation is the increase in the cost or price of specific goods or services specific to an item or class of items. Escalation includes general inflation, but is also driven by changes in technology, practices, and particularly supply-demand imbalances that are specific to a good or service in a given economy.

- **Inflation indices** account for the effects of inflation
- **Price indices** account for the effects of escalation

The rates used will have a significant impact on any budget. Historically, programs used the Office of the Secretary of Defense and Office of Management and Budget published inflation rates for use in official estimates. However, the escalation rates experienced by a specific industry can be significantly higher. As a result, contracts have been signed and executed based upon industry specific escalation rates while budgets were based on the lower OSD inflation rates, creating a risk of cost growth from the outset. A GAO case study, reported in **Reference 41**, showed that the difference in rates applied to ships, while holding all other factors constant, explained 30 percent of the $2.1 billion in cost growth for these particular ships. In February 2004, the Navy changed its inflation policy directing program offices to budget with what the Navy believes are more realistic escalation indices. This practice is becoming common in cost estimating and heavily influences the assessment of inflation uncertainty modeling. (For additional guidance, refer to your agency's policy for the mechanics of inflating using commodity price indexes to obtain a Then Year (TY) estimate and deflating using OSD rates to obtain the constant year estimate of record.)

Cost models that are heavily influenced by the uncertainty associated with labor rates, unit prices or other similar cost drivers may, in fact, already capture some degree of inflation uncertainty, depending on how their uncertainty is estimated. The point is, be careful not to double count inflation uncertainty if there are methods in the model that already capture at least some part of it. While many references propose approaches to defining inflation uncertainty, there is no current consensus on a specific approach and this handbook does not recommend one.

There are many papers on the treatment of inflation uncertainty analysis, for instance **Reference 75** and **92**. The analyst is encouraged to continue to look for additional guidance on this topic.

2.8.5 Defining Uncertainty for Cost Improvement Curve (CIC) Methodologies

Cost improvement curves (also known as learning curves) are a source of uncertainty that must be modeled. While all the uncertainty principles presented thus far in the handbook apply, there are additional issues to be considered when CIC methods are employed. For example, consider the unit theory CIC equation:

Equation 2-2 Unit Theory CIC Equation

$$\text{Unit Cost} = T1 * Q \wedge b$$

where

T1 is the theoretical first unit cost
Q is the unit number
b is the exponent associated with the given CIC slope (*b* = log(Slope)/Log(2))

As in the case with all other cost estimating methodologies, uncertainty on CIC methodologies should be based on historical data when possible. If *T1* and *b* are established by regression analysis, then the regression equation gives rise to the point estimate and the equation's uncertainty should be modeled. The statistics from the regression analysis can be used to objectively define the uncertainty of the equation as a whole as with any other CER. In this situation, however, uncertainty should not be modeled on the *T1* value or the slope value since they were solved by the regression and are not independent variables. This is how CIC uncertainty is assigned in the example model.

However, in actual practice, the estimate for *T1* and the estimate for slope are often obtained independently. In which case, the CIC equation is simply a functional relationship with a point estimate of the *T1*[6] and a point estimate of the slope as its inputs. Therefore apply uncertainty to each input value and not to the equation. Note *T1* and *b* in our example model are highly negatively correlated and should be specified as such in the uncertainty model if they are addressed separately. Omitting negative correlation between *T1* and slope may overstate uncertainty.

Here are guidelines for three common situations:

- **Obtain *T1* and obtain slope from separate sources.** If the *T1* is computed via a parametric equation, use fit statistics from the equation to model its uncertainty. If the *T1* is an analogy or expert opinion, then apply subjective uncertainty. If the *T1* source does not have an inherent slope, it is recommended to obtain a slope using historical data from multiple systems using pooled regression and model its uncertainty based on the standard deviation on the b coefficient. A high negative correlation between *T1* and slope is often appropriate.
- **Compute a *T1* from a single reference lot using a borrowed slope.** If borrowing a slope from a single analogous program or a group of similar programs, that slope was likely computed from a regressed equation. Therefore, use the standard deviation on the b coefficient to define slope uncertainty. If the slope is obtained via expert opinion, then apply subjective uncertainty. When using a known reference lot cost, use this value as a model input and insert a row or cell to compute the model's *T1* using the same slope used in the estimate. Doing so will enable each iteration of the simulation to recompute the *T1* given that iteration's draw on slope. . If the model does not estimate T1 based upon the selected slope, consider using a high negative correlation between *T1* and slope. If an adjustment factor is applied, such as a step function for transition from prototype to production, then also apply uncertainty to the factor. Again, if the factor is rooted in data, apply uncertainty as measured from the data; otherwise, apply subjective uncertainty.
- **Compute both *T1* and slope from the program's prior lots.** This is a straightforward use of regression and uncertainty should be applied to the equation using fit statistics. Do not separately put uncertainty on the slope or *T1* as these are simply regression coefficients. However, if the analyst believes the scatter in the prior lots does not fully address uncertain CIC for future lots, then subjective methods may be considered.

In summary, take care to not over specify uncertainty for CICs. Specify uncertainty on either the CIC equation, or on the *T1* and *b* pair, but not both.

[6] Throughout this section *T1* is used as the reference unit cost but refers to any reference value modeled such as *UC10* or *UC100*. Take care to use the same slope for each applicable computation (such as pivot calculations) throughout the model for each simulation draw.

Here are some additional considerations:

- **Truncating slopes**: The essence of CIC theory is decreasing cost with increased cumulative quantity. While occasionally history will show increases between lots this is likely due to unique circumstances or incomplete lot normalization. It is recommended that distributions on slopes be truncated at 100%.
- **Adjustment factors**: Often a *T1* obtained from data is further adjusted to an estimate's *T1* to account for configuration or process changes. In this case, model the factor as a multiplier and assign uncertainty to the factor. Adjustment factors are discussed further in **Section 2.8.6**
- **Broken CIC, shift, rotation, displacement, etc.**: Many programs have conditions necessitating variations to the typical CIC. For example an analyst may utilize a steep slope early in a program to account for material delays or exceptional amounts of rework with a flatter slope in later lots to account for maturing processes. While every situation cannot be foreseen and addressed here, the analyst is encouraged to model CIC uncertainty using distributions fitted to historical data whenever possible but use subjective uncertainty when applicable data is not available.
- **Rate-adjusted CIC**: Consider the rate-adjusted CIC equation:

  Equation 2-3 Rate Adjusted CIC Equation

  $$\text{Unit Cost} = T1 * Q \wedge b * R \wedge c$$

  where *T1* is the theoretical first unit cost at a rate of 1; *Q* is the unit number; *R* is the production rate; *b* is the exponent associated with the given CIC slope; and *c* is the exponent associated with the given rate slope. All of the concepts discussed above apply. Consider a high negative correlation between *T1* and each slope. All the concepts in this section also apply when modeling quantity discounts, as this is equivalent to using the above equation with b set to 1 (100% CIC).
- **Rate-adjusted CIC ridge regression**: Sometimes the multicolinearity between the cumulative lot quantity and lot rate is present in historical data and gives rise to an unreasonable pair of regressed slopes. The analyst may use ridge regression (**Reference 23** and **37**) to mitigate the effects of multicolinearity and improve the reasonableness of the results. If borrowing slopes from ridge regression results, be aware ridge regression can result in a tiny standard deviation on the resulting coefficients. Therefore, consider subjective uncertainty on those slopes.

Take care to not over specify uncertainty for CICs.
Specify uncertainty on either the CIC equation, or on the *T1* and *b* pair, but not both.
Unless there is strong evidence to do otherwise, truncate slopes at 100%.

2.8.6 Adjustment Factors

CERs are developed from historical data where some technical characteristic such as weight or power is used to estimate cost. Alternatively, subject matter experts may provide direct estimates based upon an analogy. By doing so, it is assumed that the CER or analogy is applicable to a project being estimated. If, however, there is compelling evidence that the CER or analogy on its own insufficiently estimates the cost for the new project, an adjustment may be warranted. The best way to account for such issues is to revisit the cost methods and find one that is more consistent with the new project. Unfortunately, this is not always possible or practical. In the absence of better information, and when direct modeling is not feasible, an acceptable approach is to multiply the estimate method by an uncertain factor.

In the example model, the new missile's payload composition differs from the CER dataset such that an additional 10% is added to the CER results. This situation is modeled as:

(30.15 + 1.049 * WarheadWt) * PayloadAdjustment

where PayloadAdjustment has a point estimate of 1.1 and is the mode of a triangular distribution where the minimum is 0.9 and the maximum is 1.5. The impact of the adjustment factor is illustrated in **Figure 2-21**.

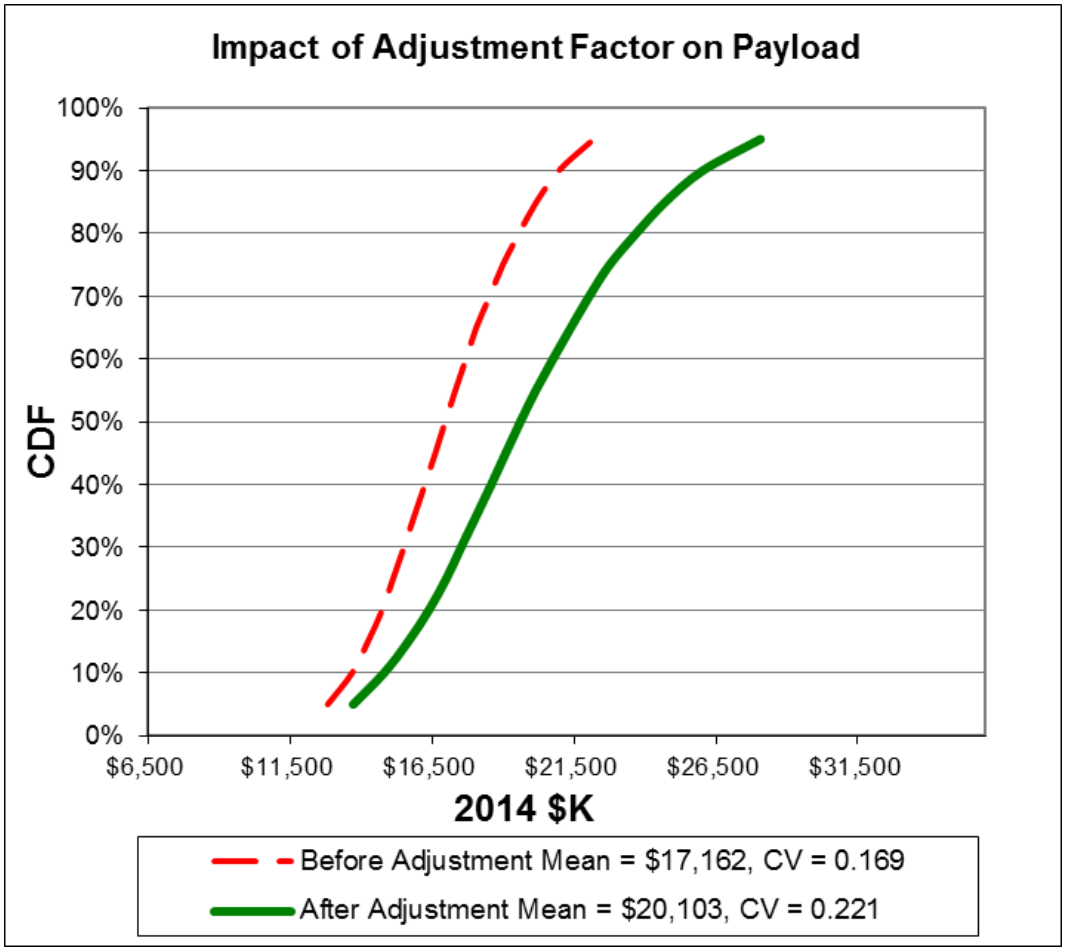


Figure 2-21 Impact of an Adjustment Factor on Cost

This approach should only be used in cases where better information is unavailable. Some organizations have developed their own standard multipliers and their uncertainty. See **Reference 43** for an example.

2.8.7 Calibrated CERs

Analysts frequently rely on published, validated CER libraries (e.g., the Unmanned Space Cost Model CER library https://www.uscmonline.com/ to obtain a relevant estimating method for one or more of the elements in their model. Whether drawing from a CER library or from an analysis of relevant historical data, an analyst may also discover a completed program from the CER's dataset that would be a suitable analogy. In these situations, the analyst may choose to calibrate the chosen CER to this known data point. For an equation of the form $Y=aX^b$ this means computing a new intercept (a) by fitting the same slope (b) through the selected analogous data point. The calibrated CER line will now pass through the analogy. Ideally the data will be available to plot this calibrated CER against the original source data to be comfortable that the shift in the CER line remains consistent with the source data. While this is a straightforward way to obtain a more defendable point estimate, how should the original CER uncertainty be adjusted to account for the recalibration? There are at least two ways to address this question:

- One view[7] is that if there is evidence to support that the analogy is more similar to the system being estimated than the average data point in the CER data set, then it may be reasonable to conclude that the CER uncertainty should be reduced. One way to do this is to make use of the original regression statistics. The uncertainty of the intercept could now be assumed to be zero. This would mean that the uncertainty of the CER is now defined solely by the uncertainty of the exponent. After all, the slope has been borrowed from the original equation, so it stands to reason the slope's uncertainty should be borrowed as well. A way to model this is to treat the exponent as another input to the CER, and then specify the exponent as the mean of a lognormal with the exponent's standard deviation. The exponent's standard deviation is typically available from the original regression report. Take care to ensure the unit space values (rather than fitted space) for the exponent and its standard deviation are retrieved from the regression report.
- Another view is that nothing has changed regarding how the CER was developed. The interaction of the intercept and the exponent uncertainty is not a simple one. The correlation between the intercept and exponent is integral to the overall CER uncertainty. Simply shifting the CER to pass through the analogy does not change the CER's uncertainty. The uncertainty applied to this new point estimate, on a relative scale, may be the same, but it will yield a different S-curve that should be more defendable.

There are other ways to estimate the uncertainty of a calibrated CER and the analyst is encouraged to explore the literature on the subject. Choosing the right path may lie, in part, by assessing where the analogous point lies within the original dataset.

2.8.8 Uncertainty on Fee

When Fee and other contract loadings such as general and administrative (G&A) and cost of money are modeled as variables they will need uncertainty assigned unless they are known to be fixed. Consider fixed bounds as these variables truly do not have long tails. Though modeling every nuance of a contract's fee plan may be challenging, strive to model the fee plan so that each simulation pass calculates the correct fee for that pass' cost. For instance, the fee (and/or award fee) and its uncertainty need to be consistent with the contract type, phase, and any contractual obligations. Fee may also be different at different points in time (e.g., the milestones).

[7] Dr. Christian Smart provided a detailed discussion on this approach on pages 97-99 of **Reference 91**.

3.0 FINISH AND ASSESS THE CISM MODEL

3.1 COMBINING UNCERTAINTIES AND INTRODUCING CORRELATION

The purpose of the simulation is to combine all the uncertainties specified in the model to estimate the total uncertainty at the parent levels. **Figure 3-1** illustrates how the simulation process combines the input uncertainty with the CER uncertainty for a specific element in the model. Each trial of the simulation makes a random draw from all the risk and uncertainties in the model to generate a point estimate. The result of each WBS element is summed to develop simulated results at the parent levels.

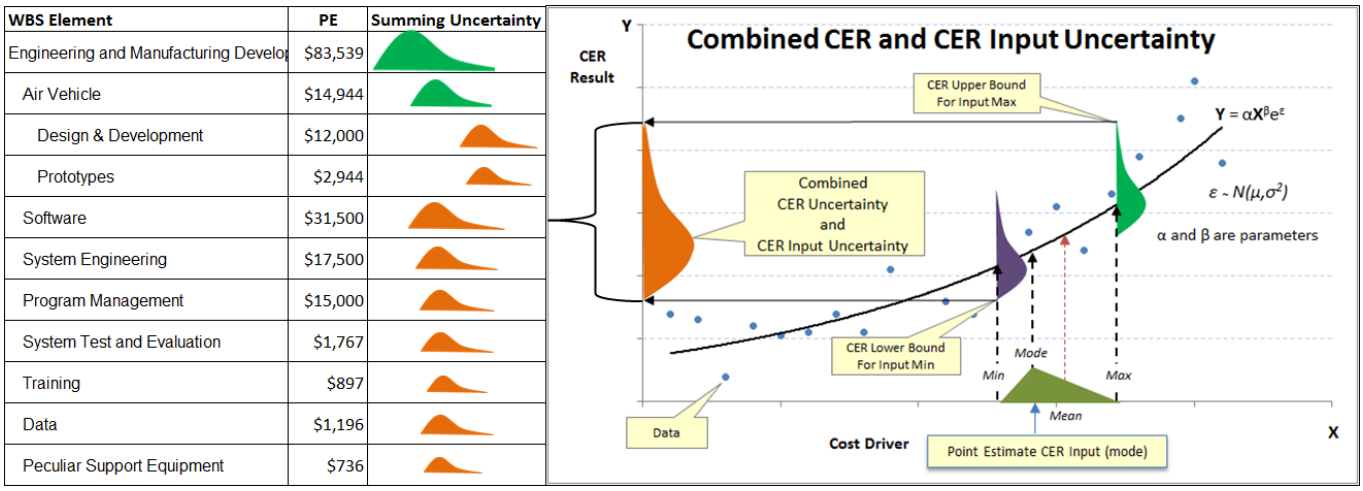


Figure 3-1 Combining Methodology and Methodology Input Uncertainties

A critical part of the simulation model is to define the correlation between each of the distributions defined in the model. Correlation is the term used to describe the degree to which variables "move together". For instance, if a random sample from one distribution is taken from the high end of the distribution, is there any reason to expect others would be drawn in a similar way? The level of correlation in a model has a profound influence on the results and is addressed in the following sections. For details, see **Appendix A.11**.

3.2 FUNCTIONAL VERSUS APPLIED CORRELATION

The term functional correlation has been in the lexicon since at least 1994 (see **Reference 17** and **19**) and yet it is largely misunderstood. Essentially, it is referring to the correlation that is developed in the simulation due to the mathematical (functional) relationships within the model. Uncertainty that is defined on a variable or assigned to a CER will be inherited by any relationship that uses them in its equation. Functional correlation can exist between:

- CER inputs if these inputs are in fact a function of each other
- CERs if the CERs share one or more common input variables. A common mistake is to assign the same CIC slope variable to a variety of elements. With this variable's uncertainty modeled only once, these elements will be inadvertently correlated in the model.
- Two or more CERs if one CER is related to other CERs (for instance through a factor relationship).
- A CER result and the uncertainty of its input(s)

A good rule of thumb is if the relationship between uncertain elements in the model is known, then capturing that in the functional relationships should be attempted. For instance, if the motor weight is known to be a function of the air vehicle weight through an engineering or regression result, then this relationship should be explicitly implemented in the model rather than allowing the elements to behave independently in the model. This simplifies what-if analysis and improves the chances of the simulation behaving properly.

If there are no known functional relationships to employ, every simulation tool will allow correlation to be applied. Applied correlation is when the analyst specifies a correlation between two or more uncertainty distributions. It is also possible to apply additional correlation across functionally correlated items. Applied correlation does not replace functional correlation. The net effect in the simulation is the combination of the applied and functional correlation, and it is not a simple sum. It is necessary to measure correlation in the simulation result before and after applied correlation to determine the applied correlation impact. Most simulation models contain a mix of both functional and applied correlation.

Functional correlation is a result of model functional relationships. Applied correlation is specified by the analyst.

3.3 MEASURE THEN APPLY CORRELATION

3.3.1 Overview of Correlation Assessment

The uncertainty analysis is not complete until correlation is addressed. If correlation is ignored, the variance at the total levels in the estimate may be understated, in many cases dramatically. The results of the first simulation run can be used to measure (see **Section 3.3.4**) the functional correlation already present across the model WBS due to mathematical relationships. If a WBS element result is a function of two or more uncertain parameters (e.g., estimating data cost by multiplying the uncertain air vehicle cost by an uncertain factor – see **Table 2-1**) then the correlation between the associated WBS elements (in this case Data and Design or Data and Prototypes) will not be perfect (correlation coefficient will be less than 1.0).

For many situations, the correlation across WBS elements will not be adequately captured through functional relationships. In these cases, additional correlation needs to be applied. A good first step is to organize the model inputs into groups that should move together (either positively or negatively) and develop a correlation matrix. Correlating cost method inputs will induce correlation across the WBS element results. Measure, apply, and then measure again to ensure adequate (see **Section 3.3.2**) correlation is present.

A correlation matrix is an efficient way to define all the correlations amongst a group of uncertainty distributions built into the model. The diagonal of the matrix (from upper left to lower right) is populated with ones to define the correlation between each distribution in the group and itself. Only half the matrix need be specified as the other half (other side of the diagonal) is a mirror image. When building the correlation matrix, there are several issues to consider:

- **Consistent Matrix**: Care must be taken to populate the matrix with a consistent set of correlations. For example, if A:B and B:C are highly correlated in a positive direction, then assigning a strong negative to A:C would cause an inconsistency.
- **Inconsistent Matrix**: When Crystal Ball or @RISK detects a correlation matrix that is inconsistent enough to prevent the simulation from running, they will offer the option to adjust correlations enough to allow the simulation to proceed (ACE does not allow the user to enter an inconsistent matrix). If a consistent matrix is not specified, then algorithms are applied to transform the specified matrix into a consistent matrix which may vary significantly from the original analyst-specified matrix. While there is no reason to doubt the algorithms, the tools use different methods and the user is not in control of the outcome. When inconsistent matrices are encountered and a manual fix can be expediently applied, it is recommended they are repaired manually rather than letting the software attempt it. Review the tool's recommendation for guidance on where to make adjustments.

Proactively investigate and repair an inconsistent matrix.

- **Empty Correlation Matrix Cells**: Popular simulation tools treat empty cells in a correlation matrix differently. Crystal Ball allows the user to leave cells in the matrix blank. Crystal Ball will "fill in" the blank cells. If the top row of the matrix is fully populated and the remainder empty, the math Crystal Ball uses to fill in blanks is known and easy to understand. If random cells are blank, it is unclear what Crystal Ball does. For that reason, @RISK requires all cells to be populated. ACE treats an empty cell as zero. It is recommended that every cell in the matrix (the half being using) be populated.
- **Linear Relationship**: Correlation is a measure of the linear relationship between random variables. Correlation does not prove a cause-and-effect relationship. Generally, there is little empirical basis for the derivation of the correlation values. Consequently, correlation is often based on subjective judgments (see **Reference 26, 27**).
- **Impact on Functional Correlation**: Most estimates do contain many elements that are functionally related through linear and non-linear methods. This often causes uncertainty distributions to be multiplied, divided, exponentiated, etc. For this reason, correlation applied on functionally related uncertainty distributions will have an impact not only on the spread of the parent, but the mean as well. This is why applying functional relationships (rather than simply adding throughputs) within a model wherever possible is so important: it may have a significant impact on the mean of the ultimate uncertainty distribution.
- **Measure First**: Before applying correlation, measure the correlation present due to functional relationships. Knowing what is present will help determine what needs to be applied.

Measure the correlation present in the model first, then apply additional correlation where required.

3.3.2 Metrics for Assessing Correlation Adequacy

In the absence of objective data, analysts are encouraged to make subjective correlation assessments using the following steps:

1. Measure the correlation present in the simulation due to functional correlation and identify those elements with a correlation of less than 0.3
2. Determine if specific elements that are currently uncorrelated should "move together," that is, be correlated either negatively or positively
3. Assign additional correlation using a correlation value between -1 and +1. **Table 3-1** provides guidance on default correlation values. Perfect correlation of +/-1.0 is discouraged.
4. Measure the correlation again to ensure elements are properly correlated

Several references (**20, 21, 22, 28,** and **52**) suggest a default correlation of 0.25 when there is no other information. However others provide evidence that 0.45 or 0.63 may be more appropriate (**References 77, 91**, **101**). **Reference 91** also reports the result of several cost data-driven investigations that supports a default closer to 0.3. The impact of correlation on the parent increases exponentially with the number of uncertain child elements, which implies the default should be based on the number of elements to be correlated. As a compromise between all the published recommendations, 0.3 is the recommended default.

Table 3-1 Default Correlation Factors

Strength	Positive	Negative
None	0.0	0.0
Weak	0.3	-0.3
Medium	0.5	-0.5
Strong	0.9	-0.9
Perfect	1.0	-1.0

In the absence of better information, 0.3 is the recommended default correlation.

3.3.3 The Impact of Correlation on a Cost Model

Only two statistics in the cost model will sum up through the WBS indenture: the mean and the variance. In the case of variance, it will only sum to the parent level variation if the uncertainty distributions are independent. However, if they are correlated either through functional, applied or both types of correlation, the total variance can be calculated using the following equation:

Equation 3-1 Total Variance Adjusted for Correlation

$$TotalVariance = \sum_{k=1}^{n} \sigma_k^2 + 2\sum_{k=2}^{n}\sum_{j=1}^{k-1} \rho_{jk}\sigma_j\sigma_k$$

Where σ is the symbol for standard deviation and ρ is the correlation coefficient. **Figure 3-2** demonstrates the impact of correlation on the sum of five random variables and compares Crystal Ball simulation results with analytical results. **Figure 3-2** contains the following:

- Analytical results for the mean and standard deviation (method of moments). See **Appendix A.6** for the formulas used in **Figure 3-2** to calculate the mean and standard deviation for these (and other) distributions.
- The top image shows the results when the correlation is forced to be zero, meaning each distribution is sampled independently from any other
- In the bottom image, a correlation of 0.3 is applied across all elements. For this example, the zero-correlation case's simple sum of the variances underestimates the total standard deviation by 46% (107 / 73)!
- Applying the correlation to these five throughput uncertainties results in no impact on the mean. In a functionally correlated model, applying correlation on top of functional correlation may influence the mean a few percent (**Reference 58**).
- Element standard deviation is adjusted for correlation using (SD*MMULT(CorrRow,SDCol))^0.5 where SD is the standard deviation in question, CorrRow is the row in the correlation matrix associated with SD, and the SDCol is the column of all five standard deviations. MMULT is an Excel function that performs the necessary matrix multiplication (row times a column).
- Note that for zero correlation (upper table) the simulation, analytical without adjustment, and analytical with adjustment all match. In the lower table, only the analytical standard deviation adjusted for correlation matches the simulation result.

Total is the sum		Parameters			Simulation			Analytical		
		Std Dev	Min	Max	Mean	Std Dev	CV	Mean	Std Dev No Adj	Std Dev Adj For Corr
Total	**500**				**575**	**73**	**0.13**	**575**	**73**	**73**
Lognormal	100	40			100	40	0.40	100	40	40
Triangular	100		75	200	125	27	0.22	125	27	27
BetaPERT	100		75	200	113	22	0.19	113	22	22
Normal	100	35			100	35	0.35	100	35	35
Uniform	100		75	200	138	36	0.26	138	36	36

Applied Correlation

0.000	LN	Tri	BPERT	Nor	Unif
LN	1.000	0.000	0.000	0.000	0.000
Tri	0.000	1.000	0.000	0.000	0.000
BPERT	0.000	0.000	1.000	0.000	0.000
Nor	0.000	0.000	0.000	1.000	0.000
Unif	0.000	0.000	0.000	0.000	1.000

Total is the sum		Parameters			Simulation			Analytical		
		Std Dev	Min	Max	Mean	Std Dev	CV	Mean	Std Dev No Adj	Std Dev Adj For Corr
Total	**500**				**575**	**107**	**0.19**	**575**	**73**	**107**
Lognormal	100	40			100	40	0.40	100	40	55
Triangular	100		75	200	125	27	0.22	125	27	42
BetaPERT	100		75	200	112	22	0.19	113	22	37
Normal	100	35			100	35	0.35	100	35	50
Uniform	100		75	200	137	36	0.26	138	36	51

Applied Correlation

0.300	LN	Tri	BPERT	Nor	Unif
LN	1.000	0.300	0.300	0.300	0.300
Tri	0.300	1.000	0.300	0.300	0.300
BPERT	0.300	0.300	1.000	0.300	0.300
Nor	0.300	0.300	0.300	1.000	0.300
Unif	0.300	0.300	0.300	0.300	1.000

Figure 3-2 Impact of Correlation of 0 and 0.3 on Elements that Sum

Correlation is an important part of the simulation model. The more distributions in the model, the more important it is to assign correlation in an appropriate manner. **Figure 3-3** illustrates the impact of breaking up an estimate into a greater number of uncertain elements on the total uncertainty. The upper tables calculate the standard deviation at the parent level assuming there is no correlation. In Excel this can be done using the formula SumSq(StdDev)^0.5 where StdDev is the Excel range containing the child element standard deviations. To preserve the CV at the total level, the CV of the child elements would have to increase. In this model, child CV would have to increase 60% going from two elements to five and another 40% to go from five elements to ten. If the elements are correlated, the effect is significantly diminished.

0.00 Correlation	Sum Independent Distributions			Sum Independent Distributions			Sum Independent Distributions		
	Mean	Std Dev	CV	Mean	Std Dev	CV	Mean	Std Dev	CV
Total	1000	176.78	0.177	1000	111.80	0.112	1000	79.06	0.079
Element 1	500	125.00	0.250	200	50.00	0.250	100	25.00	0.250
Element 2	500	125.00	0.250	200	50.00	0.250	100	25.00	0.250
Element 3				200	50.00	0.250	100	25.00	0.250
Element 4				200	50.00	0.250	100	25.00	0.250
Element 5				200	50.00	0.250	100	25.00	0.250
Element 6							100	25.00	0.250
Element 7							100	25.00	0.250
Element 8							100	25.00	0.250
Element 9							100	25.00	0.250
Element 10							100	25.00	0.250
0.30 Correlation	Sum Correlated Distributions			Sum Correlated Distributions			Sum Correlated Distributions		
	Mean	Std Dev	CV	Mean	Std Dev	CV	Mean	Std Dev	CV
Total	1000	201.56	0.202	1000	165.83	0.166	1000	152.07	0.152
Element 1	500	142.522	% Increase in CV	200	74.162	% Increase in CV	100	48.09	% Increase in CV
Element 2	500	142.522		200	74.162		100	48.09	
Element 3			14%	200	74.162	48%	100	48.09	92%
Element 4				200	74.162		100	48.09	
Element 5				200	74.162		100	48.09	
Element 6							100	48.09	
Element 7							100	48.09	
Element 8							100	48.09	
Element 9							100	48.09	
Element 10							100	48.09	

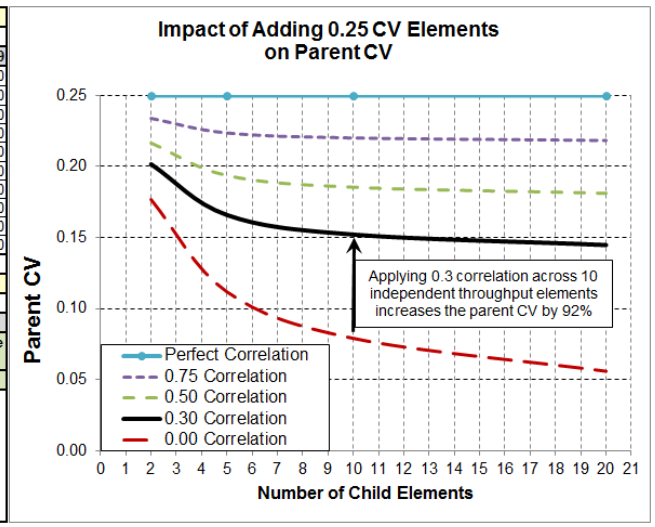


Figure 3-3 Impact of Adding Correlated Uncertain Elements

In the bottom table of **Figure 3-3**, each standard deviation is adjusted to account for correlation as described above. Adding 0.3 correlation almost doubles the CV in this example summing ten elements. The impact is even greater with larger numbers of elements.

Cost estimates are broken into a greater number of elements in order to improve the estimate accuracy and provide more flexibility for trade-off studies. However, as shown in **Figure 3-3**, doing so will cause the parent CV to reduce unless the child CVs are increased. It is reasonable to expect that breaking the estimate into more elements could reduce the overall uncertainty. However, breaking the estimate into more and more sub elements should not be done indiscriminately simply to reduce the parent CV. For instance, consider an auto fuel cost estimate where the CV was found to be 0.25. Proposing to fill half the tank in the morning and the other half in the afternoon (same location, same day) might break the estimate into two parts, but should not be used as a way to reduce the total CV to 0.18 (per **Figure 3-3**). This would be an example of where perfect correlation should be applied between the two filling events to preserve the original CV at the total.

The importance of correlation increases with the number of elements involved.

3.3.4 How to Measure Correlation

Tools such as Crystal Ball and @RISK report the correlation entered and the correlation actually used when a correction for inconsistency is required. The tools provide the capability to extract a spreadsheet containing each iteration result for any forecast in the simulation. Once these data are properly sorted in the worksheet, Excel's CORREL function can be used to calculate pairwise correlation. Performing the calculation on the actual values will return the Pearson Product Moment (PPM) correlation. Converting the data to ranks first and then using Excel's CORREL function will yield the Spearman Rank Correlation. ACE contains a correlation report that will show the PPM correlation between user-selected elements of the estimate. @RISK includes a function to measure correlation between any two distributions (input or forecast).

A correlation measuring utility is included with the handbook (see **Section 7.1.3**). This is an Excel workbook that will measure the Pearson Product Moment correlation from simulation trial data downloaded from any tool. The correlation utility was used to measure the correlation generated in the Crystal Ball example model to create the correlation examples in this handbook.

Figure 3-4 illustrates the results when measuring the correlation across WBS elements in the example model simulation before any correlation is applied. The result shows the functional correlation present in the model. The elements of Production have been rearranged to produce **Figure 3-4** in order to make it easier to see the nature of the functional correlation in this phase of the model. Training, Data, and Initial Spares were modeled as a factor of the Air Vehicle cost. Not only does this account for the functional correlation between them and the Air Vehicle elements, but since they are estimated off of a common element (Air Vehicle) they are also correlated amongst themselves. Unless there is evidence to do otherwise, since the correlations between them is very close to 0.3 (our default), no further correlation is necessary for those specific elements. It may be seen, however, that there is a need to consider correlation between the Air Vehicle elements and the elements that are estimated as a function of duration due to the lack of functional correlation. The elements that are a function of duration are not showing any correlation because in this model the leadership directed that there be no production duration uncertainty.

MEASURED CORRELATION ACROSS PRODUCTION WBS ELEMENTS

	WBS/CES	Airframe	Propulsion	Guidance	Payload	IAT&C	Training	Data	Initial Spares	Sys Eng	PM	ST&E	PSE
Air Vehicle	Airframe	1.00	0.00	0.00	0.00	0.01	0.22	0.30	0.21	0.00	0.01	0.01	0.00
	Propulsion		1.00	0.00	0.00	-0.02	0.12	0.18	0.12	0.00	-0.02	-0.01	0.00
	Guidance			1.00	0.00	0.00	0.28	0.40	0.27	0.01	0.01	-0.01	0.01
	Payload				1.00	0.01	0.13	0.15	0.13	-0.01	-0.01	0.01	0.00
	IAT&C					1.00	0.33	0.42	0.30	-0.01	0.00	0.00	-0.01
Factor of Air Vehicle	Training						1.00	0.36	0.25	0.01	0.00	0.00	0.01
	Data							1.00	0.34	0.00	0.00	-0.02	0.01
	Initial Spares								1.00	-0.01	0.01	0.00	0.01
Function of Duration	Sys Eng									1.00	0.01	0.00	0.02
	PM										1.00	0.02	0.02
	ST&E											1.00	-0.01
	PSE												1.00

Consider Adding Correlation

Additional Correlation May Not Be Needed

Consider Adding Correlation

Functional Correlation Only

Figure 3-4 Measured Production Correlation Before Correlation is Applied

3.3.5 Applied Correlation Example

In the previous section, correlation within the Production phase was measured to help identify where it may be necessary to apply additional correlation across the model's input distributions. Where possible, it is recommended that a few large matrices be created rather than many small ones. Our example model is small enough that both EMD and Production distributions can be defined in a single correlation matrix, as shown in **Table 3-2**. Guided by correlation measurement and knowledge of the model, elements should be put in an order that help define correlations where they are appropriate. It may be alarming to some to see the large number of empty cross correlation cells. In particular, in the case of the below the line (BTL) items (data, training, spares, etc.), if the same team is performing those functions, it may be appropriate to capture them in one large group by populating the gray cells in **Table 3-2**.

Table 3-2 Example Model Applied Correlation Matrix

	EMD_MthsRaw	EMD RR1 SWIncDur	EMD RR1 SWIncMM	EMD SWLaborRate	EMD_SWMonthsRaw	DesignDevPerMonth	ProdToEMD_Step	Airframe_UC1	Prop_UC1	Guid_UC1Raw	Payload_UC1	PenaltyPayload	IATC_Hrs3rdPartyTool	MfgLaborRate	ProdRR2_GuidInc	EMD_AnnualSE	EMD_AnnualPM	EMD_STE_Fac	EMD_Trng_Fac	EMD_Data_Fac	EMD_SptEquip_Fac	ProdAnnualSE	ProdAnnualPM	ProdAnnualSTE	ProdPSE_TlCost	ProdTrng_Fac	ProdData_Fac	ProdInitSpares_Fac	AirFrameWt	MotorWt	WarheadWt
EMD_MthsRaw	1.0	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
EMD RR1 SWIncDur		1.0	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
EMD RR1 SWIncMM			1.0	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
EMD SWLaborRate				1.0	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.0	0.0														0.0
EMD_SWMonthsRaw					1.0	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.0	0.0														0.0
DesignDevPerMonth						1.0	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.0	0.0														0.0
ProdToEMD_Step							1.0	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.0	0.0														0.0
Airframe_UC1								1.0	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.0	0.0														0.0
Prop_UC1									1.0	0.3	0.3	0.3	0.3	0.3	0.3	0.0	0.0														0.0
Guid_UC1Raw										1.0	0.3	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Payload_UC1											1.0	0.3	0.3	0.3	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
PenaltyPayload												1.0	0.3	0.3	0.3	0.0	0.0	0.0													0.0
IATC_Hrs3rdPartyTool													1.0	0.3	0.3	0.0	0.0	0.0													0.0
MfgLaborRate														1.0	0.3	0.0	0.0	0.0													0.0
ProdRR2_GuidInc															1.0	0.0	0.0	0.0													0.0
EMD_AnnualSE																1.0	0.3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
EMD_AnnualPM																	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
EMD_STE_Fac																		1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
EMD_Trng_Fac																			1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
EMD_Data_Fac																				1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
EMD_SptEquip_Fac																					1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
ProdAnnualSE																						1.0	0.3	0.3	0.3	0.0	0.0	0.0	0.0	0.0	0.0
ProdAnnualPM																							1.0	0.3	0.3	0.0	0.0	0.0	0.0	0.0	0.0
ProdAnnualSTE																								1.0	0.3	0.0	0.0	0.0	0.0	0.0	0.0
ProdPSE_TlCost																									1.0	0.0	0.0	0.0	0.0	0.0	0.0
ProdTrng_Fac																										1.0	0.0	0.0	0.0	0.0	0.0
ProdData_Fac																											1.0	0.0	0.0	0.0	0.0
ProdInitSpares_Fac																												1.0	0.0	0.0	0.0
AirFrameWt																													1.0	0.3	0.3
MotorWt																														1.0	0.3
WarheadWt																															1.0

Do not leave cells blank.

If a cell is not in a group, then populate the cell with zero.

If any elements of two groups should be correlated, then both groups should be combined into one.

The default 0.3 is used in this example. Actual models should contain a variety of correlations consistent with how the elements should behave.

Applying correlation in any of the tools augments, rather than replaces, the functional correlation that is already present. The upper table in **Figure 3-5** shows the functional correlation across the production elements in the missile example before any additional correlation is applied. The lower table illustrates the impact on measured correlation when the matrix in **Table 3-2** was applied. The impact on the missile production uncertainty results is also shown. Note that the functional correlation associated with Training, Data, and Initial Spares increases even though no additional correlation is applied to those elements explicitly. They increase because of the correlation applied to functionally related elements (Air Vehicle elements and their weights).

MEASURED CORRELATION ACROSS PRODUCTION WBS ELEMENTS

	WBS/CES	Airframe	Propulsion	Guidance	Payload	IAT&C	Training	Data	Initial Spares	Sys Eng	PM	ST&E	PSE
Air Vehicle	Airframe	1.00	0.00	0.00	0.00	0.01	0.22	0.30	0.21	0.00	0.01	0.01	0.00
	Propulsion		1.00	0.00	0.00	-0.02	0.12	0.18	0.12	0.00	-0.02	-0.01	0.00
	Guidance			1.00	0.00	0.00	0.28	0.40	0.27	0.01	0.01	-0.01	0.01
	Payload				1.00	0.01	0.13	0.15	0.13	-0.01	-0.01	0.01	0.00
	IAT&C					1.00	0.33	0.42	0.30	-0.01	0.00	0.00	-0.01
Factor of Air Vehicle	Training						1.00	0.36	0.25	0.01	0.00	0.00	0.01
	Data							1.00	0.34	0.00	0.00	-0.02	0.01
	Initial Spares								1.00	-0.01	0.01	0.00	0.01
Function of Duration	Sys Eng									1.00	0.01	0.00	0.02
	PM										1.00	0.02	0.02
	ST&E											1.00	-0.01
	PSE												1.00

Consider Adding Correlation

Additional Correlation May Not Be Needed

Consider Adding Correlation

Functional Correlation Only

	WBS	Airframe	Propulsion	Guidance	Payload	IAT&C	Training	Data	Initial Spares	Sys Eng	PM	ST&E	PSE
Air Vehicle	Airframe	1.00	0.28	0.21	0.34	0.27	0.40	0.50	0.39	-0.01	0.01	0.00	0.01
	Propulsion		1.00	0.22	0.34	0.28	0.35	0.44	0.34	0.00	-0.01	0.00	0.01
	Guidance			1.00	0.25	0.25	0.42	0.54	0.40	0.01	0.01	0.01	0.02
	Payload				1.00	0.33	0.38	0.47	0.38	-0.02	-0.02	0.00	0.00
	IAT&C					1.00	0.49	0.59	0.46	-0.01	0.00	0.00	0.00
Factor of Air Vehicle	Training						1.00	0.52	0.40	0.00	-0.01	0.00	0.01
	Data							1.00	0.50	0.00	0.01	0.00	0.01
	Initial Spares								1.00	-0.02	0.00	0.00	0.00
Function of Duration	Sys Eng									1.00	0.30	0.30	0.32
	PM										1.00	0.31	0.31
	ST&E											1.00	0.30
	PSE												1.00

Functional Correlation Plus Applied Correlation

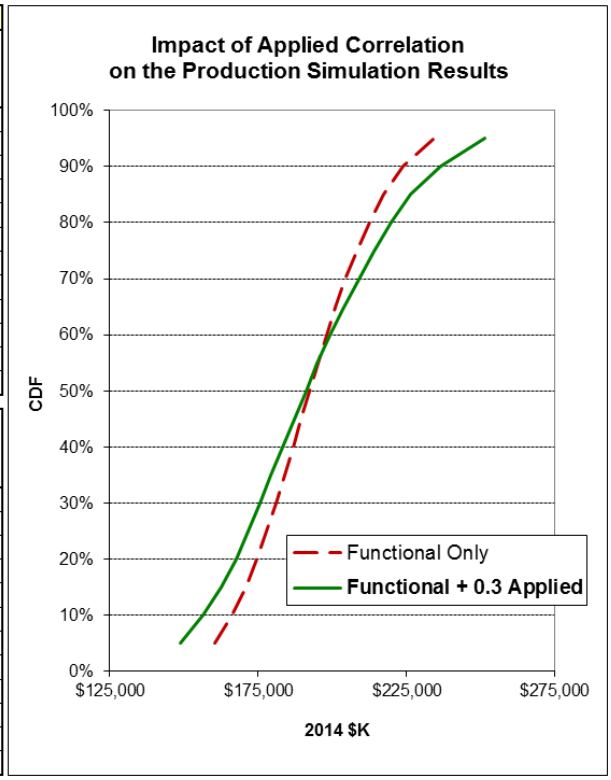


Figure 3-5 Impact of Applied Correlation On Top of Functional Correlation

Two excursions on **Table 3-2** were performed to measure the impact of adding additional correlation. The first excursion populated all the gray elements in **Table 3-2**. The original matrix contained 115 correlations. In this first excursion, an additional 71 were applied (60% more correlations). As shown in **Figure 3-6**, adding the 71 additional correlations had almost no effect on the S-curves. In the range 30 to 70 percent probability, the impact was less than 1%. This is because the use of factor methods already introduces significant functional correlation, obviating the requirement for more. In the second excursion the entire matrix was populated, adding three times the original. The impact is noticeable, but less than 3% in the 30 to 70 percent probability range. In our example, only 25% of the cells (115 out of a possible 465) capture over 95% of the fully populated scenario. A few large, well organized matrices will also help avoid applying inconsistent correlation.

Also of note, this particular example model has many factor relationships inducing significant functional correlation. The minimal impact of applied correlation demonstrates that if significant functional correlation can be built into the model, the requirement for applied correlation diminishes.

	Change to the Program Estimate					
	Estimate + 71 Corrrelations			Estimate + 350 Corrrelations		
	EMD	Prod	Total	EMD	Prod	Total
5%	-0.9%	-0.2%	-1.1%	-8.0%	-7.6%	-8.6%
10%	-1.1%	-0.2%	-0.8%	-6.5%	-6.3%	-7.1%
15%	-0.5%	-0.3%	-0.3%	-5.1%	-5.5%	-6.2%
20%	-0.8%	-0.2%	-0.2%	-3.8%	-4.6%	-4.9%
25%	-0.6%	-0.1%	-0.3%	-2.6%	-3.7%	-4.0%
30%	-0.3%	-0.2%	-0.2%	-2.0%	-3.0%	-2.7%
35%	0.1%	-0.1%	-0.2%	-1.2%	-2.2%	-2.0%
40%	0.2%	-0.1%	0.0%	-0.9%	-1.5%	-1.3%
45%	0.1%	-0.1%	0.1%	-0.8%	-0.6%	-0.7%
50%	0.2%	0.1%	-0.1%	-0.3%	0.1%	-0.4%
55%	0.3%	0.0%	0.0%	0.3%	0.9%	0.4%
60%	0.7%	0.0%	0.1%	0.6%	1.3%	1.0%
65%	0.5%	0.2%	0.2%	1.0%	1.7%	1.8%
70%	0.8%	0.0%	0.4%	1.4%	2.6%	2.7%
75%	1.0%	-0.2%	0.4%	2.3%	3.3%	3.5%
80%	0.3%	-0.2%	0.2%	2.9%	4.2%	4.3%
85%	0.6%	0.1%	0.3%	3.2%	5.4%	4.9%
90%	0.7%	0.5%	1.0%	4.2%	6.6%	6.7%
95%	0.8%	0.3%	0.3%	5.4%	7.7%	7.4%

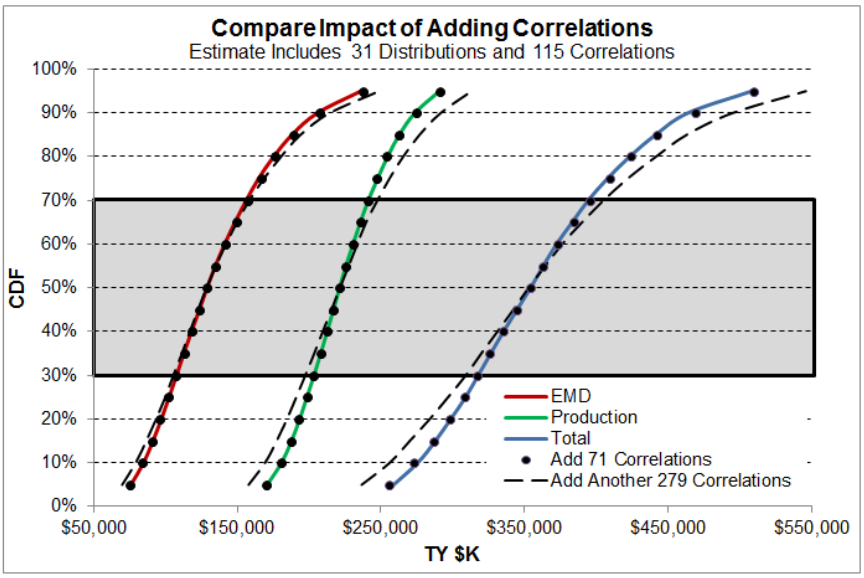


Figure 3-6 Impact of Adding Additional Correlations

Measure the correlation present in the model WBS first then determine what should be applied.
Create a few well organized, large correlation matrices rather than many small ones.
Repair inconsistent matrices rather than letting the tool attempt to find a solution.
Measure correlation again to verify expectations are met.

3.4 OTHER INFLUENCES ON SIMULATION RESULTS

3.4.1 Random Seed and Random Number Generators

The random seed is a number that initializes the selection of numbers by a random number generator. Given the same seed, a random number generator will generate the same series of random numbers each time a simulation is run. Both Crystal Ball and @RISK, by default, pick a different random seed each time the simulation runs. To avoid this, an initial random seed may be set by the user (ideally, using a seed generated by the tool). However, if the location of various assumptions is changed on the worksheet, answers will still vary. Additionally, if other workbooks are open that contain separate models, this can influence the random seed assignments. ACE assigns a random seed to every uncertainty assumption and it is saved with the model. When the assumption is moved, the random seed moves with it and, therefore, the random draw sequence is preserved.

Changing the random seed (either manually or by allowing the tool to do so) will cause the percentile results to vary on the order of 0.5%. Furthermore, it is not possible to get precise matches across tools since each uses a different random number generator and different methods for assigning random seeds.

Where possible, let the tool select a random seed initially and then fix this seed in the model.
It is important to document the random seed selected and if choices are available, the random number generator selected. To promote consistency agencies are encouraged to define a random seed and random number generator.

3.4.2 Simulation Sampling Method

Some tools allow the user to choose either Monte Carlo sampling or Latin Hypercube sampling (see **Appendix A.10** for details). Latin Hypercube draws random numbers more evenly and it will generally require fewer trials to obtain the same level of accuracy. We recommend that the number of partitions equals the number of trials when using Latin Hypercube sampling. Doing so helps ensure the entire distribution is sampled with fewer trials. @RISK and ACE do not have a user setting for the number of partitions; both fix the number of partitions to the number of trials.

3.4.3 Number of Trials (Iterations)

The number of trials required to achieve reasonable accuracy is a function of many factors including: the complexity of the functional relationships, how many distributions are defined in the model, the degree of uncertainty applied, and the number of applied correlations.

Reference 9 and **24** suggest that 10,000 trials are sufficient for cost uncertainty models and this number is a common standard throughout the cost community. Most simulation tools have a feature to stop the simulation when selected convergence criteria are met. For instance, both Crystal Ball and @RISK will test the mean, standard deviation or a selected percentile to determine when the statistic is estimated to be within a user defined percent of its actual value for a specified probability level. However there are no known standards and there is no persistent record of how the test behaved at lower or a greater number of trials.

A tool-independent Excel-based utility is available that provides a visual cue to model convergence (see **Section 7.1.4**). It begins with a 10,000 trial simulation. Then the statistics of interest (in this case the 50 percentile, 70 percentile and 90 percentile of Production) are calculated from the first 100, 200, 300 etc. trials and compared to the 10,000 trial result. The number of trials required for convergence may be visually observed from the resulting chart. A convergence criterion of 0.5% for percentiles is shown here. The concept is defined in detail in **Reference 65**. Investigation demonstrated that performing separate runs with different random seeds (rather than collecting all the data from one 10,000 trial run) made little difference to the results. The example model built in @RISK, Crystal Ball and ACE was used to demonstrate tool independence. Extracting the simulation data from each tool to the utility demonstrates similar behavior as shown in **Figure 3-7**

For the example model, 5,000 trials are required to obtain production results within 0.5% of the 10,000 trial result. However, relatively few iterations (500) still produces results within a few percent and this is more than sufficient for use while building and testing the model. Even if the model takes no time to run 10,000 trials, running the convergence utility is important to verify convergence. When CVs are very high and/or in the presence of many applied correlations, it may be necessary to run more than 10,000 trials to create the final milestone estimates.

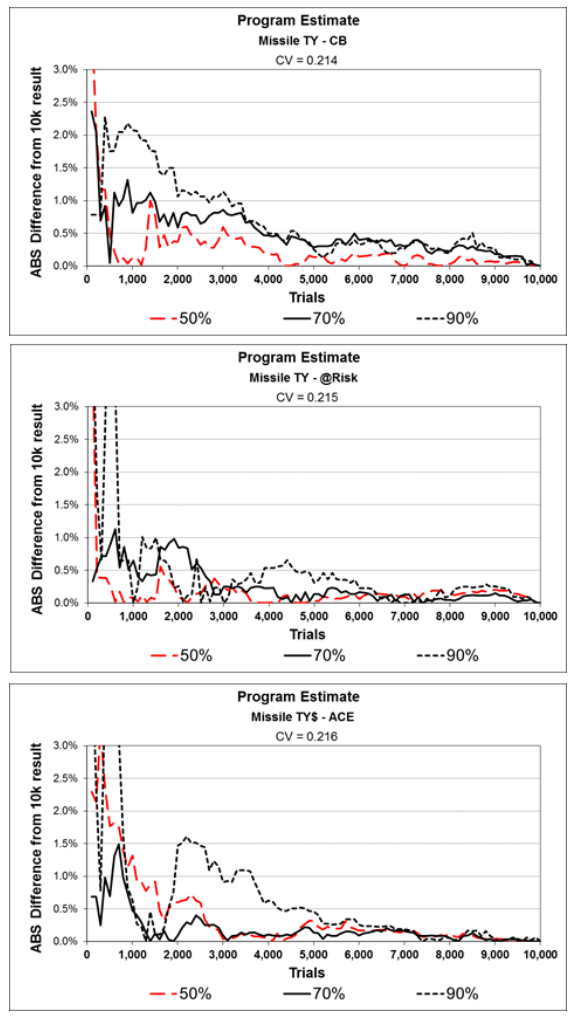


Figure 3-7 Compare Convergence Test Across Different Tools

The convergence utility will, by definition, converge at 10,000 trials. Consequently, there needs to be ample evidence of convergence by at 8-9000 trials (DoD agencies have reported that 5,000 is sufficient for many models). If not, it is recommended that the 20,000 trial version of the utility be considered. Additionally, 50/70/90 percentiles were selected because this tends to cover the range of reported results. 0.5% was selected as the threshold because **Reference 9** identified that stable percentile results may change by 0.5% by changing the random seed. There is no reference for defining standard deviation convergence, but 1.5% seems like a reasonable target as it generally takes at least a 1.5% change in standard deviation to impact percentile results by 0.5%.

While the utility gives a visual indication of convergence, it does not provide a mathematical basis for convergence confidence. Tools like Crystal Ball and @RISK provide a feature to run the simulation until predefined criteria are met on one or more of the model forecasts. The concept is to have the tool measure the chosen statistic(s) and stop the simulation when the difference is less than a specified interval at a defined confidence level. **Figure 3-8** illustrates criteria recommended by one agency: 95% confidence that the mean, standard deviation and 90 percentile are stable within 3% of their value. For the example model, both the convergence utility and the Crystal Ball precision control identified convergence near 6,450 trials (@RISK model behaved similar). If this type of feature is to be used, agencies should specify the required settings.

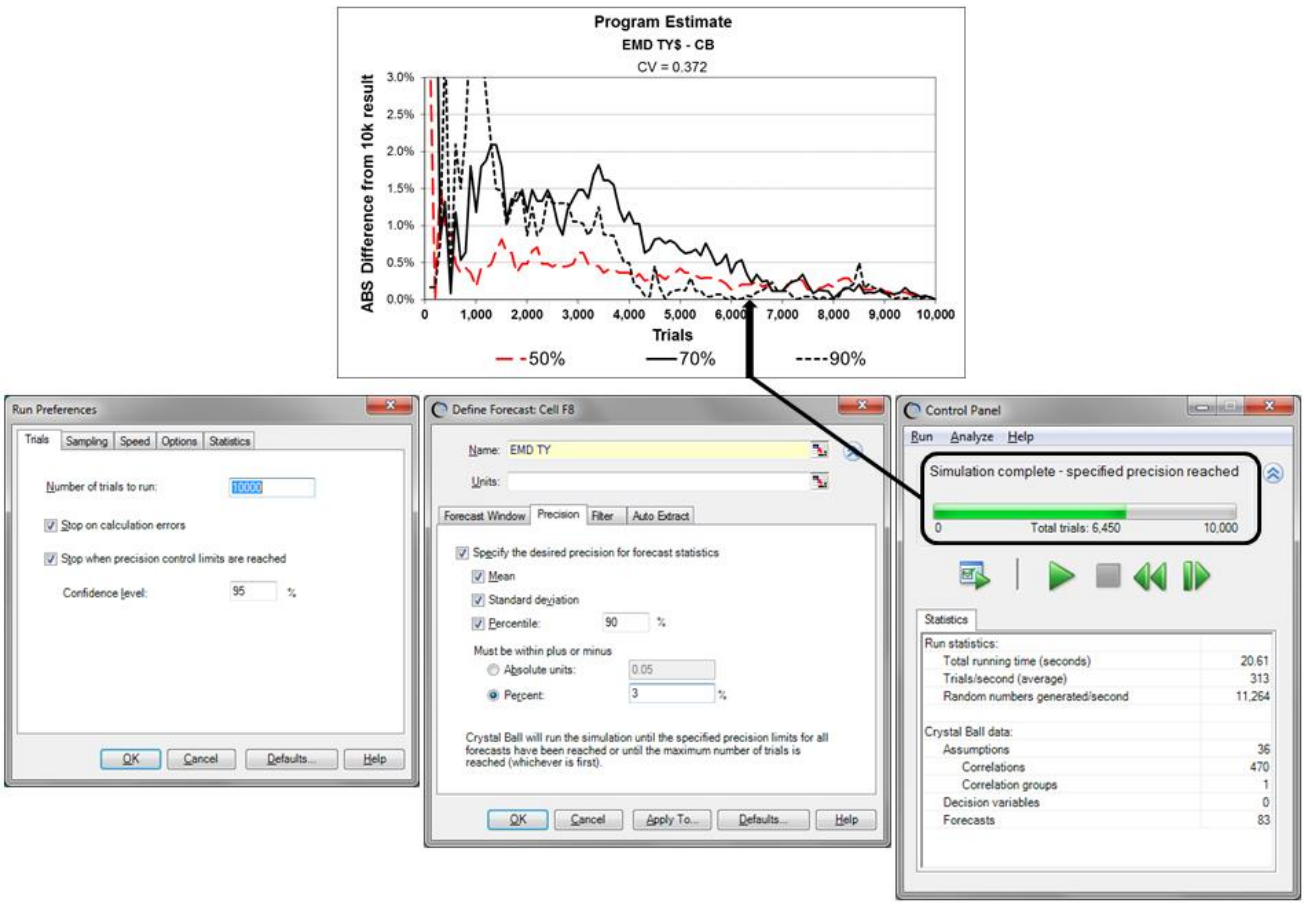


Figure 3-8 Compare Convergence Utility to Precision Control in Crystal Ball

Simulation settings such as sampling type (Monte Carlo vs. Latin Hypercube), random seed, correlation on/off and similar settings will have an impact on the simulation results.
These and similar settings may not be saved with the model file.
Agencies are encouraged to publish recommended settings for each tool that is used.
Perform a convergence analysis to verify the number of trials required to develop a stable result.

3.5 REVIEW AND INTERPRET RESULTS WITH CORRELATION APPLIED

3.5.1 Recommended Simulation Settings to Promote Consistency

Simulation tools are unique and each comes with their own various default settings and assumptions. To ensure simulation results are persistent between runs and workstations, the following default settings are recommended:

- **Sampling Method**: Latin Hypercube or Monte Carlo
- **Trials**: 10,000 trials is the industry default, but the model should be tested for convergence to verify the number of trials required to establish a stable result (see **Section 3.4.3**)
- **Latin Hypercube Partitions**: ACE and @RISK do not provide control over Latin Hypercube partitions. Both these tools automatically create the same number of Latin Hypercube partitions as trials. To facilitate comparison across tools, we recommend that as the standard.
- **Random Seed**: Every tool requires an initial random seed to begin the simulation process. Allow the tool to generate the random seeds. But when locking the estimate results is desired, select a tool-generated or a user generated value for the random seed. Unless one is selected, each simulation run will be different. For Crystal Ball and @RISK, moving cells, inserting or deleting distributions, or adding an empty correlation matrix can change the random seed assignments throughout the model. ACE assigns and retains the random seed assigned to each distribution (the user can delete them and allow ACE to repopulate the random seed assignments).
- **Stop on Convergence**: Every tool uses a different approach to assess the stability of the model. It is therefore recommended that a tool-independent utility like the one introduced in **Section 3.4.3** be deployed for use within a given agency.

If the tool's guidelines are followed, the analyst will obtain the same CISM results regardless of which tool is used. **Figure 3-9** illustrates the same results from three different tools in the form of a cumulative distribution function (CDF) for the missile model.

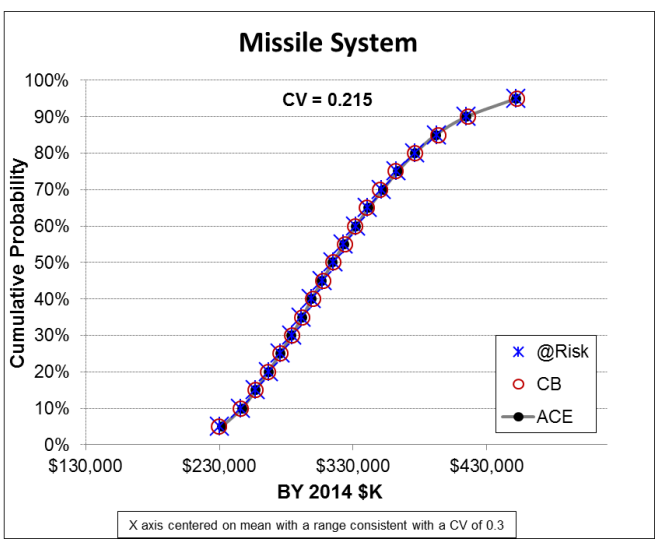


Figure 3-9 Compare Risk Analysis Results from Several Tools

3.5.2 Comparing Simulation Results

If the point estimate is found to behave appropriately, it is useful to run several simulations to get a sense of the contribution of key risk and uncertainty elements in the model. **Figure 3-10** illustrates how the S-curve gets steeper as schedule uncertainty, risk register elements, correlation, and CER uncertainty are

systematically removed from the model (the steepest curve representing uncorrelated input uncertainty). The respective CVs are: 0.216, 0.188, 0.186, 0.116, and 0.076. This type of comparison provides evidence the model is behaving as expected. **Figure 3-9** reports the CV of the Missile total to be 0.215 while **Figure 3-10** reports 0.216. This is the difference between simulation results in BY$ (**Figure 3-9**) and TY$ (**Figure 3-10**).

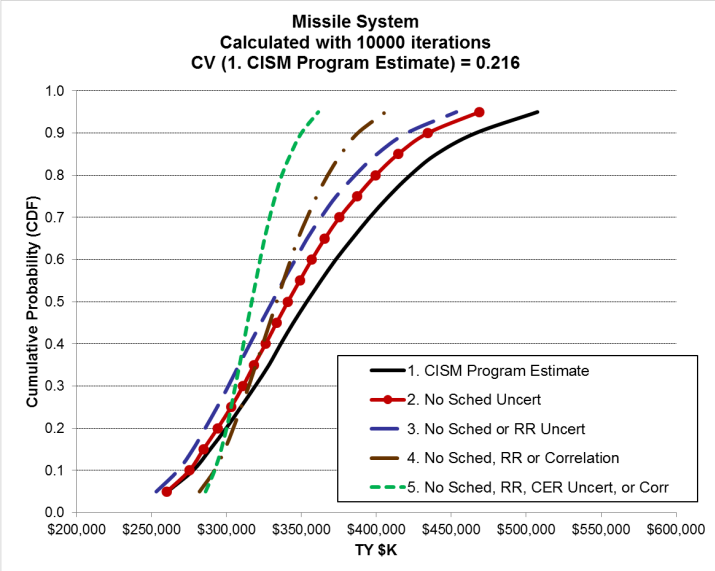


Figure 3-10 Compare Impact of Key Risk and Uncertainty Elements

Construct the model to easily switch off the schedule uncertainty, risk register, and correlation in order to investigate the impacts of these key aspects of the model.
In order to move cost and uncertainty data to a FICSM model (see **Appendix B**), it will be necessary to generate CISM results with schedule and risk register contributions removed.

3.5.3 Interpreting the Simulation Results

Building a risk and uncertainty model is an iterative process. As the process proceeds, it is useful to examine the coefficient of variation (CV) of the top-line of each phase of the estimate. Examining lower-level elements is desirable; however, the range of acceptable CVs is much broader. In general, analysts are likely to be able to compile meaningful ranges of acceptable CV for the overall estimate (by phase) by commodity.

The CV statistic is provided by all the common tools. The higher the CV, the wider the dispersion and the flatter the S-curve. A low CV is indicative of a program with low or modest risks. A high CV is indicative of a high-risk program. Often an extremely low CV is an indication of very optimistic uncertainty ranges or a lack of correlation. Likewise, extremely large CVs may be an indication of unusually broad distributions or too much correlation.

The NCCA S-curve Tool (**Reference 88**) is available to compare an estimate to historical cost growth factors (CGFs) and the CVs on the CGFs. **Table 3-3** shows sample CGFs for historical programs by Phase and Commodity developed from that source's data. These are based on comparing initial and final SARs and calculating cost growth factors (where 1.0 equals no cost growth) adjusted for quantity. The quantity adjustment utilized the Fischer method which is the square root of the product of the CGF adjusting the baseline estimate to reflect current quantities and the CGF adjusting the current estimate to reflect baseline quantities. **Table 3-4** shows the CVs on those CGFs. The analyst should examine CGFs and their CVs from these two tables and the NCCA S-curve Tool to assess the reasonableness of their estimate's CV.

Practitioners are cautioned NOT to game their model in an attempt to match any particular value in these tables because individual program conditions vary. The values are provided for cross-check guidance. A

model with different values does not mean it is wrong. But an explanation of an estimate's CV far outside the range of values on these tables is warranted. CVs for estimates at Milestone A are not tabulated due to sparse data but the source suggests CVs of 0.15 or higher than Milestone B CVs is reasonable.

Table 3-3 NCCA SAR Growth Factors: Since 1969/ Since 1980/ Since 1990 (Effective December 2011)

		Mean Cost Growth Factor, RDT&E Estimates at MS B						
		Ships/ Submarines	Aircraft	Missiles/ Guns/ Torpedoes	Space	Electronics	Vehicles	All
BY$	All	1.35 / 1.36 / 1.18	1.65 / 1.72 / 1.89	1.62 / 1.62 / 1.78	1.43 / 1.47 / 1.47	1.40 / 1.40 / 1.41	1.75 / 1.52 / 1.47	1.54 / 1.54 / 1.62
	Navy	1.35 / 1.36 / 1.18	1.99 / 2.24 / 2.63	1.23 / 1.15 / 1.21	1.17 / 1.17 / 1.17	1.45 / 1.45 / 1.38	1.89 / 1.89 / 1.89	1.55 / 1.61 / 1.71
	Air Force		1.51 / 1.44 / 1.53	1.57 / 1.19 / 1.24	1.45 / 1.53 / 1.53	1.10 / 1.10 / 1.04		1.41 / 1.30 / 1.36
	Army		1.27 / 1.32 / 1.36	1.95 / 2.12 / 2.35		1.37 / 1.37 / 1.25	1.70 / 1.40 / 1.30	1.61 / 1.62 / 1.67
	DoD					1.87 / 1.87 / 2.01		1.87 / 1.87 / 2.01
TY$	All	1.52 / 1.52 / 1.24	1.84 / 1.81 / 2.02	1.74 / 1.62 / 1.78	1.59 / 1.53 / 1.53	1.42 / 1.42 / 1.46	2.02 / 1.63 / 1.50	1.67 / 1.59 / 1.67
	Navy	1.52 / 1.52 / 1.24	2.25 / 2.44 / 2.89	1.35 / 1.16 / 1.22	1.23 / 1.23 / 1.23	1.47 / 1.47 / 1.44	2.01 / 2.01 / 2.01	1.71 / 1.72 / 1.83
	Air Force		1.69 / 1.48 / 1.58	1.80 / 1.26 / 1.32	1.63 / 1.59 / 1.59	1.10 / 1.10 / 1.04		1.56 / 1.33 / 1.41
	Army		1.33 / 1.36 / 1.40	2.02 / 2.09 / 2.29		1.38 / 1.38 / 1.29	2.02 / 1.50 / 1.29	1.69 / 1.63 / 1.67
	DoD					1.95 / 1.95 / 2.11		1.95 / 1.95 / 2.11

		Mean Cost Growth Factor, Procurement Estimates at MS B						
		Ships/ Submarines	Aircraft	Missiles/ Guns/ Torpedoes	Space	Electronics	Vehicles	All
BY$	All	1.27 / 1.30 / 1.38	1.37 / 1.36 / 1.55	1.47 / 1.33 / 1.36	1.33 / 1.42 / 1.42	0.97 / 0.97 / 1.03	1.89 / 1.46 / 1.41	1.33 / 1.24 / 1.34
	Navy	1.27 / 1.30 / 1.38	1.31 / 1.33 / 1.54	1.31 / 1.15 / 1.20	0.89 / 0.89 / 0.89	1.25 / 1.25 / 1.25	0.95 / 0.95 / 0.95	1.28 / 1.25 / 1.35
	Air Force		1.19 / 1.10 / 1.26	1.34 / 1.23 / 1.23	1.40 / 1.69 / 1.69	0.98 / 0.98 / 1.17		1.22 / 1.14 / 1.29
	Army		1.78 / 2.04 / 2.14	1.72 / 1.57 / 1.57		0.99 / 0.99 / 1.06	2.02 / 1.54 / 1.50	1.53 / 1.37 / 1.44
	DoD					0.81 / 0.81 / 0.81		0.81 / 0.81 / 0.81
TY$	All	1.47 / 1.49 / 1.78	1.55 / 1.33 / 1.51	1.89 / 1.37 / 1.38	1.48 / 1.41 / 1.41	0.98 / 0.98 / 1.06	2.38 / 1.56 / 1.48	1.57 / 1.28 / 1.38
	Navy	1.47 / 1.49 / 1.78	1.48 / 1.32 / 1.53	1.60 / 1.18 / 1.15	0.92 / 0.92 / 0.92	1.23 / 1.23 / 1.23	1.06 / 1.06 / 1.06	1.48 / 1.32 / 1.45
	Air Force		1.30 / 1.07 / 1.22	1.63 / 1.24 / 1.24	1.57 / 1.65 / 1.65	0.96 / 0.96 / 1.15		1.37 / 1.12 / 1.27
	Army		2.13 / 1.93 / 2.05	2.38 / 1.66 / 1.66		1.03 / 1.03 / 1.12	2.57 / 1.64 / 1.57	1.91 / 1.42 / 1.50
	DoD					0.80 / 0.80 / 0.80		0.80 / 0.80 / 0.80

		Mean Cost Growth Factor, RDT&E Estimates at MS C						
		Ships/ Submarines	Aircraft	Missiles/ Guns/ Torpedoes	Space	Electronics	Vehicles	All
BY$	All	1.19 / 1.20 / 1.00	1.21 / 1.21 / 1.13	1.26 / 1.27 / 1.08	1.86 / 1.94 / 1.38	1.17 / 1.19 / 1.19	1.30 / 1.30 / 1.30	1.27 / 1.28 / 1.16
	Navy	1.19 / 1.20 / 1.00	1.37 / 1.37 / 1.21	1.17 / 1.17 / 1.05	1.12 / 1.12 / 1.12	1.13 / 1.13 / 1.09	1.84 / 1.84 / 1.84	1.25 / 1.26 / 1.14
	Air Force		1.07 / 1.07 / 1.08	1.10 / 1.11 / 1.15	1.94 / 2.04 / 1.41	1.03 / 1.04 / 1.04		1.26 / 1.29 / 1.16
	Army		1.06 / 1.06 / 1.06	1.40 / 1.40 / 1.05		1.20 / 1.20 / 1.20	1.21 / 1.21 / 1.21	1.24 / 1.24 / 1.13
	DoD			2.31 / 2.31 / N/A		1.41 / 1.41 / 1.41		1.56 / 1.56 / 1.41
TY$	All	1.25 / 1.26 / 1.01	1.21 / 1.21 / 1.14	1.32 / 1.33 / 1.12	2.04 / 2.14 / 1.44	1.21 / 1.23 / 1.22	1.35 / 1.35 / 1.35	1.31 / 1.32 / 1.19
	Navy	1.25 / 1.26 / 1.01	1.36 / 1.36 / 1.23	1.10 / 1.10 / 1.06	1.13 / 1.13 / 1.13	1.18 / 1.18 / 1.13	1.91 / 1.91 / 1.91	1.26 / 1.27 / 1.16
	Air Force		1.09 / 1.09 / 1.10	1.16 / 1.18 / 1.24	2.15 / 2.26 / 1.48	1.03 / 1.04 / 1.04		1.33 / 1.36 / 1.20
	Army		1.07 / 1.07 / 1.07	1.56 / 1.56 / 1.08		1.23 / 1.23 / 1.23	1.26 / 1.26 / 1.26	1.31 / 1.31 / 1.17
	DoD			2.31 / 2.31 / N/A		1.45 / 1.45 / 1.45		1.59 / 1.59 / 1.45

		Mean Cost Growth Factor, Procurement Estimates at MS C						
		Ships/ Submarines	Aircraft	Missiles/ Guns/ Torpedoes	Space	Electronics	Vehicles	All
BY$	All	1.12 / 1.12 / 1.18	1.07 / 1.08 / 1.12	1.13 / 1.13 / 1.17	1.57 / 1.65 / 1.10	0.99 / 1.03 / 1.05	1.11 / 1.11 / 1.11	1.11 / 1.12 / 1.12
	Navy	1.12 / 1.12 / 1.18	1.09 / 1.09 / 1.06	1.12 / 1.12 / 1.08	1.03 / 1.03 / 0.94	1.06 / 1.06 / 1.12	1.36 / 1.36 / 1.36	1.10 / 1.10 / 1.11
	Air Force		1.00 / 0.99 / 1.08	1.17 / 1.20 / 1.30	1.84 / 2.06 / 1.19	0.91 / 1.08 / 1.08		1.14 / 1.19 / 1.16
	Army		1.24 / 1.42 / 1.42	1.08 / 1.08 / 1.13		0.99 / 0.99 / 1.01	1.07 / 1.07 / 1.07	1.08 / 1.09 / 1.12
	DoD					1.01 / 1.01 / 1.01		1.01 / 1.01 / 1.01
TY$	All	1.12 / 1.13 / 1.29	1.06 / 1.07 / 1.10	1.18 / 1.19 / 1.24	1.90 / 2.05 / 1.14	1.06 / 1.11 / 1.13	1.06 / 1.06 / 1.06	1.14 / 1.16 / 1.16
	Navy	1.12 / 1.13 / 1.29	1.08 / 1.08 / 1.05	1.16 / 1.16 / 1.12	1.04 / 1.04 / 0.92	1.36 / 1.36 / 1.51	1.38 / 1.38 / 1.38	1.14 / 1.15 / 1.18
	Air Force		0.99 / 0.99 / 1.08	1.25 / 1.29 / 1.41	2.33 / 2.72 / 1.25	0.89 / 1.07 / 1.07		1.22 / 1.29 / 1.19
	Army		1.19 / 1.37 / 1.37	1.14 / 1.14 / 1.21		0.95 / 0.95 / 0.96	1.01 / 1.01 / 1.01	1.06 / 1.08 / 1.10
	DoD					1.03 / 1.03 / 1.03		1.03 / 1.03 / 1.03

Table 3-4 NCCA SAR Growth Factor CVs: Since 1969/ Since 1980/ Since 1990 (Effective April 2011)

		CV on Cost Growth Factor, RDT&E Estimates at MS B						
		Ships/ Submarines	Aircraft	Missiles/ Guns/ Torpedoes	Space	Electronics	Vehicles	All
BY$	All	0.42 / 0.46 / 0.25	0.86 / 0.94 / 0.95	1.09 / 1.30 / 1.35	0.48 / 0.50 / 0.50	0.47 / 0.47 / 0.39	0.45 / 0.27 / 0.28	0.82 / 0.89 / 0.95
	Navy	0.42 / 0.46 / 0.25	1.04 / 1.09 / 1.04	0.32 / 0.23 / 0.24		0.48 / 0.48 / 0.46	0.16 / 0.16 / 0.16	0.81 / 0.89 / 0.95
	Air Force		0.52 / 0.55 / 0.57	0.54 / 0.27 / 0.27	0.49 / 0.53 / 0.53	0.40 / 0.40 / 0.21		0.51 / 0.49 / 0.49
	Army		0.35 / 0.41 / 0.44	1.34 / 1.43 / 1.44		0.48 / 0.48 / 0.25	0.52 / 0.27 / 0.26	1.04 / 1.12 / 1.23
	DoD					0.38 / 0.38 / 0.30		0.38 / 0.38 / 0.30
TY$	All	0.56 / 0.62 / 0.27	0.93 / 1.03 / 1.03	0.96 / 1.18 / 1.23	0.58 / 0.54 / 0.54	0.48 / 0.48 / 0.42	0.65 / 0.37 / 0.35	0.82 / 0.88 / 0.93
	Navy	0.56 / 0.62 / 0.27	1.08 / 1.16 / 1.10	0.42 / 0.22 / 0.23		0.49 / 0.49 / 0.49	0.17 / 0.17 / 0.17	0.88 / 0.98 / 1.03
	Air Force		0.65 / 0.60 / 0.62	0.65 / 0.40 / 0.40	0.60 / 0.57 / 0.57	0.46 / 0.46 / 0.21		0.64 / 0.54 / 0.54
	Army		0.34 / 0.41 / 0.44	1.17 / 1.31 / 1.33		0.47 / 0.47 / 0.28	0.75 / 0.43 / 0.35	0.94 / 1.01 / 1.12
	DoD					0.40 / 0.40 / 0.32		0.40 / 0.40 / 0.32

		CV on Cost Growth Factor, Procurement Estimates at MS B						
		Ships/ Submarines	Aircraft	Missiles/ Guns/ Torpedoes	Space	Electronics	Vehicles	All
BY$	All	0.36 / 0.40 / 0.45	0.44 / 0.51 / 0.45	0.37 / 0.43 / 0.43	0.55 / 0.86 / 0.86	0.41 / 0.41 / 0.25	0.66 / 0.24 / 0.25	0.59 / 0.47 / 0.44
	Navy	0.36 / 0.40 / 0.45	0.50 / 0.58 / 0.52	0.43 / 0.43 / 0.48				0.42 / 0.46 / 0.47
	Air Force		0.41 / 0.43 / 0.39	0.26 / 0.31 / 0.31	0.55 / 0.95 / 0.95	0.40 / 0.40 / 0.25		0.40 / 0.48 / 0.44
	Army		0.28 / 0.23 / 0.24	0.36 / 0.46 / 0.46		0.45 / 0.45 / 0.20	0.64 / 0.19 / 0.20	0.52 / 0.45 / 0.40
	DoD					0.32 / 0.32 / 0.32		0.32 / 0.32 / 0.32
TY$	All	0.72 / 0.77 / 0.78	0.50 / 0.52 / 0.47	0.60 / 0.44 / 0.46	0.59 / 0.99 / 0.99	0.43 / 0.43 / 0.31	0.99 / 0.28 / 0.28	0.74 / 0.55 / 0.53
	Navy	0.72 / 0.77 / 0.78	0.55 / 0.61 / 0.55	0.79 / 0.44 / 0.52				0.66 / 0.63 / 0.64
	Air Force		0.52 / 0.44 / 0.41	0.36 / 0.38 / 0.38	0.58 / 1.13 / 1.13	0.41 / 0.41 / 0.27		0.49 / 0.52 / 0.50
	Army		0.28 / 0.23 / 0.23	0.54 / 0.43 / 0.43		0.48 / 0.48 / 0.30	0.97 / 0.25 / 0.26	0.74 / 0.44 / 0.39
	DoD					0.38 / 0.38 / 0.38		0.38 / 0.38 / 0.38

		CV on Cost Growth Factor, RDT&E Estimates at MS C						
		Ships/ Submarines	Aircraft	Missiles/ Guns/ Torpedoes	Space	Electronics	Vehicles	All
BY$	All	0.42 / 0.43 / 0.11	0.56 / 0.56 / 0.22	0.43 / 0.43 / 0.16	0.89 / 0.90 / 0.36	0.19 / 0.19 / 0.19	0.22 / 0.22 / 0.22	0.63 / 0.55 / 0.23
	Navy	0.42 / 0.43 / 0.11	0.71 / 0.71 / 0.29	0.25 / 0.25 / 0.09		0.19 / 0.19 / 0.18		0.52 / 0.53 / 0.25
	Air Force		0.15 / 0.15 / 0.15	0.21 / 0.23 / 0.24	0.89 / 0.90 / 0.37	0.06 / 0.08 / 0.08		0.69 / 0.72 / 0.27
	Army		0.09 / 0.09 / 0.09	0.56 / 0.56 / 0.09		0.12 / 0.12 / 0.12	0.15 / 0.15 / 0.15	0.37 / 0.37 / 0.13
	DoD					0.24 / 0.24 / 0.24		0.30 / 0.30 / 0.24
TY$	All	0.62 / 0.63 / 0.12	0.56 / 0.56 / 0.24	0.54 / 0.54 / 0.22	1.00 / 1.01 / 0.37	0.21 / 0.22 / 0.22	0.24 / 0.24 / 0.24	0.69 / 0.63 / 0.25
	Navy	0.62 / 0.63 / 0.12	0.72 / 0.72 / 0.30	0.11 / 0.11 / 0.11		0.23 / 0.23 / 0.22		0.57 / 0.57 / 0.26
	Air Force		0.17 / 0.17 / 0.18	0.30 / 0.31 / 0.33	1.00 / 1.00 / 0.38	0.06 / 0.08 / 0.08		0.81 / 0.84 / 0.30
	Army		0.11 / 0.11 / 0.11	0.70 / 0.70 / 0.12		0.14 / 0.14 / 0.14	0.18 / 0.18 / 0.18	0.49 / 0.49 / 0.15
	DoD					0.27 / 0.27 / 0.27		0.31 / 0.31 / 0.27

		CV on Cost Growth Factor, Procurement Estimates at MS C						
		Ships/ Submarines	Aircraft	Missiles/ Guns/ Torpedoes	Space	Electronics	Vehicles	All
BY$	All	0.19 / 0.20 / 0.23	0.27 / 0.27 / 0.20	0.19 / 0.19 / 0.18	0.70 / 0.74 / 0.14	0.46 / 0.45 / 0.48	0.16 / 0.16 / 0.16	0.55 / 0.35 / 0.27
	Navy	0.19 / 0.20 / 0.23	0.23 / 0.23 / 0.06	0.15 / 0.15 / 0.18	0.13 / 0.13 / N/A	0.69 / 0.69 / 0.82		0.28 / 0.28 / 0.30
	Air Force		0.26 / 0.27 / 0.22	0.26 / 0.26 / 0.21	0.71 / 0.73 / 0.07	0.32 / 0.23 / 0.23		0.49 / 0.50 / 0.21
	Army		0.41 / 0.30 / 0.30	0.14 / 0.14 / 0.10		0.37 / 0.37 / 0.39	0.14 / 0.14 / 0.14	0.27 / 0.26 / 0.27
	DoD					0.35 / 0.35 / 0.35		0.35 / 0.35 / 0.35
TY$	All	0.36 / 0.37 / 0.42	0.32 / 0.31 / 0.20	0.24 / 0.24 / 0.24	0.97 / 0.99 / 0.18	0.69 / 0.68 / 0.73	0.17 / 0.17 / 0.17	0.64 / 0.53 / 0.39
	Navy	0.36 / 0.37 / 0.42	0.32 / 0.32 / 0.05	0.20 / 0.20 / 0.21	0.16 / 0.16 / N/A	0.95 / 0.95 / 1.09		0.45 / 0.46 / 0.51
	Air Force		0.29 / 0.30 / 0.24	0.30 / 0.30 / 0.25	0.95 / 0.93 / 0.08	0.36 / 0.29 / 0.29		0.72 / 0.73 / 0.26
	Army		0.39 / 0.25 / 0.25	0.22 / 0.22 / 0.22		0.37 / 0.37 / 0.39	0.13 / 0.13 / 0.13	0.28 / 0.27 / 0.28
	DoD					0.39 / 0.39 / 0.39		0.39 / 0.39 / 0.39

Another approach to assessing CV reasonableness is to do so in the context of the Nunn-McCurdy breach. For a detailed discussion, including how breach values may appear on a program's S-curve, see **Appendix A.1.6**.

Table 3-5 shows the CV results for the missile example. If the mean or standard deviation and the CV is unavailable, the analyst (or reviewer) can estimate the CV by calculating the percentage difference between the 80% and the 50% probability level. In the case of the missile example, only twice is the estimate for CV different than the actual CV by more than 0.03. This rule of thumb is a handy way to quickly estimate or validate CV.

Another indicator of the quality of the uncertainty assessment is the probability level of the point estimate. In **Table 3-5**, the probability level of the point estimate is reported in parentheses. The point estimate generally falls in the 10% to 30% probability range. When the point estimate probability level is very low this is often an indication that the CV may also be very low (i.e. insufficient uncertainty). When the point estimate probability level is very high, this can be an indication that the point estimate may already be padded with some amount of uncertainty.

Table 3-5 Example Model CV Results

Shaded cells mark estimated CVs that are more than 0.03 different than the actual.

WBS	Point Estimate	Mean	Std Dev	CV	80% / 50%	50.00%	80.00%
Missile System TY$	$ 276,893 (10%)	$365,333	$78,884	0.22	0.20	$353,524	$422,525
EMD TY$	$ 88,102 (11%)	$140,417	$52,654	0.38	0.34	$130,228	$174,642
Production & Deployment TY$	$ 188,792 (16%)	$224,916	$37,027	0.17	0.15	$220,985	$254,080
Missile System	$ 246,836 (10%)	$325,183	$70,260	0.22	0.20	$314,740	$376,245
Engineering and Manufacturing De	$ 83,539 (12%)	$130,683	$47,692	0.37	0.33	$121,567	$162,083
Air Vehicle	$ 14,944 (24%)	$28,615	$21,080	0.74	0.72	$22,954	$39,533
Design & Development	$ 12,000 (26%)	$24,380	$20,409	0.84	0.85	$18,752	$34,725
Prototypes	$ 2,944 (20%)	$4,235	$1,516	0.36	0.36	$3,975	$5,412
Software	$ 31,500 (33%)	$44,497	$23,620	0.53	0.52	$39,641	$60,344
System Engineering	$ 17,500 (9%)	$27,113	$8,293	0.31	0.30	$25,693	$33,350
Program Management	$ 15,000 (14%)	$20,528	$5,395	0.26	0.26	$19,672	$24,762
System Test and Evaluation	$ 1,766 (8%)	$3,654	$1,683	0.46	0.48	$3,289	$4,866
Training	$ 897 (16%)	$2,038	$1,551	0.76	0.74	$1,618	$2,822
Data	$ 1,196 (17%)	$2,714	$2,054	0.76	0.75	$2,156	$3,775
Peculiar Support Equipment	$ 736 (8%)	$1,524	$703	0.46	0.47	$1,379	$2,021
Production & Deployment	$ 163,297 (16%)	$194,499	$31,984	0.16	0.15	$191,119	$219,721
Air Vehicle	$ 104,826 (18%)	$127,291	$25,142	0.20	0.18	$124,490	$147,286
Airframe	$ 21,651 (35%)	$25,477	$7,692	0.30	0.30	$24,570	$32,055
Propulsion	$ 21,849 (32%)	$24,056	$4,541	0.19	0.16	$23,617	$27,360
Guidance	$ 27,810 (19%)	$37,322	$10,274	0.28	0.28	$35,949	$46,000
Payload	$ 17,246 (31%)	$19,607	$4,195	0.21	0.20	$19,204	$22,954
Air Vehicle IAT&C	$ 16,269 (41%)	$20,830	$11,501	0.55	0.54	$18,270	$28,199
System Engineering	$ 12,000 (40%)	$12,442	$1,363	0.11	0.11	$12,347	$13,682
Program Management	$ 10,000 (40%)	$10,369	$1,136	0.11	0.11	$10,289	$11,402
System Test and Evaluation	$ 5,000 (33%)	$5,369	$690	0.13	0.13	$5,296	$6,000
Training	$ 4,193 (8%)	$6,587	$1,972	0.30	0.29	$6,295	$8,133
Data	$ 4,193 (8%)	$6,036	$1,475	0.24	0.22	$5,855	$7,165
Peculiar Support Equipment	$ 7,634 (50%)	$7,634	$2,039	0.27	0.22	$7,635	$9,350
Initial Spares and Repair Parts	$ 15,451 (33%)	$18,771	$6,090	0.32	0.30	$17,821	$23,238

Select a benchmark CV using the NCCA S-curve Tool to assess an estimate's overall dispersion.

3.6 ALLOCATE AND TIME PHASE PROBABILITY ADJUSTMENT DOLLARS

3.6.1 What is a Probability Adjustment?

For the purposes of convenience and to serve as a reference terminology for future calculations described in this handbook, **probability adjustment (PA)** is defined as the amount of funds needed to bring the point estimate value up to a selected probability level from the simulation results. The difference between the point estimate and selected estimate, probability adjustment, is illustrated in **Figure 3-11**.

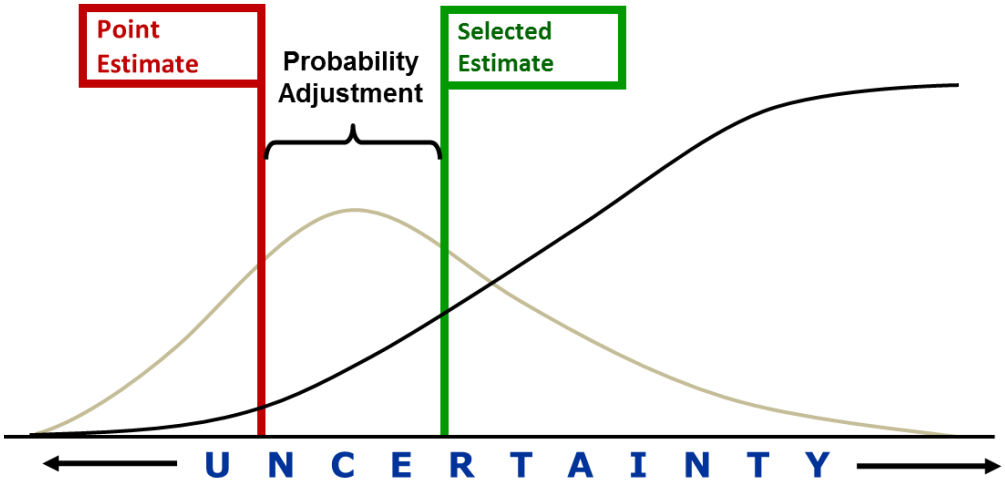


Figure 3-11 Defining "Probability Adjustment" Based on Point Estimate

To calculate PA dollars, a desired probability level or specific budget must be selected. This value may be mandated or it may be simply an organizational practice. A common practice is to select the mean. An advantage of selecting the mean is that it will sum through the WBS without the need for an allocation process. A consequence is that every element in an estimate, each alternative, and each project mean result will be at a different probability level.

3.6.2 Purpose of Allocating PA Dollars

Summing child WBS elements, all at a specific probability level, into parent elements will result in unintended probability levels at the parent levels throughout the WBS as illustrated in **Table 3-6**. This is because once costs are modeled in probabilistic terms, summing elements is a matter of summing distributions – not values. First column 1 sums the point estimate through the WBS elements. Note however each element's point estimate result is at a different probability level (the probability is shown in brackets). When summing distributions, only the means of distributions will sum to a total mean (column 2). But note the probability of each mean result in column 2. The means sum, but each element is at a different probability. Column 3 shows the result of a given probability from the simulation result (in this case the 58% EMD and 58% Production) and then the child elements are adjusted such that they sum to their parent. Note the range of probabilities across all the elements of the allocated results (column 3) is less than the range of probabilities when reporting the mean (column 2). The remaining columns in **Table 3-6** illustrate that selecting a given probability level does not address the situation. Column 4 is the 30% simulation result from every element in the WBS. In column 5 each parent row is the sum of column 4's 30% child elements and note that the resulting sum is different than the 30% parent value. For probabilities less than the mean, the sum will be less than the simulation result. Columns 6 and 7 illustrate the same situation if a probability level near the mean is chosen and columns 8 and 9 illustrate the same situation if a probability level above the mean is chosen. Near the mean, the difference gets smaller (compare the difference between columns 4 and 5 with the difference between 6 and 7). Above the mean, the sum of the children returns a result higher

than the simulation result (see columns 8 and 9). The following section explains how to achieve allocated results.

> PA dollars are the difference between the point estimate and a selected result.
> The point estimate and mean sum; percentile results must be allocated to sum.

Table 3-6 The Requirement to Allocate Illustrated

	Elements Sum			Elements Do Not Sum					
Column	1	2	3	4	5	6	7	8	9
BY $2014	Point Estimate	Mean	Allocated 58% From Level 2	All at 30%	Sum of Children	All at 60%	Sum of Children	All at 80%	Sum of Children
Missile System	$246,836 (10%)	$325,183 (56%)	$328,430 (~58%)	$283,940	$254,462	$332,166	$331,032	$376,245	$407,615
Engineering and Manufacturing De	$83,539 (12%)	$130,683 (58%)	**$130,903 (58%)**	$101,840	$92,091	$132,976	$130,521	$162,083	$172,077
Air Vehicle	$14,944 (24%)	$28,615 (64%)	$27,172 (60%)	$16,668	$16,274	$27,005	$26,869	$39,533	$40,137
Design & Development	$12,000 (26%)	$24,380 (64%)	$22,814 (61%)	$12,990		$22,517		$34,725	
Prototypes	$2,944 (20%)	$4,235 (57%)	$4,357 (60%)	$3,284		$4,352		$5,412	
Software	$31,500 (33%)	$44,497 (59%)	$45,130 (60%)	$30,275		$45,072		$60,344	
System Engineering	$17,500 (9%)	$27,113 (56%)	$27,908 (60%)	$21,911		$27,907		$33,350	
Program Management	$15,000 (14%)	$20,528 (57%)	$20,963 (60%)	$17,201		$20,978		$24,762	
System Test and Evaluation	$1,766 (8%)	$3,654 (59%)	$3,699 (60%)	$2,612		$3,704		$4,866	
Training	$897 (16%)	$2,038 (64%)	$1,917 (61%)	$1,168		$1,900		$2,822	
Data	$1,196 (17%)	$2,714 (64%)	$2,565 (60%)	$1,559		$2,545		$3,775	
Peculiar Support Equipment	$736 (8%)	$1,524 (59%)	$1,550 (60%)	$1,091		$1,546		$2,021	
Production & Deployment	$163,297 (16%)	$194,499 (54%)	**$197,527 (58%)**	$175,603	$162,371	$199,220	$200,511	$219,721	$235,538
Air Vehicle	$104,826 (18%)	$127,291 (54%)	$129,286 (57%)	$112,408	$103,893	$131,213	$131,208	$147,286	$156,568
Airframe	$21,651 (35%)	$25,477 (54%)	$26,204 (58%)	$20,683		$26,610		$32,055	
Propulsion	$21,849 (32%)	$24,056 (54%)	$24,460 (58%)	$21,596		$24,674		$27,360	
Guidance	$27,810 (19%)	$37,322 (54%)	$38,350 (58%)	$30,487		$38,936		$46,000	
Payload	$17,246 (31%)	$19,607 (54%)	$20,045 (58%)	$17,187		$20,237		$22,954	
Air Vehicle IAT&C	$16,269 (41%)	$20,830 (60%)	$20,228 (58%)	$13,940		$20,751		$28,199	
System Engineering	$12,000 (40%)	$12,442 (53%)	$12,629 (57%)	$11,645		$12,730		$13,682	
Program Management	$10,000 (40%)	$10,369 (53%)	$10,524 (57%)	$9,704		$10,609		$11,402	
System Test and Evaluation	$5,000 (33%)	$5,369 (54%)	$5,445 (57%)	$4,943		$5,498		$6,000	
Training	$4,193 (8%)	$6,587 (56%)	$6,672 (57%)	$5,359		$6,800		$8,133	
Data	$4,193 (8%)	$6,036 (55%)	$6,104 (57%)	$5,155		$6,215		$7,165	
Peculiar Support Equipment	$7,634 (50%)	$7,634 (50%)	$8,014 (57%)	$6,565		$8,151		$9,350	
Initial Spares and Repair Parts	$15,451 (33%)	$18,771 (57%)	$18,854 (57%)	$15,107		$19,300		$23,238	

3.6.3 How to Allocate PA Dollars

The first step in the PA allocation process is to determine at what level in the WBS the PA dollars are to be managed. That is, to select the level in the WBS where dollars cannot be shared with other elements at the same level (e.g., development dollars cannot be shared with production dollars and vice versa). In our example model, the appropriation level is selected. Others may choose to manage PA dollars at a lower level (e.g., at the prime contractor level in development and/or production if there are two or more).

By choosing to manage the PA dollars at the appropriation level, it means the simulation statistical results at this level will be the basis for calculating PA dollars and starting point to perform PA allocation should budgets need to be set at lower levels. Allocation is not necessary when the mean is the selected estimate.

Once the WBS levels from which the PA will be managed are determined, select the desired probability level of the simulation results. The difference between the point estimate and the simulation result at the selected probability level are the PA dollars to be allocated to lower-level elements. Put another way, the child elements need to sum to the selected probability level. While manual, ad hoc changes to child elements can achieve this purpose, it is preferred to follow a defendable and repeatable allocation scheme that allocates PA dollars to lower-level elements in a consistent manner. Methods vary from simple business rules to elaborate mathematical allocation schemes. The "Needs" (**Reference 54**) allocation approach is one such method and

it is summarized in **Appendix A.12.2**. Whichever PA allocation method is chosen, it should be consistent with the agency's objectives for the process. The following objectives were selected for the example model presented in this handbook.

- This example allocates PA dollars from the appropriation level. This means the results for the EMD and Production total will be the simulation result for the selected probability level. Allocating from the total overall would imply that EMD and Production funds can be exchanged. The goal is to allocate from the level where PA dollars are managed. In this case, that is at the appropriation totals.
- The lower-level EMD and Production results at the selected probability level will not sum. Only the mean will sum. If a probability level is selected, the lower-level elements need to be adjusted to cause them to sum.
- Some methods allocate PA dollars to the point estimate at lower levels. This method will adjust the lower-level probability results directly. For example, if the result is requested at the 85% probability level, begin with the simulation results for all elements at that probability level.
- The allocation method should be influenced by the uncertainty and correlation of lower WBS levels
- Some methods are designed to not allow any WBS element result to be less than the point estimate. This method adopts a different premise. It will treat all elements the same, meaning all elements will gravitate towards the selected probability level, even if it is lower than the point estimate. This significantly simplifies the algorithm.

A simple allocation scheme that is consistent with these objectives is to make the adjustment directly to the simulation probability results (not the point estimate). This process follows steps illustrated in **Figure 3-12**:

- Select the level in the WBS from which PA dollars will be allocated (e.g., EMD and Production)
- Generate the simulation results in BY dollars for all levels in the WBS (1)
- Sum the immediate subordinate probability results (2)
- Compute the difference between the sum of the children and the parent value (3). When performed at the level from which PA dollars are allocated, this is the percentile result minus the sum of children (1-2). At lower levels in the WBS this is the allocated result minus the sum of children (7-2). The difference represents the dollars to be "allocated" to the subordinates to cause them to sum.
- Using the standard deviation (4), prorate (6) the amount to allocate (3) to compute the adjustment for each child (6)
- Apply the adjustment (6) to the element percentile result (1) to develop the allocated result (7). Complete the allocated results by summing to the parent levels.
- Levels above the WBS chosen as fixed are merely the sum of their children. In this case the total Missile System will now be the sum of the EMD and Production statistical result, meaning it will no longer match its statistical result. And it should not be expected to match since the EMD and Production dollars have been fixed to their statistical results.

Allocation Process	Column	1	2	3	4	5	6	7	8	9
80% Allocated from EMD and Production BY 2014 $K		Percentile Results	Sum of Child Percentiles	Amount To Allocate (1-2 or 7-2)	Std Dev or Corr Adj Std Dev	Stdev/ Sum(Stdev)	Child Adjustment (3 * 5)	Allocated Result (1 + 6)	Point Estimate	PA Dollars (7-8)
Missile System	MissileSys	$376,127						$381,908	$246,836	$135,071
Engineering and Manufacturing Dev	EMD	**$163,168**	$170,462	-$7,294				$163,168	$83,539	$79,629
Air Vehicle	AV_EMD	$39,012	**$39,507**	**-$2,831**	20,400	0.3202	-$2,335.92	**$36,676**	$14,944	$21,732
Design & Development	DesignDev_EMD	**$34,083**			**19,708**	**0.9288**	**-$2,629.11**	$31,454	$12,000	$19,454
Prototypes	Proto_EMD	**$5,424**			**1,510**	**0.0712**	**-$201.43**	$5,222	$2,944	$2,278
Software	SW_EMD	$60,064			23,706	0.3721	-$2,714.50	$57,349	$31,500	$25,849
System Engineering	SysEng_EMD	$33,317			8,332	0.1308	-$954.08	$32,363	$17,500	$14,863
Program Management	PM_EMD	$24,677			5,433	0.0853	-$622.16	$24,055	$15,000	$9,055
System Test and Evaluation	STE_EMD	$4,852			1,686	0.0265	-$193.04	$4,659	$1,767	$2,892
Training	Trg_EMD	$2,794			1,472	0.0231	-$168.57	$2,626	$897	$1,729
Data	Data_EMD	$3,723			1,978	0.0310	-$226.46	$3,496	$1,196	$2,301
Peculiar Support Equipment	PSE_EMD	$2,023			695	0.0109	-$79.62	$1,944	$736	$1,207
Production & Deployment	Production	**$218,740**	$225,199	-$6,460				$218,740	$163,297	$55,442
Air Vehicle	ProdAV	$146,105	**$155,931**	**-$13,887**	24,904	0.6286	-$4,060.57	**$142,044**	$104,826	$37,219
Airframe	Airframe	**$31,853**			**7,598**	**0.2004**	**-$2,783.44**	$29,070	$21,651	$7,419
Propulsion	Propulsion	**$27,414**			**4,549**	**0.1200**	**-$1,666.23**	$25,748	$21,849	$3,898
Guidance	Guidance	**$45,416**			**10,046**	**0.2650**	**-$3,680.17**	$41,736	$27,810	$13,925
Payload	Payload	**$23,037**			**4,272**	**0.1127**	**-$1,564.83**	$21,472	$17,246	$4,226
Air Vehicle IAT&C	IAT&C	**$28,211**			**11,445**	**0.3019**	**-$4,192.44**	$24,019	$16,269	$7,750
System Engineering	SysEng	$13,682			1,363	0.0344	-$222.24	$13,459	$12,000	$1,459
Program Management	PM	$11,401			1,136	0.0287	-$185.20	$11,216	$10,000	$1,216
System Test and Evaluation	STE	$6,000			689	0.0174	-$112.41	$5,887	$5,000	$887
Training	Trg	$8,160			1,990	0.0502	-$324.41	$7,835	$4,193	$3,642
Data	Data	$7,176			1,466	0.0370	-$239.06	$6,937	$4,193	$2,744
Peculiar Support Equipment	PSE	$9,351			2,041	0.0515	-$332.78	$9,018	$7,634	$1,384
Initial Spares and Repair Parts	InitalSpares	$23,325			6,030	0.1522	-$983.24	$22,342	$15,451	$6,891

Figure 3-12 A PA Dollar Allocation Process

3.6.4 How to Time Phase Allocated PA Dollars

Once the PA dollars have been calculated and allocated, the next step is to determine how the PA dollars should be time-phased. A common approach is to layer the PA dollars on top of the phased point estimate results (see **Reference 84**). Having calculated PA dollars for each WBS element and chosen that they can be phased over the point estimate schedule, there are several approaches to phase the PA dollars.

- **Algorithm at Lowest Levels**: The analyst may consider developing phasing methods at the lowest level that are influenced by the probability level requested. Specifically, the method can cause the schedule to contract or expand with the probability level requested.
- **Specific time**: The analyst may consider time-phasing the PA dollars after a specific "risky" event. An example might be after the critical design review or the first flight test of a missile system. If a program is underway and the budget in the near-term years is inflexible, then the analyst can append the dollars to years beyond the current time-phased point estimate. This is particularly well-suited for situations where the uncertainty issues, when manifest, will require additional schedule (i.e. longer program duration) to address.
- **Backload**: If the near-term budget is set or there is little chance of consuming PA dollars early in the project, it is advisable to backload PA dollars into the later years of the phase
- **Frontload**: When it is apparent that the greatest uncertainty is early in the project, it is advisable to frontload the PA dollars
- **Prorate**: The analyst needs to make an effort to identify when the uncertainty will occur and choose one of the previous methods. When there is no evidence to do otherwise, prorating PA dollars across the point estimate phased result is recommended. Proration is a common approach for the Production estimate.

It is often inappropriate to spread the PA dollars across the PE schedule. For example, the mean cost may not be executable over the PE schedule. Where possible, consideration should be made to phase the PA

dollars over a commensurate schedule (for instance, phase mean dollars over the mean schedule). If costs are to be phased over shorter than appropriate schedules, a cost penalty factor should be considered.

Table 3-7 illustrates the a two-step process to arrive at PA allocated TY phased results. In this case, the PA BY dollars are prorated over the point estimate schedule. The BY to TY factor is applied by year to arrive at the allocated TY phased result.

Table 3-7 PA Dollars Phased over the Point Estimate Schedule

80% BY 2014 $K Allocated from Second Level. PA dollars Phased Across the Point Estimate Schedule	Total BY 2014	2014	2015	2016	2017	2018	2019	2020	2021	2022
Missile System	$381,908	$26,310	$31,589	$32,793	$35,608	$31,589	$54,007	$58,904	$56,274	$54,833
Engineering and Manufacturing Development	$163,168	$26,310	$31,589	$32,793	$35,608	$31,589	$5,279			
Production & Deployment	$218,740						$48,728	$58,904	$56,274	$54,833

BY to TY Factors RDTEN		1.01414	1.03225	1.05069	1.06954	1.08875	1.10830	1.12821	1.14847	1.16910
BY to TY Factors WPN		1.02922	1.04775	1.06661	1.08581	1.10535	1.12525	1.14550	1.16612	1.18711

80% TY $K Allocated from Second Level. PA dollars Phased Across the Point Estimate Schedule	Total TY	2014	2015	2016	2017	2018	2019	2020	2021	2022
Missile System	$425,094	$26,682	$32,608	$34,455	$38,084	$34,393	$60,682	$67,475	$65,622	$65,093
Engineering and Manufacturing Development	$172,072	$26,682	$32,608	$34,455	$38,084	$34,393	$5,851			
Production & Deployment	$253,022						$54,831	$67,475	$65,622	$65,093

3.6.5 Key Modeling Considerations When Time Phasing PA Dollars

A first step is to determine which elements of the model will be subject to schedule uncertainty. The next decision is to determine how (or if) elements are linked. Our example model includes two overarching elements: EMD and Production. Should they both be subject to schedule uncertainty? Should Production start be linked to EMD end? These are key modeling considerations. Once a result is obtained, the analyst has to decide how the PA dollars should be spread, and over what schedule in order to develop a budget estimate. There are many possible combinations, three are described here and summarized in **Table 3-8**:

1. **EMD Duration Uncertainty, Prorate over PE_S only**
   - **Simulation**: RDTE and Production schedules are disconnected in the simulation model. RDTE slips do not influence start of Production in the simulation. No attempt to influence Production rate or duration through risk or uncertainty. (Our example model is built this way.)
   - **Budget Estimate**: RDTE PA dollars are spread across the RDTE PE_S. Production dollars are spread across Production PE_S.
2. **EMD Duration Uncertainty, Influence Prod Start, Prod Duration Uncertainty**
   - **Simulation**: RDTE and Production schedules are connected in the simulation model. RDTE slips do influence the start of Production in the simulation. Production duration and production rates are influenced by risk and uncertainty.
   - **Budget Estimate**: RDTE PA dollars are spread across the RDTE mean schedule. Production starts in the same year EMD ends. Production PA dollars are spread across its mean schedule.
3. **Full Cost/Schedule Integration**: This is the same as the second option, except care is taken to separate time dependent and time independent costs. This is only possible in a full FICSM model (see **Appendix B)**.

Table 3-8 Summary of Some Simulation and PA Dollar Phasing Options

		Simulation Influenced by Phase duration			Phasing of PA Dollars Prorated over ? Schedule			
	Approach	**EMD**	**EMD Link to Prod**	**Prod**	**EMD**	**Prod Start**	**Prod**	**Type of Model**
1	EMD Duration Uncertainty, Prorate over PEs only	Yes	No	No	PEs	PEs	PEs	CISM
2	EMD Dur Uncertainty, Influence Prod Start, Prod Dur uncertainty	Yes	Yes	Yes	Mean	End EMD	Mean	CISM
3	Fully Integrated Cost Schedule Method	Yes	Yes	Yes	Integrated result			FICSM

For this handbook, approach 1 is suggested as the preferred way to fully simulate the program, particularly for early programs where the EMD start is far in the future. The example CISM model models EMD start and EMD duration uncertainty. It has a fixed start and finish date for production, and production start is not linked to EMD end. However, the model was designed with a switch to allow Production start to be linked with EMD end so that when the simulation draws long EMD durations, Production starts later. It would also be simple enough to design the model such that annual buy quantities have an uncertain production rate and thus introduce duration uncertainty into production phase. While CISM models based on approach 1 may be appropriate for developing budget estimates, approach 2 or variations on it should be explored (and as appropriate presented to decision makers) to gain a sense of how duration uncertainty may affect both the total cost and the overall schedule. Depending on the history, situation, and practical realities for the project, a defendable case can likely be made for variations on any of these approaches.

4.0 HOW TO PRESENT THE CISM RISK AND UNCERTAINTY STORY

This section presents example charts and tables to help explain the risk and uncertainty analysis to two different groups: technical reviews with engineers and other cost analysts; and management reviews with program staff and decision makers. Each type of review requires a different level of detail to present the risk and uncertainty analysis and the impact of the analysis on cost estimating results. Technical reviews focus on a presentation of the data, the analysis methods, the estimate drivers, and the quality of the results. Management reviews include less technical detail, and focus more on the cost results as modified with risk and uncertainty.

The charts and tables shown below are representative examples only. The evolving needs of each project and each organization will dictate specific chart requirements.

4.1 TECHNICAL REVIEW

4.1.1 Overview

Technical reviews are a process of subjecting an estimate to the scrutiny of others. Technical reviews are used to communicate status, identify estimate weaknesses, suggest alternative approaches, and coordinate activities within multi-disciplinary or interagency estimating teams. Reviews may also allow peers from outside the project to bring objectivity and a fresh viewpoint to the estimate. The detailed story told by the charts described in this section enables consensus-building within the estimating team which then forms the basis for the simple, yet complete overview prepared for decision makers. Reporting to colleagues or technical management allows the analyst to choose from a wide variety of the analyst's favorite and most technically compelling charts and tables. As a minimum, the technical presentation should provide an explanation of the risk analysis as described on **Table 4-1**.

Table 4-1 Technical Review Content and Explanation

CONTENT	EXAMPLE CHART
Overall Review of the Estimate (Impact on Total Cost or Cost by Phase)	S-curve showing multiple curves to convey impact of key uncertainty elements (e.g., **Figure 4-1**) Scatter plot of cost vs. schedule to convey dispersion breadth (e.g., **Figure 4-2**)
Uncertainty Drivers • By WBS Element • By Input Variable or CER	Pareto chart to identify the most expensive lowest-level WBS elements (e.g., **Figure 4-3**) Tornado chart to identify the uncertain variables that most influence the phase total (e.g., **Figure 4-5**) Distributions used in the estimate and their parameters (e.g., **Table 2-10**)
Sensitivity Analysis	Sensitivity charts to identify the elements that contribute the most to phase (e.g., Development, Production or Operating and Support) uncertainty (e.g., **Figure 4-6**)

4.1.2 S-curve and Scatter Plots

The S-curve plot shown in **Figure 4-1** (a repeat of **Figure 3-10**) provides an overview of the impact of broad categories of uncertainty elements. This can form the starting point for discussing the structure of the model and its behavior. Specifically, charts that follow should defend the modeling techniques, CERs, duration impacts, uncertainty, and correlation choices.

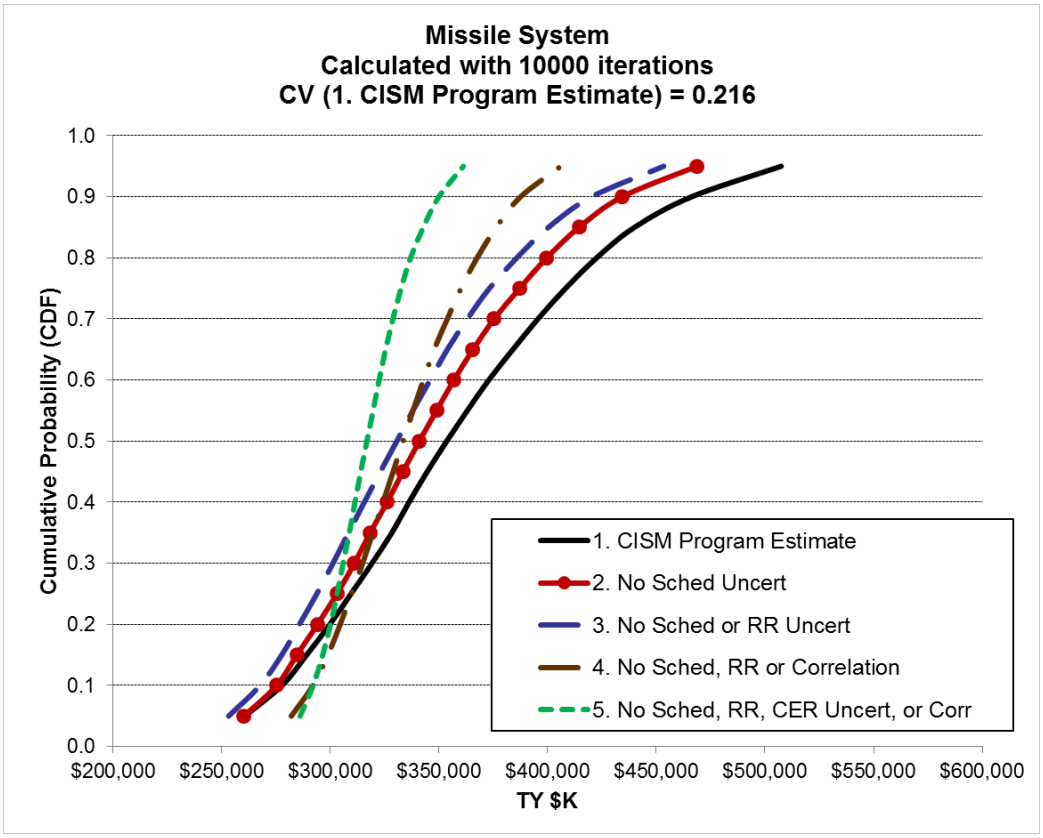


Figure 4-1 Compare Impact of Key Risk and Uncertainty Elements

Another useful chart to support an overview discussion of the uncertainty in a model is the scatter plot to provide a visual reference for the dispersion of the results from multiple simulation runs. For example: **Figure 4-2** illustrates two scatter plots where each draw's EMD cost is plotted against EMD end date (on the left panel) and also EMD duration (on the right panel) in months. The CV for date is remarkably low because a date is based upon a number like 43,909 (19 March 2020, the mean date). Note: Even a tiny CV of 0.011 represents a standard deviation of 483 days. The right image is the same chart with EMD duration expressed in months with a CV of 0.207 which represents a standard deviation of almost fifteen months. A utility to create these charts is provided with this handbook. See **Section 7.1.5** for details.

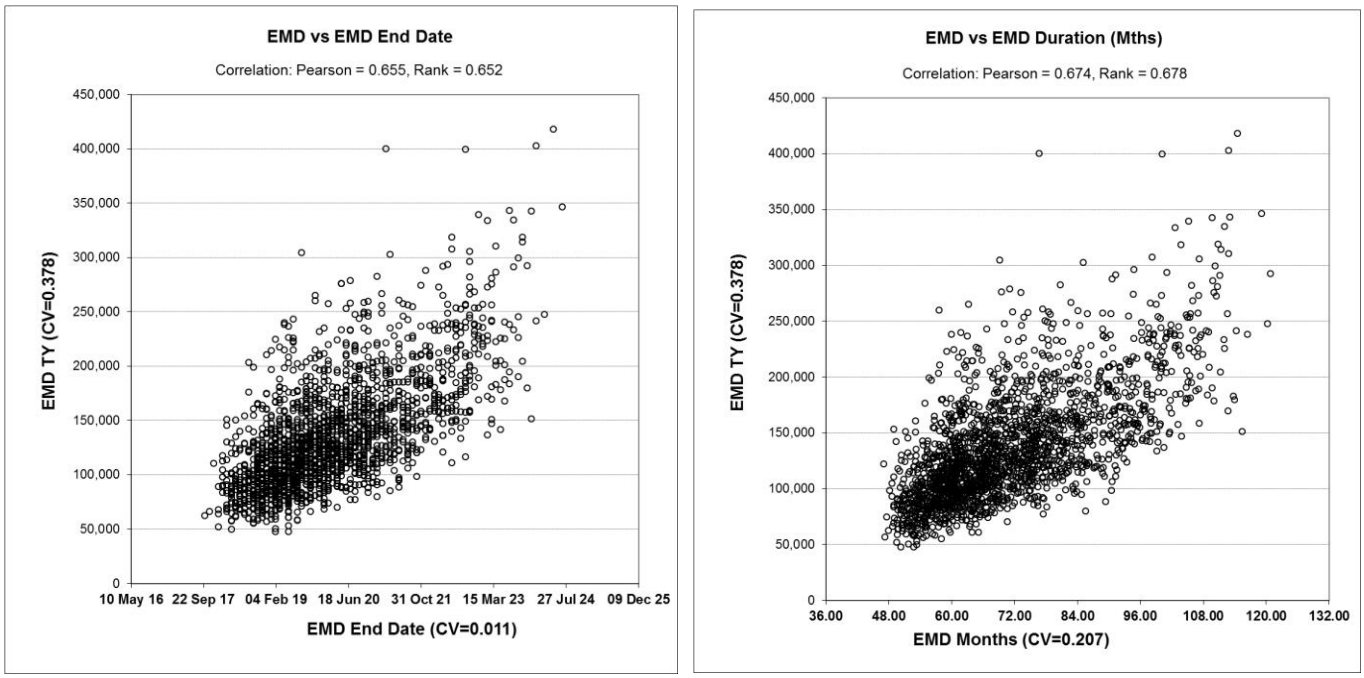


Figure 4-2 Scatter Plot Examples

4.1.3 Driver Analysis Overview

There are several reporting tools available to help identify cost and schedule drivers. There are various options for running these tools and the most fundamental is which estimate should be the subject of the analysis. Too often driver analysis is performed on the point estimate in constant year dollars because that is what most tools will do by default. The analyst is encouraged to investigate both BY and TY driver analysis.

The following tools will be discussed:

- **Pareto**: identifies those WBS elements that contribute the most to the target WBS total
- **Tornado**: identifies the uncertain variables that most influence the target WBS **total**
- **Sensitivity**: identifies the elements that contribute the most to the target row **uncertainty.** The sensitivity analysis can focus on finding the child WBS elements (**contributors**) that contribute the most to the total uncertainty or finding the estimating method inputs (**drivers**) that contribute most to the total uncertainty.

4.1.4 Pareto Chart

The Pareto chart identifies the most expensive lowest level elements in the estimate WBS to convey specific activities, resource or program functions that drive overall program cost. An example Pareto based upon a point estimate in BY dollars and the mean estimate in TY dollars is illustrated in **Figure 4-3**. Note: As can be seen in this figure, the chart will yield a different message depending on the estimate under scrutiny. The

TY Pareto captures the impact of inflation based on how the estimate is phased. The analyst is encouraged to consider both results to assess the impact of how the dollars are phased influences the results.

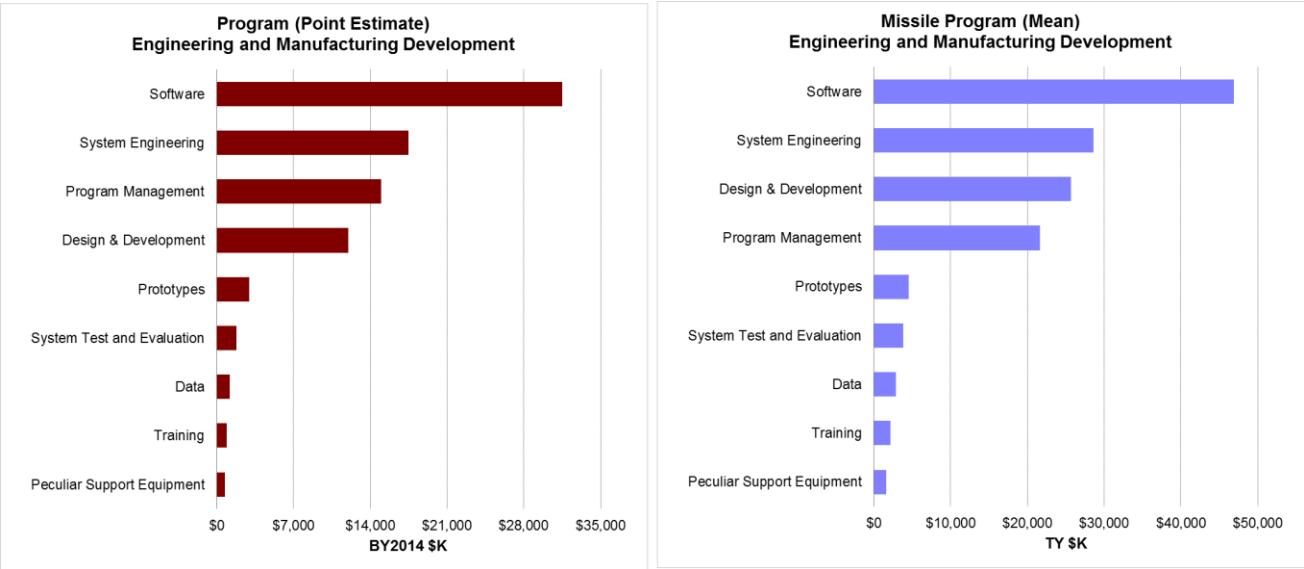


Figure 4-3 Pareto Charts of the BY Point Estimate and TY Mean

A typical way to speak to **Figure 4-3** is:

- Since the budget is based upon the mean result in TY dollars, a Pareto chart was developed based upon the mean TY dollar result.
- Software is by far the most expensive single element in the EMD phase.
- The top four items (Software, System Engineering, Design and Development, and Program Management) account for the vast majority of the EMD cost. These are the elements that should be the focus of any cost model refinement or risk mitigation plans.

Figure 4-4 is an example of a variation on the Pareto chart. The concept was obtained from the 2010 Army Cost Analysis Handbook (**Reference 69**). The chart provides insight on how the probability selected for the estimate will influence the importance associated with subordinate WBS elements. Note that the rank order below EMD Design changes when the estimate is considered in TY$. For instance, EMD System Engineering falls from fourth in the BY chart to sixth in the TY chart. For this reason, it is recommended that these charts be considered in TY$. It is further recommended that basing this chart on TY allocated results should be considered if something other than prorate is used to phase the PA dollars.

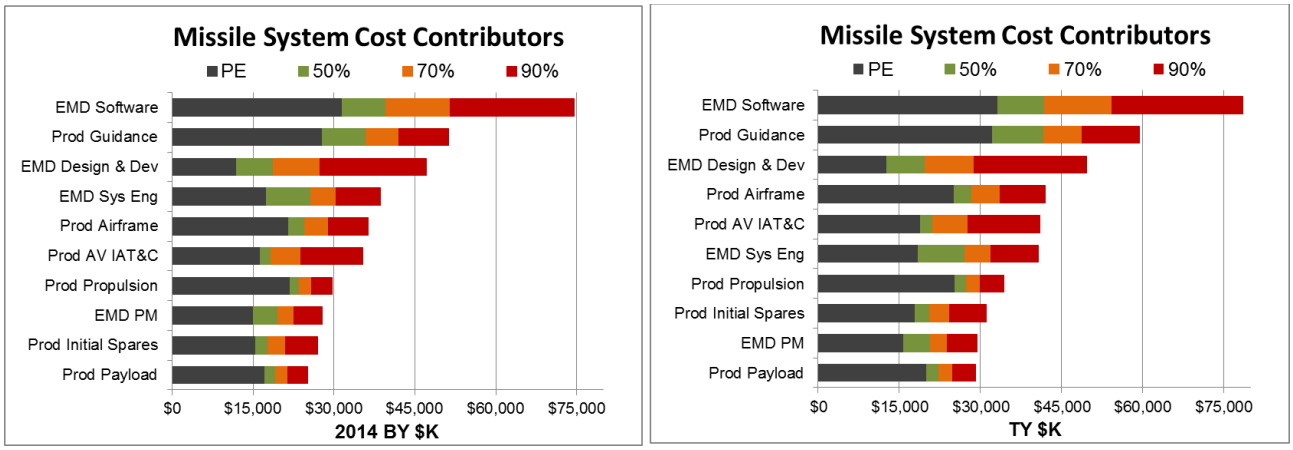


Figure 4-4 Stacked Pareto Charts Based on BY and TY Simulation Statistical Results

A typical way to speak to **Figure 4-4** would be:

- This chart (the TY version) reports the statistical results (not the allocated results) for the top contributors to the total missile system cost[8]
- The colors on the bars give a sense of how many dollars are required to move from one probability level to the next
- The WBS elements in **Figure 4-4** are sorted on the 90% probability result. By doing so, the impact of low probability risk register items is captured.
- If chart was sorted on the point estimate or 50% results (rather than the 90%), EMD Design and Development falls from third to tenth. By sorting on a high percentile, the chart reveals those items that warrant close attention in order to avoid potentially serious cost overruns.

4.1.5 Tornado Chart

Tornado charts rank the uncertainty distributions that have the most influence on a specific element's point estimate result. The Tornado chart illustrates the swing between the maximum and minimum forecast values for each variable. Input distributions that have the largest impact on the distribution of the selected output will generate the longest bars in the graph. The variable that causes the largest swing is displayed at the top and the variable that causes the smallest swing is displayed at the bottom. This chart helps identify the distributions that have the most impact on the total. However, the analysis is performed one element at a time. For instance, when payload weight is tested, other weights (like airframe or propulsion) are not changed to be consistent. For this reason, the Tornado chart alone is insufficient to find uncertainty drivers.

Crystal Ball begins by finding all the distributions defined in the model. @RISK and ACE begin with just those distributions that are functionally related to the element under analysis (target). Crystal Ball and @RISK begin the Tornado analysis process by determining the 10 and 90 percentile of each distribution to be evaluated. ACE takes it a step further by obtaining the 10 and 90 percentile of each element that is functionally related to the target from the simulation. This means the combined applied and inherited uncertainty is considered when defining the 10 and 90 percentile bounds for the analysis. So in Crystal Ball and @RISK, the most important distributions are found. In ACE, the most important variables are found.

The Tornado analysis generates two point estimates for each driver it finds, one using the variable at the low end and another at the high end of the specified range (by default, the 10 and 90 percentile). All other variables are held constant. The idea is to find the single variable that has the most influence on the total. **Figure 4-5** illustrates the Tornado chart developed in BY and TY dollars. In this model, the rank order does not change. But that is not always the case. Since budgets are reported in TY$, it is recommended that Tornado analysis be performed on TY results using the 10/90 uncertainty range.

[8] It is more appropriate to build this chart on a specific phase like development, production or operations and support. The example is built against the total system cost for illustrative purposes only.

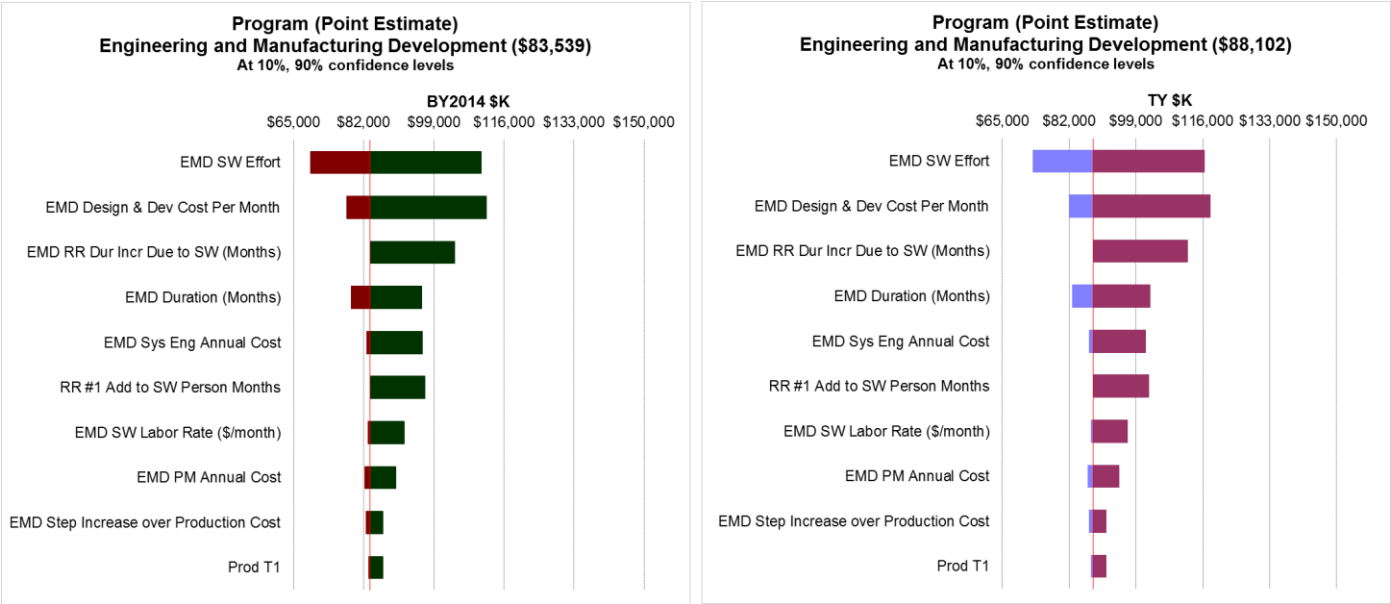


Figure 4-5 Compare Tornado Performed on PE BY and PE TY

A typical way to speak to a Tornado chart is:

- This chart identifies the uncertain variables in the model that have the most influence on the total cost
- Uncertainty distributions associated with CERs are filtered out of this analysis in order to focus on finding the cost drivers (input variables) that have the most impact on the total cost, in this case EMD
- The analysis is performed on the TY point estimate results in order to ensure any cost impact due to phasing is captured
- The analysis calculates the EMD TY point estimate for each variable at the 10 percentile and 90 percentile values[9]. The range of the TY point estimate is the basis for the way the results are sorted. Hence, the variable that has the most impact is listed at the top of the chart. Lesser impacts are plotted in order below giving the chart a "tornado" appearance.
- The Tornado analysis is driven by modeled uncertainty and does give a sense of which drivers have the most influence on total uncertainty. A sensitivity analysis will support (or not) these findings.

4.1.6 Sensitivity Chart

The objective of a Sensitivity Analysis is to find the most important contributors to the target element uncertainty. Use these charts to determine which elements in the model warrant detailed discussion. This may or may not report the same elements as the Tornado. There are two types of uncertainty elements decision makers may be interested in:

- **Cost Contributor Uncertainty**: Find WBS elements that contribute the most to total uncertainty. There is a closed form solution to calculate the contribution of each WBS element uncertainty to the total. It is based on the fact that the sum of the variances, adjusted for the Pearson Product Moment correlation between them, will sum to the simulation total variance in a WBS. Consequently, it is possible to calculate the correlation adjusted (combined functional plus applied) contribution by element and thus directly calculate each WBS element's contribution to the total. ACEIT provides this chart, the other tools do not, but it is simple to create.

[9] Note that for Crystal Ball and @RISK, this is the 10/90 percentile of the uncertainty distribution associated with the variable. For ACE, it is the 10/90 percentile that is the result from all distributions and any inherited uncertainty associated with the variable.

- **Cost Driver Uncertainty**: Find input variables that contribute the most to total uncertainty. Every tool provides this feature and they all approach the problem differently. That means there will be no way to obtain consistent results across tools without specifying a specific process. Crystal Ball, @RISK and ACE all have the ability to perform the analysis by measuring the correlation between defined distributions and the target total. @RISK version 6 introduced a new method that is their default and ACE also provides an optional alternative approach.

This discussion will focus on the Cost Driver Uncertainty. For consistency across tools, it is recommended that the rank correlation view is selected. Cost driver distributions that correlate most with the target are deemed to be the most important. But take care to look for distributions that have nothing to do with the target appearing on the chart if they happen to be highly correlated with distributions that are important. Crystal Ball and @RISK will alert the user when correlated distributions are displayed on the report rendering the results potentially misleading (see footnote on the left chart in **Figure 4-6**). A workaround is to run the analysis with applied correlation disabled to see if the results change.

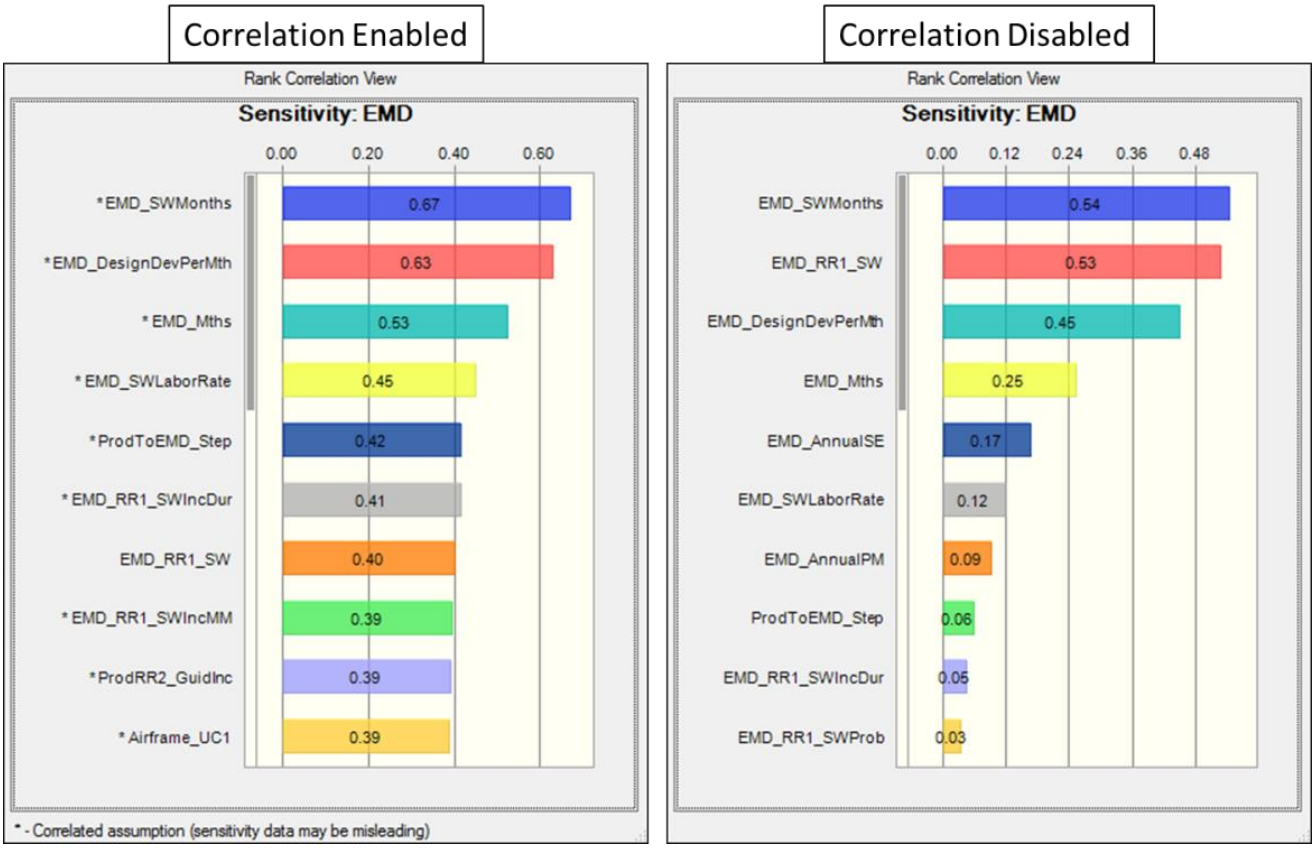


Figure 4-6 CB Sensitivity Result with Correlation Enabled and Disabled

Indeed the results are quite different when applied correlation is disabled. For instance, the correlation of Airframe_UC1 to the EMD uncertainty changes from 0.39 to less than 0.03 since it does not appear on the correlation disabled chart. Also, note that the scale on the horizontal axis changed, a consideration when comparing charts like this, especially if the tool does not permit the user to correct the x-axis scale.

ACE provides an option to engage a method (see **Reference 40**) that will account for applied correlation. **Figure 4-7** illustrates that impact on the Sensitivity analysis.

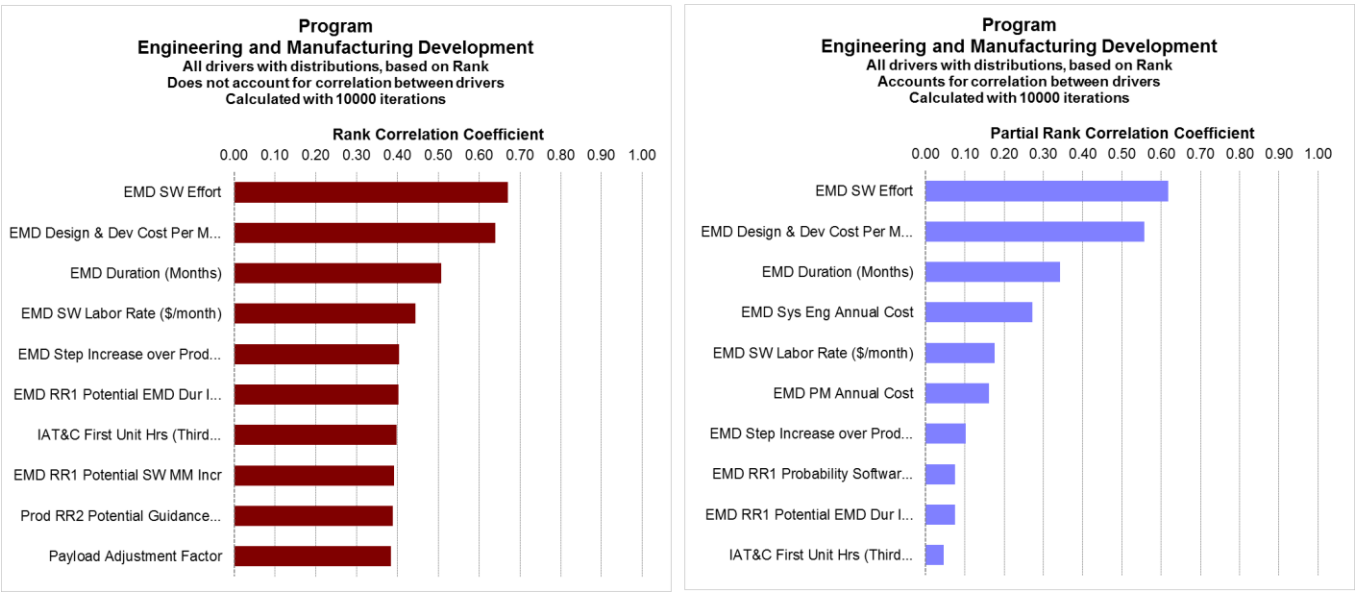


Figure 4-7 ACE Sensitivity with Correlation Ignored and Correlation Accounted for

@RISK 6.0 introduces another method for finding those distributions that have the most impact on a particular element. The process is summarized as follows:

- Collect the trial data for the target element and all of the related distributions defined in the model
  - Distributions associated with cells that do not link (via the model's formulas) to the target cell are removed from the Sensitivity analysis to avoid spurious results (this feature can be disabled if necessary).
- Sort the trial data by the first variable and divide into bins
- Calculate the statistic of interest (such as the mean or a percentile) for each of the bins
- The highest and lowest value calculated for any bin is stored and compared to the results of the same process for every other distribution considered in the analysis

The user is permitted to change the number of bins used to perform the analysis. Doing so will change the results considerably and, the smaller the trial-to-bin ratio, the more unstable the results. If this method is to be used, agencies are highly encouraged to promote use of a consistent number of trials and bins, for instance 10,000 trials and 10 bins. However, 1,000 trials per bin may still yield unstable results. Consulting a convergence test result (**Section 3.4.3**) may help with determining the number of trials per bin that are required for stable results.

The sensitivity report remains somewhat controversial because there are so many variations and each have their own particular shortcomings. Typically, however, most analysts continue to use whatever the tool default happens to be. As of this writing, Crystal Ball uses correlation to identify uncertainty drivers and reports when applied correlation is encountered by adding a warning note to the chart (see **Figure 4-6**); ACE uses correlation but includes a method to adjust for applied correlation; and @RISK 6.0 has a completely different approach. If analysts are going to use tool sensitivity reports as the basis for identifying drivers that have the most influence on total uncertainty, it is imperative that they understand the method the tool is using including any shortfalls and build speaking points accordingly.

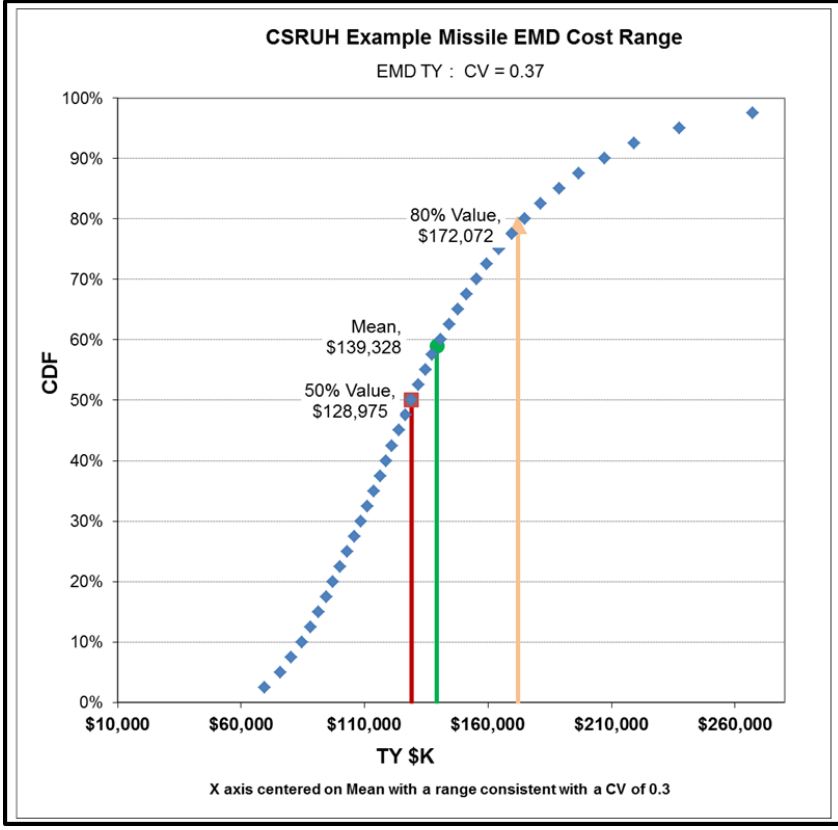
Technical Review Charts:
Use multiple methods (Pareto, Tornado, Sensitivity, etc.) to identify cost, duration and uncertainty drivers. This is not an all-encompassing list of useful charts.

4.2 REPORTING TO DECISION MAKERS

Reporting to senior leadership does not typically require the type of detailed charts shown to colleagues or technical management. Presenting the risk story to senior leadership or to a review agency requires presentation of the S-curve as shown in **Figure 4-8**. A utility to generate this figure is available with this handbook. See **Section 7.1.6** for details.

In the upper left is the S-curve with markers for individual points of interest such as the mean, the 80% or a particular scenario. Agencies may choose other estimates to include on the S-curve such as budgets, CAPE estimates, and high and low scenarios. If doing so, be aware that outside estimates placed onto your curve will likely convey a different probability level than that of the estimate's proponent. To the right of that is a decile table showing each 10% increment of probability and its corresponding value. Markers may be optionally used to show other estimates on the decile chart. Directly below the S-curve is the CV to convey the estimate's dispersion. At the bottom right is a list of the major drivers of risk in the model. Optionally, parameters for the low and high scenario may be shown. This is to give context to the overall curve in words and parameters that the decision maker can understand. This chart must be repeated for each phase or appropriation on which uncertainty analysis was performed. For an on-going program this S-curve is the to-go cost, but the sunk cost needs to be shown on the chart. Optionally, a second S-curve containing both sunk and to-go cost may be presented in which case both sunk cost and to-go costs must be clearly noted.

Strive for a consistent x-axis range throughout a given presentation and even throughout each organization. It is suggested that the x-axis be centered on the mean with a range consistent with a CV of 0.3.



Probability	TY K$
90%	$207,255
80%	$172,072
70%	$155,334
60%	$140,795
50%	$128,975
40%	$118,768
30%	$108,375
20%	$97,122
10%	$84,554

Cost Uncertainty Drivers
- SW Months
- Design Cost/Mth
- SW labor Rate
- EMD Duration

Figure 4-8 Sample S-curve Presentation Chart

The second necessary chart is the time-phased estimate by program phase. This will show the TY estimate at the selected cumulative probability by year. An example is shown in **Figure 4-9**.

80% TY $K Allocated from Second Level. PA dollars Phased Across the Point Estimate Schedule	Total TY	2014	2015	2016	2017	2018	2019	2020	2021	2022
Missile System	$425,094	$26,682	$32,608	$34,455	$38,084	$34,393	$60,682	$67,475	$65,622	$65,093
Engineering and Manufacturing Development	$172,072	$26,682	$32,608	$34,455	$38,084	$34,393	$5,851			
Production & Deployment	$253,022						$54,831	$67,475	$65,622	$65,093

Figure 4-9 Sample Phased Estimate by Appropriation Presentation Chart

Note that the allocated total for EMD is slightly different than the S-curve value at 80%. The reason for this is that the S-curve is developed directly from the simulation which includes influence from duration uncertainty. The risk allocation approach is performed on constant year dollars, phased and then inflated to the TY. As shown in **Figure 4-10**, the 80% TY results from the simulation and allocation differ by about 1.6% for EMD. Production is almost identical because we did not include duration uncertainty in Production. The BY results match each other identically, except for the total. The total should not match since the simulation is the statistical result, not the sum of EMD and Production, while the allocated result is the sum. It may be useful to know the percentile of the current budget.

80% Simulation and Allocated Results	BY Results		TY Results	
	Simulation Result	Allocation Result	Simulation Result	Allocation Result
Missile System	$376,127	$381,908	$421,719	$421,268
Engineering and Manufacturing Development	$163,168	$163,168	$174,934	$172,072
Production & Deployment	$218,740	$218,740	$253,240	$253,022

Figure 4-10 Comparing Simulation and Allocated TY Results

In addition to the S-curve and phased charts, a complete presentation of the risk analysis must contain the following items:

- The contents of the point estimate
- General approach of how the uncertainty was defined and, in the case of the simulation method, how the bounds and distributions were chosen
- Identify the most important contributors to the cost estimate uncertainty and any risk mitigation initiatives captured by the estimate
- Identify the cost drivers that have the most impact on the cost estimate
- The key point is to list those topics that have meaning to the decision maker

Presenting the risk story to leadership:
Time-phased TY estimate by appropriation
S-curve summary for each program phase.
S-curves should report CV, mean, and other points of interest.
S-curve x-axis range should be conditioned on a CV when comparing two or more S-curves.
Technical review charts should be in backup to support a brief explanation of the drivers to decision makers.

5.0 ALTERNATIVES TO THE CISM APPROACH

5.1 OVERVIEW

Although this handbook focuses on how to build a detailed model there are acceptable alternatives including outputs-based, scenario-based or method of moments. Considerations in choosing one of these approaches over CISM include: available data, available resources, available schedule, the complexity of the estimate, and the consequences of "less precise results." These methods are meant to complement rather than replace CISM. Each method has its strengths and weakness. A best practice is to employ more than one technique and to compare results. If the aggregate results of the alternative procedures are far apart, further work is required to determine why.

This handbook recommends CISM for conducting cost risk and uncertainty analysis and, if time and resources permit, a FICSM model. A sound rationale is required if something other than CISM is chosen as the primary method.

5.2 ENHANCED SCENARIO-BASED METHOD

5.2.1 Overview

The Scenario-Based Method (SBM), published in 2008 (**Reference 66)**, offered a simpler analytical alternative to advanced statistical methods or simulation for generating measures of cost risk. Since 2008, enhancements to SBM were made that included integrating historical cost performance data into its cost risk analysis algorithms and providing a perspective on its use in terms of the 2009 Weapon Systems Acquisition Reform Act. Together, these improvements define the enhanced scenario-based method (eSBM), which was published in 2012 (**Reference 86[10])**.

An objective of an eSBM is to assess the impact of various scenarios against a program baseline. Consequently, the baseline scenario is often based on the Cost Analysis Requirements Description (CARD) parameters. Rather than building up risk and uncertainty element by element as in Monte Carlo simulation, eSBM instead shifts attention to the identification and quantification of what can go right and what can go wrong with an acquisition program from a high-level management point of view. CISM and eSBM both yield S-curves. Yet, the two techniques are fundamentally different in approach, CISM applies risk and uncertainty at low levels in the program cost element structure while eSBM is more of a top-down approach. Proponents of eSBM believe that it can ease the mathematical burden on analysts, focusing instead on defining and analyzing risk scenarios as the basis for deliberations on the amount of cost reserve needed to protect a program from unwanted or unexpected cost increases. The development of eSBM scenarios, executed in conjunction with the requirements, acquisition, engineering, and resourcing communities, promotes a complete, understandable, and integrated view of cost risk and savings opportunities.

5.2.2 Application of the eSBM

Detailed eSBM steps and a worked numerical example can be found in **Appendix A.13**. The NCCA S-curve tool (available for download from www.ncca.navy.mil/tools/tools.cfm) can be used as an alternative to the mathematics described in **Appendix A.13** to develop a total cost distribution,. The S-curve Tool allows practitioners to:

- Enter data to create up to two S-curves
- Use historical CVs and cost growth factors (CGFs) to generate S-curves
- Plot alternative point estimates on an S-curve

[10] Authors grant permission to liberally quote from their referenced work for this section/appendix without detailed citations.

Figure 5-1 outlines the S-curve tool process flow. A user's guide and complete details are available on the web site. For the CSRUH example model, one could select parametric, lognormal, specify the PE cost is in TY, the PE probability, and the CV. The tool will generate the S-curve and plot additional points, if so desired. The tool allows comparisons of the example model S-curve to historical S-curves for the commodity of interest.

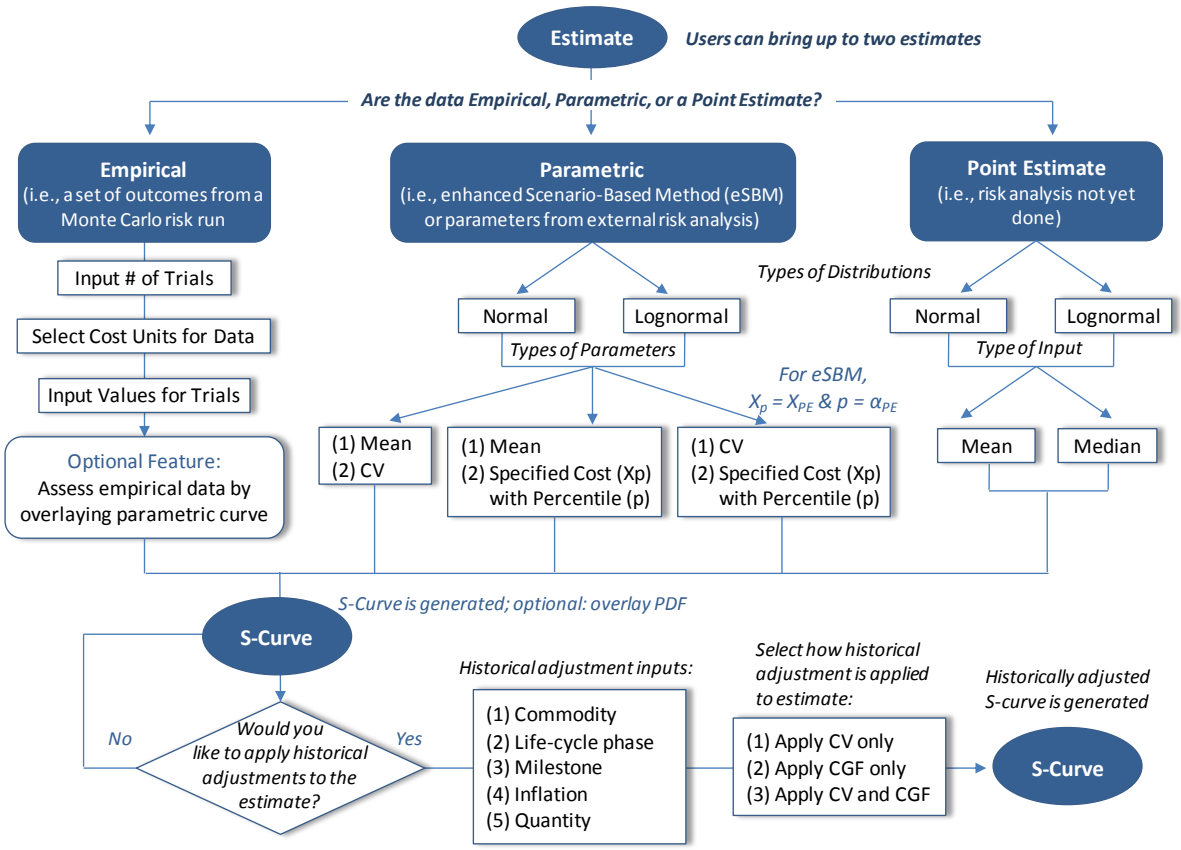


Figure 5-1 NCCA's S-curve Tool Process Flow

5.2.3 eSBM Example Application

An essential step in the eSBM process is to conduct a thorough review of major elements of risk. Elements of risk are identified and scoped based on:

- A general knowledge of defense acquisition programs and the discipline of defense cost analysis
- Site visits to the facilities of the prime contractor
- Extensive discussions with the program office
- Repeated interface with other major stakeholders including the Service and Joint-Staff requirements' community, budget analysts, Service and OSD acquisition officials, and the logistics community

The missile example model WBS and point estimate are the basis of the eSBM example. eSBM begins with a vigorous discussion among stakeholders to identify and analyze various scenarios. For example, expert stakeholders may determine that the research that led to **Figure 5-2** makes it a reasonable reference from which a low and high bound for software person months could be estimated. However, eSBM typically continues with an in-depth investigation into the nature and underlying causes of potential code growth. Results of the investigation can serve as a risk mitigation strategy for the program office.

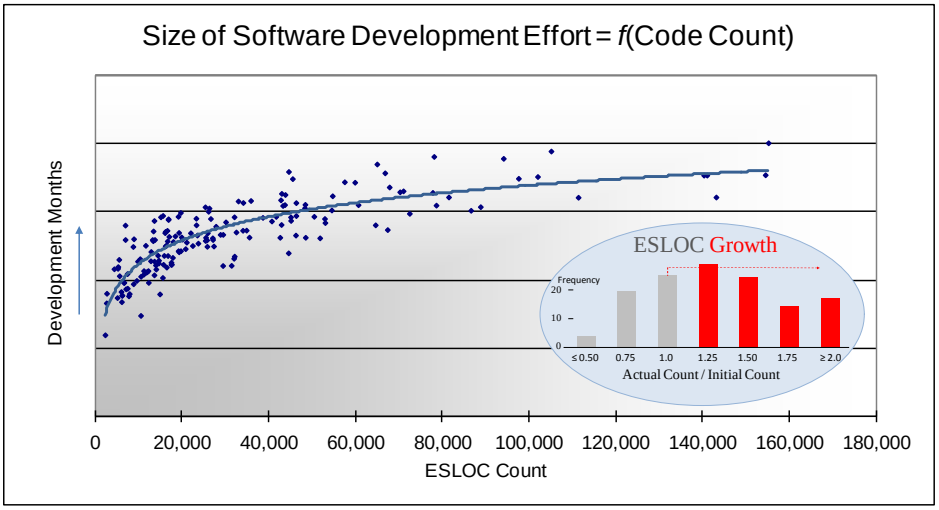


Figure 5-2 eSBM Example Software Development Uncertainty

Ideally, the expert stakeholders will identify data-driven methods to develop ranges on the key cost drivers they have identified in the model. This is one of the great benefits of the eSBM method, developing a deep understanding of the program's uncertainties through a detailed and systematic assessment of the program elements. The result of such discussions should lead to a summary of values that can be used to create different scenarios as illustrated in **Table 5-1**.

Table 5-1 eSBM Example Model Scenarios

Cost Driver	Optimistic	Point Estimate	Pessimistic
EMD Duration (Months)	54	60	72
EMD RR1 Dururation Increase (Months)			24
EMD Software Effort (Person Months)	1,130	2,100	3,150
EMD RR1 Increase SW Effort (Person Months)			800
Prod RR#2 Incr to Guidance First Unit Cost			50
Prod Airframe Weight (lbs)	182	330	855
Prod Motor Weight (lbs)	280	290	350
Prod Warhead Weight (lbs)	20	25	35
Prod IAT&C First Unit Hrs (Third Party Tool)	240	450	675

It is necessary in eSBM to anchor the point estimate to a point on a the selected distribution. A wide variety of choices are available including the use of historical cost growth data to estimate the point estimate position. The point estimate, in any event, should be developed with its corresponding value on the S-curve in mind. For the missile example, the unadjusted baseline cost estimate of $277M (TY$) is regarded as falling at the 30th percentile on the S-curve based on expert opinion regarding estimates of this type. The last piece of the puzzle is to estimate the CV from historical data. Ideally, data related to the program can be found and analyzed to estimate the CV. Alternatively, **Table 3-4** or **Figure 5-3** (see **Reference 88**) can be used to estimate the CV for the missile production cost.

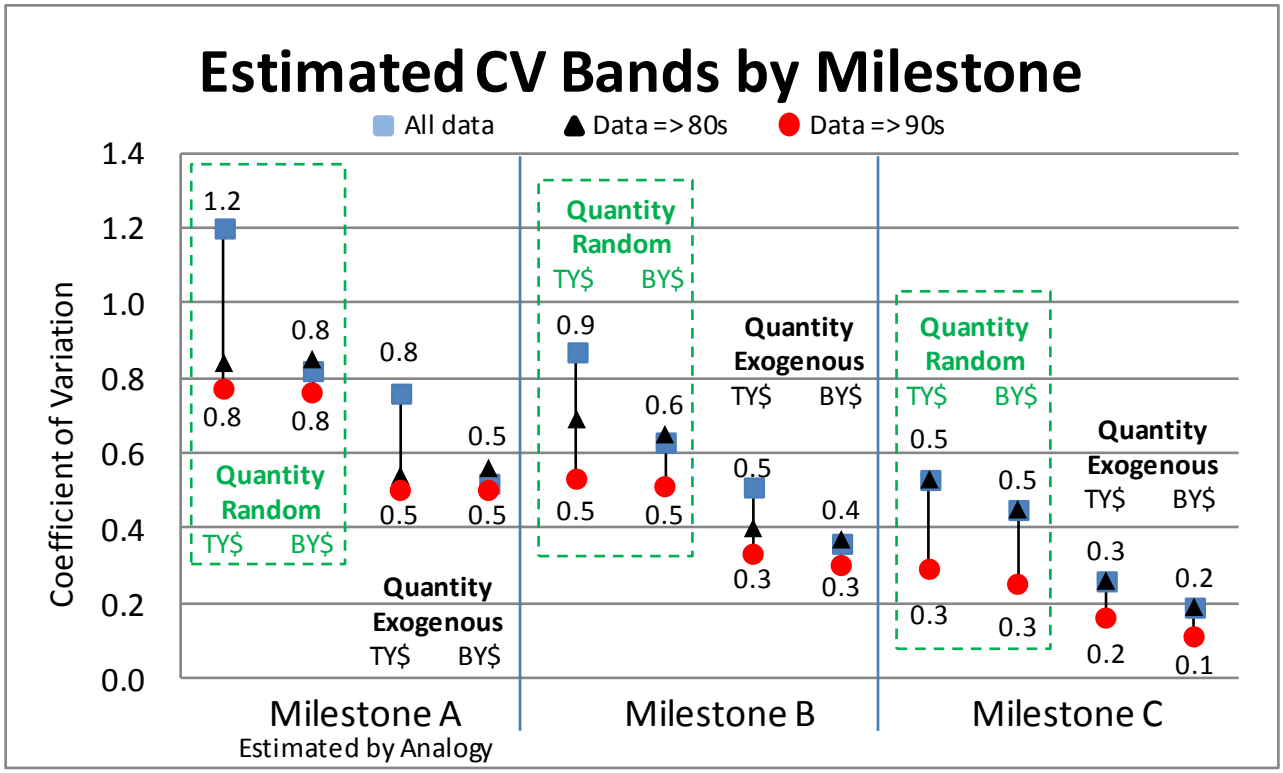


Figure 5-3 Estimated CVs By Milestone for Navy Major Programs

In the missile example, a CV of 40% is chosen, which is the midpoint CV value at MS B for Navy programs (**Figure 5-3**). In this case, the 40% CV is a value based on acquisition outcomes expressed in Then-Year dollars but adjusted for changes in quantity. By using TY dollars, it captures, to some degree, the variability of the rate of inflation over time. Quantity changes, in many cases, are best handled as what-if excursions with their own S-curves. **Table 5-2** shows the calculations to convert the PE, CV, and PE probability to unit space mean and standard deviation to model a lognormal distribution.

Table 5-2 eSBM Example Calculations

Element	ID	Value	Comment/Formula
Point Estimate	PE	$277	TY $M
Coefficient of Variation	CV	0.4	SAR Growth Study
Probability of PE	ProbPE	0.300	Expert Opinion
Standard Normal	SN	-0.524	NORM.S.INV(ProbPE)
LN Mean	LMean	5.826	LN(CV)-SN*SQRT(LN(1+CV^2))
Unit Space Mean	Mean	$365	EXP(LMean+0.5*LStdev^2)
LN Std Dev	LStdev	0.385	SQRT(LN(1+CV^2))

The calculations in **Table 5-2** completely define the S-curve. **Figure 5-4** illustrates the results with the baseline PE and the scenarios also plotted.

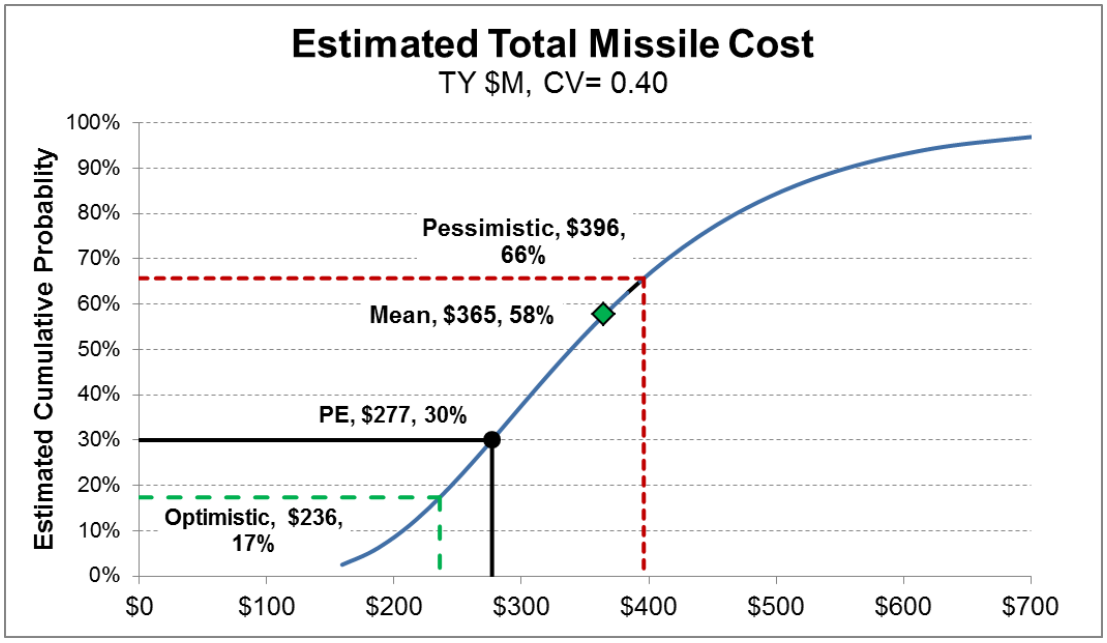


Figure 5-4 Example eSBM Result

5.3 METHOD OF MOMENTS (COST UNCERTAINTY)

An analytical method to estimate total uncertainty in a cost estimate is called the "method of moments[11]". The method of moments in the context of cost uncertainty analysis is the estimation of a total-level mean and variance from the sum of the subordinate elements. The mean and variance are the first and second moment of a random variable, hence the name method of moments. With knowledge of the mean and standard deviation for each element and how they are correlated, the mean and standard deviation at the parent levels can be calculated without the need for simulation.

In **Section 3.3.3** the method of moments is used to analytically calculate the total mean and standard deviation from the sum of five different correlated distributions. What is not known is the distribution shape at the parent level. However, a reasonable approach is to assume a lognormal distribution in order to estimate values at specific probabilities. Method of moments is a convenient approach when the model is a simple sum of uncertain elements, particularly if there are a large number of them. However, there are several complications, including:

- the variance sum must be adjusted for correlation
- distributions at the parent levels are assumed rather than derived
- efforts to combine uncertainties (such as the uncertainty of the cost driver and the CER) can lead to complex calculations
- simulation tools come with an abundant supply of ready to use charts and tables to analyze and report results. Those building custom method of moments models need to also develop the necessary charts and tables.

While, the method of moments is not recommended for general use, it has been found to be a simple, accurate alternative by some expert analysts. Method of moments has been implemented in the NASA/Air Force Cost Model and in Missile Defense Agency cost models. Most analysts, however, will find it challenging to develop custom method of moments models for every estimate. The use of simulation tools enables an analyst who is not an expert in the mathematics of method of moments to develop sophisticated,

[11] Method of Moments (Statistics) and Method of Moments (Probability) are examples of two other uses of the term.

credible uncertainty analyses. But if the model is simple, particularly when summing a very large number of correlated throughputs, the method of moments can be an effective approach. For more information on the method of methods, see **Reference 11**.

5.4 OUTPUTS-BASED SIMULATION METHOD

5.4.1 Outputs Modeling Based on Historical Data References

The outputs-based method applies uncertainty directly to the results (cost model outputs) rather than to the model's inputs. The analyst selects uncertainty distributions for the WBS element outputs to address the combined uncertainty of the cost method and the cost method inputs. **Figure 5-5** shows the point estimate results for the EMD phase of the example model. To illustrate the mechanics of preparing an outputs-based simulation, the uncertainty defined in **Figure 5-5** is drawn from either CRUAMM **(Reference 80** and **Attachment 1)** or **Table 2-9** as noted. Further, these elements have 0.3 correlation applied as depicted in **Figure 5-6**.

						Distribution Parameters					
DETAIL ESTIMATE based on PEs	Point Estimate	Forecast BY 2014	Distribution Form	Point Estimate is:	Uncertainty	Min or 15%	Most Likely	Mean	Max or 85%	Std Dev	Source
Missile System											
Engineering and Manufacturing De	$83,539	$83,539									
Air Vehicle	$14,944	$14,944									
Design & Development	$12,000	$12,000	Lognormal	Median	1			1.338		1.189	CRUAMM
Prototypes	$2,944	$2,944	Lognormal	Median	1			1.315		1.123	CRUAMM
Software	$31,500	$31,500	Lognormal	Median	1	0.696		1.063	1.437		Last Resort Table
System Engineering	$17,500	$17,500	Triangular	Mode	1	0.119	1.000		2.074		CRUAMM
Program Management	$15,000	$15,000	Triangular	Mode	1	0.876	1.000		1.914		Last Resort Table
System Test and Evaluation	$1,767	$1,767	Lognormal	Median	1			1.366		1.271	CRUAMM
Training	$897	$897	Lognormal	Median	1	0.627		1.107	1.594		Last Resort Table
Data	$1,196	$1,196	Lognormal	Median	1			1.904		3.086	CRUAMM
Peculiar Support Equipment	$736	$736	Triangular	Mode	1	0.876	1.000		1.914		Last Resort Table

Figure 5-5 Outputs-Based Uncertainty

	DesignDev_EMD	Proto_EMD	SW_EMD	SysEng_EMD	PM_EMD	STE_EMD	Trg_EMD	Data_EMD	PSE_EMD
DesignDev_EMD	1.000	0.300	0.300	0.300	0.300	0.300	0.300	0.300	0.300
Proto_EMD		1.000	0.300	0.300	0.300	0.300	0.300	0.300	0.300
SW_EMD			1.000	0.300	0.300	0.300	0.300	0.300	0.300
SysEng_EMD				1.000	0.300	0.300	0.300	0.300	0.300
PM_EMD					1.000	0.300	0.300	0.300	0.300
STE_EMD						1.000	0.300	0.300	0.300
Trg_EMD							1.000	0.300	0.300
Data_EMD								1.000	0.300
PSE_EMD									1.000

Figure 5-6 Outputs-Based Correlation Matrix

5.4.2 Outputs Model Based on Risk Scoring

Risk scoring methods estimates the uncertainty of a top level WBS cost element (e.g., Development, Production, etc) through a process that maps an uncertainty distribution to a risk assessment ordinal score. The first step is to develop one or more tables to define the level of risk for a variety of project characteristics. These tables range from simple scores of low, medium, or high to complex scoring matrices with several levels of granularity for multiple categories. **Table 5-3** and **Table 5-4** (from **Reference 68**) present two example Risk Scoring Matrices used in a methodology developed by the Intelligence Community Cost Analysis Improvement Group (IC CAIG) and the Missile Defense Agency (MDA). In this particular application the risk assessment scores run from 0 to 10. The next step is to use historical data to

develop a way to establish a growth factor and its uncertainty from projects with similar scores. For an estimated program, the risk assessment is performed in collaboration with the technical personnel who are familiar with the development and the production phases of the subject program. For more detail on this approach, including sample tables and distribution mapping, please see **Reference 33** and **68**.

Table 5-3 Hardware Risk Scoring Matrix

	Risk score: 0 = low, 5 = medium, 10 = high				
Risk category	**0**	**1–2**	**3–5**	**6–8**	**9–10**
1. Technology advancement	Completed, state of the art	Minimum advancement required	Modest advancement required	Significant advancement required	New technology
2. Engineering development	Completed, fully tested	Prototype	Hardware and software development	Detailed design	Concept defined
3. Reliability	Historically high for same system	Historically high on similar systems	Modest problems known	Serious problems known	Unknown
4. Producibility	Production and yield shown on same system	Production and yield shown on similar system	Production and yield feasible	Production feasible and yield problems	No known production experience
5. Alternative item	Exists or availability on other items not important	Exists or availability on other items somewhat important	Potential alternative in development	Potential alternative in design	Alternative does not exist and is required
6. Schedule	Easily achieved	Achievable	Somewhat challenging	Challenging	Very challenging

Source: © 2003, Society of Cost Estimating and Analysis (SCEA), "Cost Risk Analysis."

Table 5-4 Software Risk Scoring Matrix

	Risk score: 0 = low, 5 = medium, 10 = high				
Risk category	**0**	**1–2**	**3–5**	**6–8**	**9–10**
1. Technology advancement	Proven conventional analytic approach, standard methods	Undemonstrated conventional approach, standard methods	Emerging approaches, new applications	Unconventional approach, concept in development	Unconventional approach, concept unproven
2. Design engineering	Design complete and validated	Specifications defined and validated	Specifications defined	Requirements defined	Requirements partly defined
3. Coding	Fully integrated code available and validated	Fully integrated code available	Modules integrated	Modules exist but not integrated	Wholly new design, no modules exist
4. Integrated software	Thousands of instructions	Tens of thousands of instructions	Hundreds of thousands of instructions	Millions of instructions	Tens of millions of instructions
5. Testing	Tested with system	Tested by simulation	Structured walk-throughs conducted	Modules tested but not as a system	Untested modules
6. Alternatives	Alternatives exist; alternative design not important	Alternatives exist; design somewhat important	Potential for alternatives in development	Potential alternatives being considered	Alternative does not exist but is required
7. Schedule and management	Relaxed schedule, serial activities, high review cycle frequency, early first review	Modest schedule, few concurrent activities, review cycle reasonable	Modest schedule, many concurrent activities, occasional reviews, late first review	Fast track on schedule, many concurrent activities	Fast track, missed milestones, review at demonstrations only, no periodic reviews

Source: U.S. Air Force.

6.0 PORTFOLIO LEVEL CONSIDERATIONS

This section of the handbook explores the impact of project probability selection in a portfolio of projects situation. Please see **Reference 30** for further information.

The simplest way to generate the uncertainty distribution at the portfolio level is to model each project using their median and 85% result to define a lognormal distribution (see **Section 2.4.4**), a fitted distribution (see **Section 2.4.3**) or an empirical distribution (see **Section 2.4.3.6**). It is then necessary to assign correlation between the projects. In the absence of any other rationale, assigning the default 0.3 is appropriate.

Table 6-1 defines the uncertainty distributions for eight example programs in a portfolio. In this example, each project's point estimate represents very different probabilities. The first four are at the median; the last four are at the 25% probability (consistent with a right skew triangular distribution see **Section 2.5.5**). The point estimate is not the proposed budget, simply a selected point on each distribution that best fits the source model distribution. With these very different distribution shapes and dispersion, the following explores the effects of correlation on various percentile results at the parent level.

Table 6-1 Distributions Fitted to Project Uncertainty

WBS/CES	Point Estimate	Distribution	Point Estimate Position	CV	Low %	High %	Low Percentile	High Percentile
Portfolio Total								
Lognormal High	100	LogNormal	Median	0.35				
Lognormal Low	100	LogNormal	Median	0.15				
Normal High	100	Normal	Mean	0.35				
Normal Low	100	Normal	Mean	0.15				
Triangular High RIght	100	Triangular	Mode		90.3	171.1	15	85
Triangular Low RIght	100	Triangular	Mode		95.9	130.5	15	85
BetaPert High RIght	100	BetaPert	Mode		90.3	171.1	15	85
BetaPert Low Right	100	BetaPert	Mode		95.9	130.5	15	85

Table 6-2 illustrates the difference between the simulation result and the sum of the projects when they are budgeted to a specific probability (each block is a different project probability scenario). Additionally, each column identifies the correlation applied across all the projects.

Table 6-2 Portfolio Total Results at Various Project Probabilities

Correlation of Projects:	0.00	0.30	0.50	0.90
Probability Level	30%	30%	30%	30%
Portfolio Total at Selected Probability Level	$846	$835	$820	$775
Sum of Projects at Selected Probability Level	$762	$762	$762	$762
Probability of Sum of Projects	5%	9%	15%	27%
Probability Level	50%	50%	50%	50%
Portfolio Total at Selected Probability Level	$887	$886	$883	$871
Sum of Projects at Selected Probability Level	$866	$866	$866	$866
Probability of Sum of Projects	40%	42%	44%	49%
Probability Level	60%	60%	60%	60%
Portfolio Total at Selected Probability Level	$908	$911	$915	$922
Sum of Projects at Selected Probability Level	$923	$923	$923	$923
Probability of Sum of Projects	67%	64%	62%	60%
Probability Level	70%	70%	70%	70%
Portfolio Total at Selected Probability Level	$930	$939	$951	$978
Sum of Projects at Selected Probability Level	$987	$987	$987	$987
Probability of Sum of Projects	89%	84%	78%	71%
Probability Level	90%	90%	90%	90%
Portfolio Total at Selected Probability Level	$994	$1,019	$1,051	$1,148
Sum of Projects at Selected Probability Level	$1,176	$1,176	$1,176	$1,176
Probability of Sum of Projects	100%	100%	98%	92%

This example assumes that the dollars saved from one project can be moved to another. Put another way, the PA dollars are managed at the portfolio level. There are several key observations from this example:

- Budgeting all projects at 50% leads to a portfolio budget that has a probability of less than 50%, even at very high correlation between projects
- Budgeting all projects at 70% provides a much higher probability of success at low correlation levels (which is likely realistic)
- Budgeting all projects at 60% leads to a portfolio probability near 60% at all levels of correlation. This is because 60% tends to be close to the mean of most project simulation results (the CISM model is near 57%). Since means sum, the sum of percentiles close to the mean will approximate the parent mean.
- If all projects are budgeted less than the mean, the total at the portfolio level will be at a lower probability than the projects. The opposite is true if projects are budgeted above the mean.

The 60% values will not be much higher than the mean values for most projects. Setting 60% as a budgeting goal treats each project the same. Budgeting to the mean results in projects having different probabilities (albeit not very different).

Budgeting all projects to the same probability treats each project the same.
Budgeting all projects to their respective means sets them to different probabilities.

7.0 CSRUH UTILITIES AND SUPPORT FILES

Several Excel based utilities and support files were delivered with the CSRUH and should be available from the same location as the handbook. The files were developed and tested in the following applications:

- Microsoft Office Professional Plus 2010 (14.0.7109.5000)
- Oracle Crystal Ball Fusion Edition Release 11.1.2.2.000 (32-bit) Build 11.1.2926.0 14 March 2012
- Palisade Corporation @RISK for Excel Version 6.0.0 Professional Edition 2012
- Automated Cost Estimating Integrated Tools (ACEIT) 7.3a Build #54 12 March 2012

7.1 CSRUH UTILITIES

Six CSRUH Excel utilities were developed to address key uncertainty analysis and reporting processes and were used to create many of the figures and tables in this handbook. They are listed in the order that they appear in the handbook. The order of appearance should not be interpreted as order of importance.

7.1.1 CSRUH Best Fit Utilities

The CSRUH_U1a_CrystalBallBestFitUtility.xlsm utility can be used to compare several distribution fits to sample data. See **Section 2.4.3** for details on the distribution fitting process and guidance on how to interpret results. Crystal Ball must be installed to use this utility. The CSRUH_U1b_CRUAMMBestFitUtility.xlsm is a more sophisticated alternative that does not require Crystal Ball. It operates within Excel using Solver or optionally using ACEIT's CO$TAT application.

7.1.2 CSRUH Adjust for Skew Utility

The CSRUH_U2_AdjustForSkew.xlsx utility can be used to calculate the minimum and maximum of a triangular, uniform and betaPERT distribution based upon SME inputs. For details, see **Section 2.5.4**.

7.1.3 CSRUH Correlation Utility

The CSRUH_U3_CorrelationUtility.xlsx is a tool-independent utility that can be used to measure the Pearson Product Moment correlation developed in the simulation. For details see **Section 3.3.4**.

7.1.4 CSRUH Convergence Utility

The CSRUH_U4_ConvergenceUtility.xlsx provides a tool-independent method to determine the number of trials required to achieve a stable result. Many tools offer alternatives. For details see **Section 3.4.3**. If convergence is not achieved with the 10k version, a 20k version is also available. Due to the size of these files, it is recommended that all other Excel files should be closed if possible.

7.1.5 CSRUH Scatter Plot Utility

The CSRUH_U5_ScatterPlotUtility.xlsx is a tool-independent utility that can be used to plot any two random variables from the uncertainty model. Each dot on the plot represents one trial result. Typical plots are cost (y-axis) vs. duration (x-axis). For an example, see **Section 4.1.2**. Cost vs. cost (e.g., production cost vs development cost) or cost vs. driver (e.g., cost vs. weight) are examples of other useful scatter plots.

7.1.6 CSRUH S-curve Utility

The CSRUH_U6_SCurveUtility.xlsm (this utility includes macros) is a tool-independent utility that can be used to construct the S-curve including various data points plotted on the curve. The utility features a method by which the x-axis range can be conditioned to a specific CV rather than let Excel auto fit the range. For an example, see **Section 4.2**.

7.2 CSRUH SUPPORT FILES

Thirteen CSRUH support files were developed to build the figures and tables in this handbook not addressed by the utilities. They are listed in the order that they appear in the handbook. The order of appearance should not be interpreted as order of importance.

7.2.1 CSRUH Illustration Files

The following files contain the bulk of the illustrations in the CSRUH:

- **CSRUH 1 Illustrations.xlsx**: The first sheet in this workbook contains an index of all the illustrations in the CSRUH and where they can be found amongst the utility and support files. Additionally, this file can be referenced to review the mathematics associated with the figures and tables in the CSRUH.
- **CSRUH 1 Illustrations.pptx**: This PowerPoint presentation contains many of the illustrations in the CSRUH

7.2.2 CSRUH Example Model Files

The example model used to generate most of the figures and tables in the main body of the CSRUH was developed in three separate tools to demonstrate the CSRUH guidance is tool independent. The following files are available:

- **CSRUH 2 Missile CB Model.xlsm**: This file contains the Crystal Ball example model. It also contains a chart that can be used to review how all three tools compare for each WBS element (see **Figure 3-9**). Additionally, the sample calculation for all three allocation methods (see **Section 3.6** and **A.12**) is contained in this file. Two other CB models (2a and 2b) are provided to demonstrate how to address sunk costs (see **Section 2.8.2**) and outputs based modeling (see **Section 5.4.1**).
- **CSRUH 3 Missile @RISK Model.xlsm:** This file contains the @RISK example model. There is also a "swap out" version of this file (2a) that can be read by users without @RISK on their machines.
- **CSRUH 4 Missile Model ACEIT_73a.ACEIT:** This file contains the ACEIT example model
- **CSRUH 5 Missile CER Data.xlsx**: This file contains the source data, regression analysis and distribution fitting analysis used to support the CSRUH example model and associated figures and tables.
- **FICSM Model files**: There are four files to support the FICSM (see **Appendix B**) discussion. Two are associated with Microsoft Project (MSP) 2007 and two are associated with MSP 2010.

8.0 REFERENCES

References are listed in chronological order rather than alphabetical to make it easier to identify the most current sources for the handbook. The references are also provided in alphabetical order (but the numbering remains the same). Most of the references are available from the same location as the handbook. In some cases, a DOI number, JSTOR or EverySpec web site is provided:

- DOI (Digital Object Identifier www.doi.org/) system provides a technical and social infrastructure for the registration and use of persistent interoperable identifiers for use on digital networks.
- JSTOR (www.jstor.org/) originally containing digitized back issues of academic journals, it now also includes books and primary sources, and current issues of journals.
- EverySpec (www.everyspec.com/) provides free access to Military, DoD, Federal, NASA, DOE, and Government specifications, standards, handbooks, and publications

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APPENDIX A TERMINOLOGY AND DETAIL

This appendix presents definitions for technical terms used throughout this handbook and some additional detail on specific concepts and process. Many publications contain definitions of each of these terms and processes, often in conflict with one another. Therefore, this appendix elaborates on many of these concepts mathematically and graphically to clarify their use in this handbook.

A.1 DEFINITIONS

A.1.1 Deterministic

Deterministic refers to events that have no random or probabilistic aspects but proceed in a predictable fashion. A deterministic model consists of an exact relationship; for example, if the labor rate (LabRate$) is known and the man-hours (Mhrs) are known, then the cost is known by calculating LabRate$*Mhrs. Unlike a stochastic model, a deterministic equation has no random error.

A.1.2 Stochastic

Stochastic refers to patterns or processes resulting from random variables. Unlike a deterministic model, the equations and/or their parameters are not known with certainty but only with some amount of probability. For example, the labor rate and the man-hours are only known within some degree of probability and therefore the cost calculated by LabRate$*Mhrs is only known within a probability distribution defined by the combined input uncertainty. In other situations, there could be random errors associated with the equation, too. A parametric cost estimating method is an example of a method where uncertainty of the equation and its inputs must be combined to arrive at the total uncertainty for the element in question.

A.1.3 Coefficient of Variation (CV)

CV is the standard deviation divided by the mean. The result is a unitless measure of the distribution dispersion.

A.1.4 Confidence Interval and Confidence Level

A confidence interval is an interval estimate for an unknown population parameter in a probability distribution. A confidence interval is used to indicate the reliability of a point estimate—the tighter the interval, the more reliable the point estimate is at a given confidence level. For example, when estimating the mean of a normal distribution, the sample mean is a point estimate of (or best guess) for the value of the mean. However, this estimate is almost surely not equal to the population mean. A confidence interval identifies a range around the estimate for the population mean. The width of that range is a function of the *confidence level*. The confidence level associated with the interval (e.g., 80%, 90%, or 95%) is the proportion of times in which the interval will contain the true value of the unknown parameter.

A.1.5 Cost Contributors and Cost Drivers

When asked to identify the cost drivers in a cost estimate, those asking the question may define cost driver different than the person answering the question. For some, the project cost drivers are the WBS elements (generally at a lower level) that have the highest cost. For others, project cost drivers are the cost estimating method input variables that have the most influence on the total cost of interest. To draw a distinction between these two concepts, the following terms are defined for this handbook:

- **Cost Contributors**: Child WBS elements that influence the total cost or total uncertainty of interest
- **Cost Drivers**: Cost estimating method inputs that influence the total cost or total uncertainty of interest

A.1.6 Nunn-McCurdy Breach

The following is an extract from **Reference 74**:

"For more than 25 years, the Nunn-McCurdy Act (10 U.S.C. § 2433) has served as one of the principal mechanisms for notifying Congress of cost overruns in Major Defense Acquisition Programs (MDAPs). Nunn-McCurdy establishes different thresholds to determine if a MDAP or designated major subprogram of a MDAP experiences a cost overrun (for purposes of this report, the term program will refer to MDAPs as well as designated major subprograms). These thresholds are based on a comparison between a program's actual costs and the current baseline estimate or the original baseline estimate (defined below). A program that has cost growth that exceeds any of these thresholds is said to have a Nunn-McCurdy breach and must notify Congress of the breach.

There are two categories of breaches: significant breaches and critical breaches. As shown in **Table A-1**, a "significant" Nunn-McCurdy breach occurs when the Program Acquisition Unit Cost (PAUC- defined as the total cost of development, procurement, and construction divided by the number of units) or the Procurement Unit Cost (PUC- defined as the total procurement cost divided by the number of units to be procured) increases 15% or more over the current baseline estimate or 30% or more over the original baseline estimate. A "critical" breach occurs when the PAUC or PUC increases 25% or more over the current baseline estimate or 50% or more over the original baseline estimate.

Table A-1 Nunn-McCurdy Breach Thresholds

	Significant Breach	Critical Breach
Current Baseline Estimate	≥15%	≥25%
Original Baseline Estimate	≥30%	≥50%

Source: 10 U.S.C. § 2433.

According to Title X of the U.S. Code, the Department of Defense (DOD) is required to establish a baseline description of all major defense acquisition programs when the program is officially started. This baseline description includes information on the program's planned cost, schedule, and performance. The cost information is referred to as the "baseline estimate". The baseline description (including the cost estimate) is contained in the Acquisition Program Baseline (APB).

APBs are required to initiate a program, and can only be revised:

- at the milestone reviews or when full rate production begins
- if there is a major program restructuring that is fully funded, or
- as a result of a program breach if the breach is primarily the result of external causes beyond the control of the program manager

Under current DoD policy, current APBs cannot be revised just to avoid a Nunn-McCurdy breach. An original baseline estimate is the cost estimate included in the original (first) APB that is prepared prior to the program entering "engineering and manufacturing development" (also known as "Milestone B"), or at program initiation, whichever occurs later. An original baseline estimate can only be revised if the program has a critical Nunn-McCurdy breach (see **Table A-1**).

A current baseline estimate is the baseline estimate that is included in the most recently revised APB. If the original baseline estimate has not been revised, the original baseline estimate is also the current baseline estimate."

Reference 59 defines Major Automated Information System (MAIS) and provides an overview of congressional reporting requirements. Senior officials must review the MAIS Program Manager's quarterly

reports to determine whether a significant or critical change has occurred. A significant change is when any of the following has occurred:

- **Schedule**: a delay of more than six months but less than a year in any program schedule milestone or significant event from the initial baseline
- **Cost**: estimated program development cost or full life-cycle cost for the program has increased by at least 15 percent, but less than 25 percent, over the original estimate
- **Performance**: a significant, adverse change in the expected performance of the major automated information system to be acquired

A critical change to a MAIS program has occurred if:

- **Time certain development**: failure to achieve IOC within five years of milestone A approval
- **Schedule**: a delay of one year or more in any program schedule milestone or significant event from the original baseline
- **Cost**: the estimated program development cost or full life-cycle cost for the program has increased by 25 percent or more over the original baseline
- **Performance**: a change in expected performance that will undermine the ability of the system to perform the functions anticipated in the original baseline

To put the cost values into perspective, **Figure A-1** illustrates them plotted against lognormal cumulative probability curves (S-curves). The labels identify the probability the program will come in at or below these values. For instance, if the program's ***baseline*** S-curve has a 0.10 CV, there is almost no chance for a significant or critical breach. Even when the CV is 0.3, the implication is that there is only a 6% chance of a critical breach. While comforting, these results are not consistent with history. Many projects have suffered a breach. **Figure A-1** suggests that CVs less than 0.15 at Milestone B may be too optimistic. For further discussion on how excessive optimism and unrealistic cost estimates have been blamed for various breaches, please read **Reference 74.**

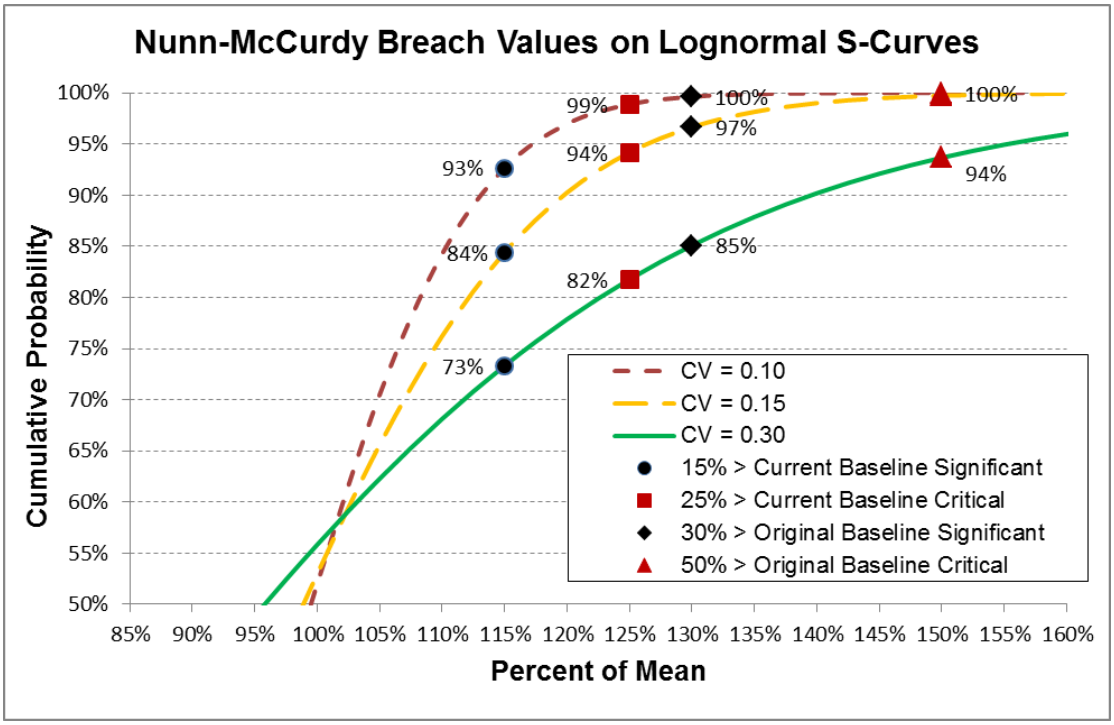


Figure A-1 Nunn-McCurdy Breach Values on Lognormal S-curves

A.1.7 Acquisition Program Baseline Deviation

The contents of this section are taken from **Reference 102**. Further direction can be found in 10 U.S.C. 2435, 10 US.S.C. 2445c and DoD 5000.01.

Milestone B is normally the formal initiation of an acquisition program with the Milestone Decision Authority (MDA) approval of the Acquisition Program Baseline (APB). An APB is required beginning at EMD or program initiation, whichever occurs later (see Table 3 of **Reference 102**). The APB is the agreement between the MDA and the Program Manager and his or her acquisition chain of command that will be used for tracking and reporting for the life of the program or program increment. Deviations from the approved APB will be immediately reported to the MDA. Deviations have specified default thresholds for schedule and cost of:

1. Objective schedule value plus 6 months
2. Objective cost value plus ten percent

Within 30 days of an APB deviation, the PM has to inform MDA for the reason for deviation, and within 90 days a proposed revised APB shall be submitted or Component-level review shall be held to review the program. An APB deviation is more common than a Nunn-McCurdy breach since they have lower thresholds; however they still require a great deal of work to remedy.

If the probability of an APB deviation is low (e.g., plotted very high on an S-curve), that is an indication the uncertainty in the model may be understated, since, APB deviations do happen.

A.1.8 Risk

Risk is the possibility of incurring loss or misfortune. In the context of cost estimating, risk is the possibility the program will not be able to complete on budget. Too often analysts will use the terms risk and uncertainty interchangeably. In this handbook, “risk” is derived from an “uncertainty” analysis. Having developed the uncertainty distribution for the project total cost and duration, the analyst can estimate the risk in terms of probability of the completed project exceeding a specified budget and/or schedule. Opportunity, on the other hand, is the possibility of a favorable outcome. In the context of cost estimating, opportunity is typically an item of discrete uncertainty that results in a cost avoidance.

A.1.9 Standard Error

The standard error is a common terminology for the standard deviation of a sample that is used to estimate the population standard deviation. Two types of standard errors commonly found in cost analysis include:

- **Standard error of the mean**: is the estimated standard deviation of the sample mean. The standard error of the mean can be used to build a confidence interval for the mean. This confidence level defines the range about the sample mean that the true mean can be found within a specific probability.
- S**tandard error of estimate (SEE)**: is an estimate of the standard deviation about the regression line. It measures the amount of dispersion of the CER errors (or it provides one-sigma spread of the CER errors). It provides a measure of “average” distance of the sample data from the regression equation. To be used in an uncertainty analysis, the SEE should be converted to a prediction interval to account for sample size, degrees of freedom and the distance of the estimate from the center of the CER dataset. Many papers, texts and tools use Standard Error (SE) rather than SEE to describe the CER error. SEE and SE are used interchangeably in this handbook. For multiplicative error CERs it is often called the standard percent error (SPE).

A.1.10 Integrated Master Plan (IMP) and Schedule (IMS)

The IMP defines a hierarchy of program events, in which each event is supported by specific accomplishments, and each accomplishment is based on satisfying specific criteria to be considered complete. The IMS is an integrated, networked schedule containing all the detailed discrete work packages and planning packages (or lower-level tasks of activities) necessary to support the events, accomplishments, and criteria of the IMP.

The IMP and IMS are business tools to manage and provide oversight of acquisition, modification, and sustainment programs. They provide a systematic approach to program planning, scheduling, and execution. They are equally applicable to competitive and sole source procurements with industry, as well as to government-only, in-house efforts. They help develop and support program/project budgeting, and can be used to perform "what-if" exercises and to identify and assess candidate problem workarounds. Finally, use of the IMP/IMS focuses and strengthens the interaction between the government and contractor teams with respect to program execution.

The IMP should provide sufficient definition to track the step-by-step completion of the required accomplishments for each event, and to demonstrate satisfaction of the completion criteria for each accomplishment. Events in the IMP are not tied to calendar dates; they are tied to the accomplishment of a task or work package as evidenced by the satisfaction of the specified criteria for that accomplishment. The IMS should be defined to the level of detail necessary for day-to-day execution of the program.

To build a reasonable IMP and IMS, you need to estimate the attributes of work products and tasks, determine the resources needed, estimate a schedule, and identify and analyze program risks. Accomplishments in the IMP should have criteria for determining completion with clear evidence so that the entire program team can understand the progress.

The IMS and IMP should be traceable to the work breakdown structure (WBS), and be linked to the statement of work and ultimately to the earned value management system (EVMS). The WBS specifies the breakout of work tasks on which the IMP and IMS should be built on and on which the EVMS should report on. A good WBS includes key work efforts partitioned into discrete elements that result in a product (i.e., document, software item, test completion, integrated product) or in measurable progress (percent complete is not recommended when the end state is not completely quantifiable—an issue in software development, test procedures, or training materials). With a good WBS foundation, both the IMP and IMS can be more useful tools; with the IMP integrating all work efforts into a defined program plan, and the IMS summarizing the detailed schedule for performing those work efforts. The IMP is placed on contract and becomes the baseline execution plan for the program/project. Although fairly detailed, the IMP is a relatively top-level document compared to the IMS. The IMP has several levels of detail.

- Level 1 of the IMP is Program Events (PE). These are the major milestones and activities that occur on a program. Generally, there should not be more than about 10 - 20 PEs on a program.
- Level 2 of the IMP is Significant Accomplishments (SA). These are the major items a project must accomplish to meet the PE.
- Level 3 of the IMP is Accomplishment Criteria (AC). These are the more-detailed items or checklists of how a project meets its SAs.
- Level 4 of the IMP is generally the detailed tasks of each SA. These details are what will become the IMS tasks.

For further detail, see **Reference 47**.

A.2 POINT ESTIMATE

A.2.1 Why a Point Estimate is Uncertain

Unless each child of the cost estimate is known with certainty, the point estimate represents only one of many possible outcomes. A decision-maker who must decide on the "official" budget should make that decision in the context of the point estimate with respect to all other outcomes. That is the central point of risk and uncertainty analysis -- quantifying the possible outcomes and their likelihood so that an informed choice can be made. Recognizing that there is more than one outcome raises two interesting questions. One, what causes there to be more than one outcome and, two, if there is more than one outcome, which one does the aggregated point estimate represent?

The answer to the first question is simply that estimating techniques are not sufficiently precise to capture all the vagaries associated with producing an estimate. To the extent historical data is used, uncertainty creeps in because no two programs or projects are identical: each is unique unto itself. The technology, schedule, contractor(s), and the budget climate all contribute to the unique character of each program and its data points. Moreover, even if the past is perfectly known, the future is not. Considering the educated guesses that an analyst must make in developing an estimate, it is no surprise that a cost estimate is just that: an estimate.

The second question can be even more perplexing. If the point estimate is composed of several subsidiary estimates, what is the likelihood at the total level? There are many possible results; here are three:

- **Consistently Symmetric Uncertainty**: If each cost element point estimate is the center of any symmetric distribution shape (that is, for each element, the mean, median, and mode are the same), the sum of the point estimates will be the center of a symmetric distribution at the total level. However, if even one distribution is skewed, then the sum will no longer be at the center.
- **Summing Modes**: If each cost element point estimate is the mode of a skewed distribution, the sum of the point estimates will not sum to the mode of the total distribution
- **Summing Means, Medians, Modes or Something Else**: This is the most common situation. Even if the estimating method is known to return the mean of the CER, if it is being influenced by uncertain inputs it is unclear what the point estimate result represents.

For cases 2 and 3 (which are the most common in cost estimates), the resulting sum is not at the 50/50 overrun/underrun position nor is the overrun/underrun position of that sum known or easily deduced. The likelihood of the point estimate is unknown and the likelihood of the other outcomes is also unknown. Risk and uncertainty analysis is used to estimate the likelihood of any point estimate.

A.2.2 Content of a Point Estimate

Typical contents of a cost point estimate include:

- Work Breakdown Structure (WBS) to identify all the elements requiring a cost estimate
- Technical parameters that properly define the system to be estimated
- Rates and factors such as labor rates, head counts, fee, overhead, operating hours, or man-hour rates, and other programmatic cost drivers
- A schedule that properly reflects how the project will unfold
- An assessment of the risk register that will list events that, should they occur, will have an impact on the project cost and/or schedule. The impact could be positive (opportunity) or negative (risk). It is recommended that the point estimate for risk register items are zero. For a high probability item, an alternative is to include the consequence in the point estimate and define a low probability risk register item that represents the saving (opportunity) that the item does not occur.

A.3 UNCERTAINTY

A.3.1 Overview

Uncertainty is defined as a potential deficiency in any phase or activity of the cost estimating process due to a lack of knowledge or due to random variations in the cost estimating process. The analyst's challenge is to adequately capture and model the complete uncertainty associated with the cost estimate. This includes the impact of schedule and risk register uncertainty.

A.3.2 Objective Uncertainty

Objective uncertainty associated with cost model parameters is a measure of source data variability. If the cost estimating relationship (CER) or the input(s) to the CER are derived from statistical analysis of relevant historical data, the uncertainty associated with the cost estimate can often be characterized "objective," meaning derived using a repeatable, proven process. The basis for the uncertainty calculation is a function of how the estimate is derived. In the sections that follow, the most common methods for deriving CERs are introduced with their associated "objective" uncertainty. The uncertainty model is designed to capture and combine both CER and the CER input uncertainty.

A.3.3 Distribution Boundaries for Objective Distributions

If statistical methods have been used to generate the CER, the analyst should have the necessary information to replicate the uncertainty in the cost model. Ideally, the CER statistics will provide the bounds for a specific confidence level. More often than not, however, the analyst will be provided with other data such as the standard deviation for a specific position within the data set. As illustrated in **Figure A-2**, uncertainty will increase (standard deviation gets larger) as the point estimate moves towards (and beyond) the data boundaries. The minimum information required is the CER result and the standard deviation for that result. Analysts should take care when interpreting a CER result. For instance, a loglinear CER developed using ordinary least squares will give the median of the distribution, not the mean. Also, it is common for the standard deviation to be expressed in log space and it would need to be converted to unit space for use in the cost model. See **Appendix A.6.4** for details.

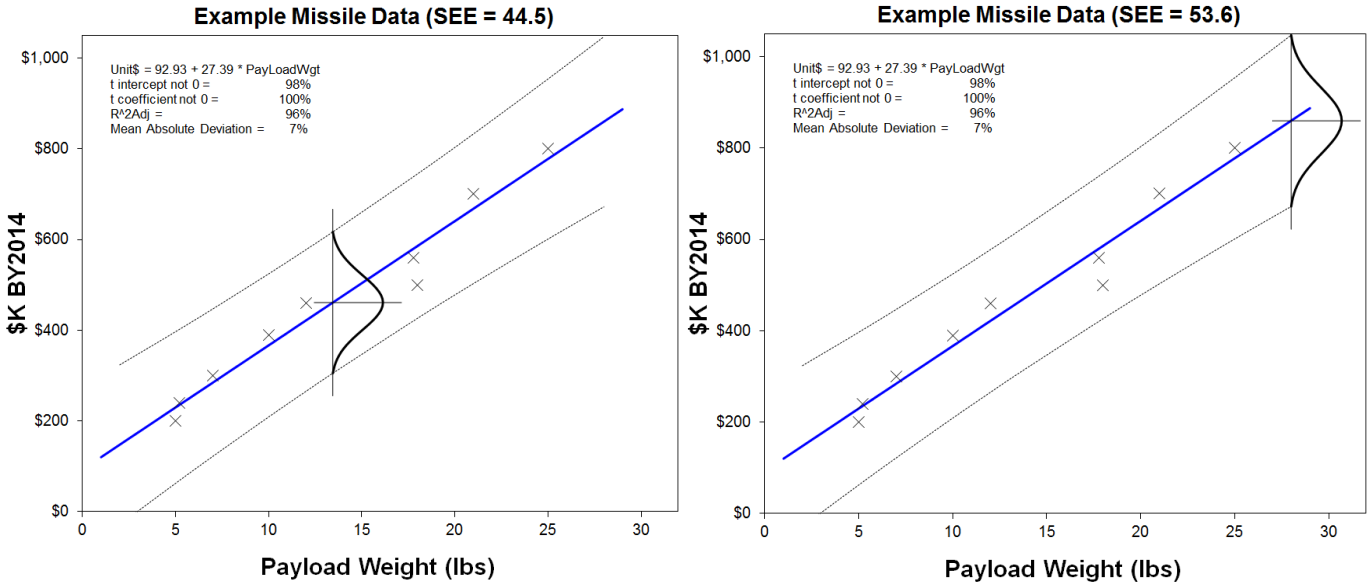


Figure A-2 Objective bounds a function of distance from data center

A.3.4 Subjective Uncertainty

In the context of cost and schedule estimating, many decisions that heavily influence the risk analysis will be subjective in nature, meaning they are based more on "expert opinion" than rigorous statistical analysis. Uncertainty is characterized as "subjective" when there is a lack of information to characterize it objectively. Subjective uncertainties have long been criticized for their lack of rigor and have relatively poor standing in fields driven by empirical study, where matters of precision and repeatability are considered paramount. Nevertheless, subjective uncertainty plays a large role in cost and schedule estimates due to limitations in time, resources and relevant data.

A.3.5 Cost Estimating Uncertainty

Cost estimating uncertainty is the uncertainty associated with the estimating method that is employed. General cost estimating methods include: analogy, engineering build-up, and parametric. Each methodology and/or technique has its own, unique level of uncertainty. If the method is merely the sum of labor and material costs, the analyst must assess the cost uncertainty to capture rate, technical, configuration, and schedule uncertainty. When the method is parametric or a single point analogy, there is technical, configuration, and schedule uncertainty embedded in the underlying data used to create the method. For parametric CERs, the uncertainty is also a function of where the point estimate will fall in the data range (the further the input variable is from the center of mass data used to derive the CER, the greater the uncertainty of the CER). Several other subjective factors may influence how the analyst should adjust (subjectively) the objective uncertainty associated with the method, such as: deficiencies in the quality of the data due to variations in contractors' accounting practices; assumptions made to normalize the data; and other cost estimating influences not captured by the cost estimating method.

A.3.6 Technical Uncertainty

Ideally, to generate a parametric CER, the analyst will select and use relevant historical data that were successful in meeting similar technical challenges to the program being estimated. By doing so, the technical uncertainty is at least somewhat captured by the analysis of that data. However, if the program to be estimated is facing unusual technical challenges, then an adjustment to the CER uncertainty distribution is required. The same can be said about an analogous estimate.

A.3.7 Configuration Uncertainty

Configuration uncertainty is the variation in the fundamental technical cost drivers of a WBS element. Configuration uncertainty is a form of technical uncertainty. From the cost estimator's perspective, configuration uncertainty falls into two categories: uncertainty in input parameters that are captured by the cost model and uncertainty in parameters or basic configuration features that cannot be addressed without modifying the structure of the cost model. An example of the first type of uncertainty event would be design changes that reduced the weight of an antenna by 12 lb. An example of the second kind of uncertainty event would be a requirements change that switched from a mechanically-steered antenna to a phased array antenna.

A.3.8 Schedule Uncertainty

Schedule uncertainty is the variation in the possible key dates associated with a WBS item. The elemental units of a program's schedule are called activities. A WBS item normally comprises several schedule activities. The duration of the individual schedule activities is driven by three factors: the technical difficulty of the work to be performed, the qualifications of the people performing the work, and the availability of an adequate number of people/resources to do the work. In other words, schedule uncertainty is driven by technical uncertainty. Schedule activities influence each other through precedence relationships (e.g., Activity C cannot begin until Activities A and B are complete). Schedule precedence relationships often cut

across WBS items. Because of these interrelationships, schedule slips in one WBS item can impact the duration of activities in other WBS items, and can actually increase the labor hours in the impacted items. Programs that have a high degree of technical interrelationship between activities and a high degree of concurrent activity inherently have high schedule uncertainty. In other words, the topology of the program schedule has a strong impact on the schedule uncertainty.

A.4 DESCRIPTIVE STATISTICS

In this section we will provide the definitions for the most common statistics used to quantitatively describe the main features of a collection of data. A univariate analysis is the process of describing the distribution of a single variable, including its central tendency (including the mean, median, and mode) and dispersion (including measures of dispersion (spread) such as the variance and standard deviation and the range and quartiles of the data set). The shape of the distribution may also be described via indices such as skewness and kurtosis (peakness).

A.4.1 Expected Value, Average or Mean

The expected value is the arithmetic average or mean of the distribution of possible values for a variable. For a given set of n values ($y_1, y_2, \ldots, y_n$), the mean ($\bar{y}$) is defined to be the arithmetic average of these n values. In mathematical notations, it is given by

Equation A-1 Arithmetic Mean

$$\bar{y} = \frac{\sum_{i=0}^{n} y_i}{n}$$

The arithmetic mean is a composite measure and has the following characteristics:

- The most widely known and used average
- It is an artificial value, since it may not coincide with any actual value
- It is affected by the value of every item, but may be unduly affected by extreme values especially in small data sets

Figure A-3 shows how the expected value is simply the sum of all values divided by the number of values. Expected values have an important mathematical property: the sum of the expected values of a set of variables is equal to the expected value of the sum of the set of variables. In other words, when summing the expected values of a number of WBS items, the result will be the expected value of the sum of the WBS items.

A.4.2 Geometric Mean

It is not common for statistical packages to report the geometric mean by default or as part of a descriptive statistics summary (for a discussion on the difference between the arithmetic and geometric mean in the context of cost analysis, see **Reference 73**). It is, however, an important concept for cost analysis as it takes the effect of compounding into account. The geometric mean is used when the average rate of growth is to be measured (as opposed to the arithmetic mean that is appropriate when the average of a number of values is of interest). The geometric mean is given by:

Equation A-2 Geometric Mean

$$\bar{y}_g = \sqrt[n]{x_1 * x_2 * x_3 \ldots * x_n}$$

To illustrate the proper use of the geometric mean, consider a situation where $100 is inflated from FY 2000 for ten years. We are accustomed to simply referring to inflation tables to extract and use the total inflation factor. What if we were interested in calculating the mean inflation factor over the ten-year period? **Table A-2** shows the explicit calculation of applying the OSD raw inflation year by year to arrive at the value at the end of 10 years of inflation (column 3 and 4). Three different ways to calculate the mean inflation are illustrated in columns 5 – 7: two are incorrect and one is correct.

- Method A calculates the difference between the ending and beginning values; divides by the original value (100); and further divides that by the number of periods (1) to arrive at a mean inflation
- Method B is the arithmetic mean of the ten inflation factors
- Method C is the geometric mean of the ten inflation factors and is the correct method

Method A, in this case, returns the least accurate result. Method B may be intuitive for some, and in this case it is near the correct result, but it is nonetheless wrong. Method C delivers the correct result, demonstrating that when the result depends on individual rates being *multiplied together*, the geometric mean is the correct approach.

Table A-2 Geometric Mean Example

	1	2	3	4		5	6	7
End of FY	**OSD Inflation**	**Inflation Factor**	**Inflate by Year**	**Direct Calculation**		**Inflate Using Mean From: A**	**B**	**C**
2000				100.00		100.00	100.00	100.00
2001	1.80%	1.018	100.00 * 1.018 =	101.80		102.06	101.89	101.89
2002	0.80%	1.008	101.80 * 1.008 =	102.61		104.15	103.82	103.81
2003	1.00%	1.010	102.61 * 1.010 =	103.64		106.29	105.78	105.77
2004	2.00%	1.020	103.64 * 1.020 =	105.71		108.48	107.78	107.76
2005	2.80%	1.028	105.71 * 1.028 =	108.67		110.71	109.81	109.80
2006	3.10%	1.031	108.67 * 1.031 =	112.04		112.98	111.89	111.87
2007	2.70%	1.027	112.04 * 1.027 =	115.07		115.31	114.00	113.98
2008	2.40%	1.024	115.07 * 1.024 =	117.83		117.68	116.16	116.13
2009	1.50%	1.015	117.83 * 1.015 =	119.60		120.09	118.35	118.32
2010	0.80%	1.008	119.60 * 1.008 =	120.5532		122.5621	120.5912	120.5532
		Total Inflation	120.55 - 100.00 =	20.55				
		Mean Inflation	1 + 20.55/100/10 =	1.0206	A			

	Inflation Factor	
Arithmetic Mean	1.01890	**B**
Geometric Mean	1.01887	**C**

A.4.3 Median

The median is the point in a distribution where half the observed values will be lower and half will be higher (the 50th percentile). In other words, this is the point where the actual cost is just as likely to be higher as it is to be lower. For a finite number of observations, if the sample size is odd, the median is the middle value. If the sample size is even, the median is the average of the middle two values. The sum of the medians of all lower level WBS elements is not equal to the median of the parent WBS element, except in the unusual case in which the distributions of all the WBS items are perfectly symmetrical.

A.4.4 Most-likely Value (Mode)

The mode is the most probable single value for a variable (the peak of the distribution, see **Figure A-3**). The independent variables of a cost estimating relationship (CER) are often interpreted as most-likely values.

However, the CER's result (i.e. the point estimate) for a WBS item is typically interpreted as the mean or median. Even if all the WBS element point estimates were modes, their sum would not equal the most-likely value of total result, except in the unusual case in which the distributions of all the WBS items are symmetric.

A.4.5 Skewness vs Skew

A distribution is said to be skewed if one of its two tails is longer than the other. For example, if there is a long tail to the right of the distribution, then it is said to be positively skewed (or skewed right). This means that the distribution has a long tail in the positive direction. Similarly, if there is a long tail to the left, then the distribution is said to be negatively skewed (or skewed left). If the distribution is symmetrical, then the distribution has no skew. For example, the normal distribution has a skewness value of 0 since it is a symmetric distribution.

For a random variable Y, the measure of skewness is a parameter that describes asymmetry in the probability distribution of Y. It is defined by

Equation A-3 Skewness

$$Skewness(Y) = E\left(\frac{y-\mu}{\sigma}\right)^3$$

where μ and σ are the mean and standard deviation of the random variable Y, respectively. As for a sample data set of n values ($y_1, y_2, \ldots, y_n$), the formula to compute the skewness factor is given below:

Equation A-4 Skewness for a Sample

$$Skewness\,(Y) = \frac{n}{(n-1)(n-2)}\sum_{i=1}^{n}\left(\frac{y_i-\bar{y}}{s}\right)^3 = \frac{n}{(n-1)(n-2)}\frac{\sum_{i=1}^{n}(y_i-\bar{y})^3}{\left(\frac{1}{n-1}\sum_{i=1}^{n}(y_i-\bar{y})^2\right)^{1.5}}$$

where s is used to denote the sample standard deviation. Note that Equation A-4 is the adjusted Fisher-Pearson standardized moment coefficient. It is used to calculate the skewness measure in Excel and several statistical packages, including Minitab, SAS, and SPSS. As a general rule, the mean is larger than the median in positively skewed distributions and less than the median in negatively skewed distributions. Although counter examples can be found, they are rare in real data.

Another definition of skew, used in this handbook and commonly used elsewhere, is the measure of the area to the left of the mode of the distribution. If the area is half the total area, the distribution is symmetric. A skew of 0.25 would mean that 75% of the distribution is to the right of the mode, making it a skewed right distribution. In the case of triangular distributions the skew can be calculated from the min, mode and max given the area under any probability distribution is equal to 1. The formula reduces to:

Equation A-5 A Simpler Skew Formula Applicable to Triangles

Triangular Skew = (Mode-Min)/(Max-Min)

The same equation can be used for a uniform distribution. Since there is no mode in a uniform distribution, the point estimate position in the uniform distribution can be used in place of the mode in the skew equation. That is, the formula describes the probability to the left of the point estimate for a uniform distribution. This concept is used to calculate the adjustment for skew described in **Section 2.5.4** for a uniform distribution.

A.4.6 Variance

The variance is the average squared distance of each value from the mean, but it is not expressed in the units of measure of the mean or the original data. Rather, the data is expressed in units of measure of the original data squared. The measure of variance is greatly affected by extreme values. To calculate the variance, first calculate the arithmetic mean and then for each data point, find the difference between the point and the mean. Next, square all these differences and sum them. This sum is then divided by the number of items in the data set (if the data is from a sample rather than the entire population, the sum is divided by the number of items minus one).

A.4.7 Standard Deviation

The standard deviation is one of the most widely used statistics for measuring the spread, or dispersion, of values in a population of data. For a given set of n values ($y_1, y_2, \ldots, y_n$), the standard deviation (Stdev or S) is defined by

Equation A-6 Standard Deviation For a Sample or Population

$$s = \begin{cases} \sqrt{\dfrac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n-1}} & \text{if data is from a sample} \quad -\text{Std. Dev. (Sample)} \\ \sqrt{\dfrac{\sum_{i=1}^{n}(y_i - \bar{y})^2}{n}} & \text{if data is from a population} \quad -\text{RMS (Population)} \end{cases}$$

In essence, the standard deviation measures the amount of spread about the mean in the data set. It is also greatly affected by extreme values. The standard deviation is the square root of the variance. However, unlike the variance, the standard deviation is in the same unit of measure as the data, and this is a primary reason for its popularity. By default, tools like Excel calculate the sample standard deviation.

A.4.8 Coefficient of Variation

The coefficient of variation (CV) of a distribution is defined as the ratio of the standard deviation to its mean.

Equation A-7 Coefficient of Variation

$$\text{CV} = \text{Stdev/Mean}$$

CV is a relative measure of dispersion because it expresses the standard deviation as a percentage of the mean. CV is one of the more recognized metrics to characterize the spread in a sample or a population. Whenever an S-curve is displayed, the CV should also be displayed. Without it, the scale of the chart can provide a misleading sense of the dispersion. CV is also known as a normalized measure of dispersion for data that can only take non-negative values. The measure of CV would be meaningless for a data set running from a negative region to a positive region.

A.4.9 Inter-quartile Range

The inter-quartile range is the length of the interval that contains the middle fifty percent of the values in an ordered data set. The ordered data is broken into four roughly equal groups. The first quartile separates the lowest valued quarter from the second quarter. The second quartile (the median) separates the second quarter from the third quarter. The third quartile separates the third quarter from the last quarter. The inter-quartile range is the difference between the end of the first quartile and the beginning of the third quartile, which covers the middle fifty percent of the values in the data set.

A.5 PROBABILITY

A.5.1 Overview

Probability is the relative frequency of an outcome of a repeatable, observable experiment. Probability is measured on a scale between 0 and 1. Probability is assigned to each outcome of an experiment based on its relative frequency where 1 represents always and 0 represents never.

A.5.2 Probability Distribution

A probability distribution is a mathematical formula that describes how the relative frequency of occurrence is assigned to the real numbers in the range of a random variable. The distribution may be described by either a density function p(x) or a cumulative probability function F(x). These functions are two different presentations of the same data. In **Figure A-3,** the dark, curved line represents the statistical distribution underlying the sample data shown in the table at the left. This type of curve is also called a Probability Density Function (PDF).

Computer Prices

Vendor 1	976.07	
Vendor 2	980.13	
Vendor 3	1,098.79	
Vendor 4	1,173.85	
Vendor 5	1,186.67	
Vendor 6	1,188.86	
Vendor 7	1,238.30	
Vendor 8	1,242.35	
Vendor 9	1,305.64	
Vendor 10	1,347.96	
Vendor 11	1,357.49	Median
Vendor 12	1,374.73	
Vendor 13	1,499.69	
Vendor 14	1,539.91	
Vendor 15	1,586.31	
Vendor 16	1,726.37	
Vendor 17	2,044.31	
Vendor 18	2,074.57	
Vendor 19	2,219.39	
Vendor 20	2,392.14	
Total	29,553.55	
Expected Value (Total / 20)	1,477.68	
Median	1,352.73	
≅ (1,347.96 + 1,357.49) / 2		

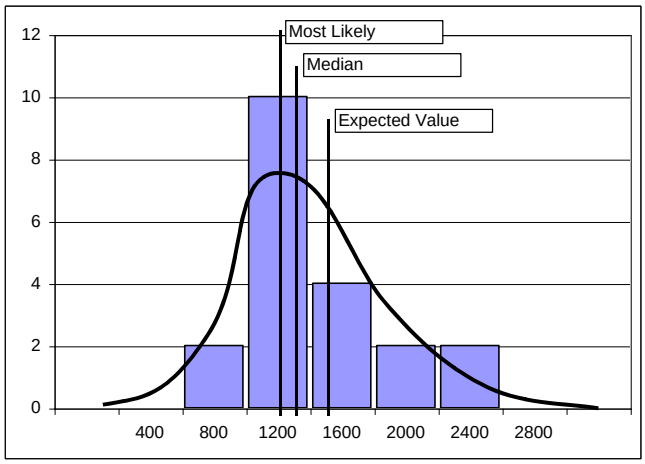


Figure A-3 Distribution Example

A.5.3 Probability Density Function (PDF)

A continuous PDF is a function that describes the relative likelihood of a random variable to be a given value. The area under any PDF is equal to 1. The area under a pdf for a specific interval identifies the probability associated with that interval.

A.5.4 Cumulative Distribution Function (CDF)

The CDF is a mathematical curve that identifies the probability that the actual value will be less than or equal to the given value. When shown graphically, the CDF is an S-shaped curve. The term S-curve is used synonymously with CDF. In mathematical terms, the definition of the cumulative distribution function of a random variable X gives the probability of obtaining a value equal to or less than x. The value of X is also called the x percentile.

Equation A-8 Cumulative Distribution Function (CDF)

$$P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$$

The value of a cumulative distribution function is bounded between 0 and 1, with 0.5 indicating the median of the population as illustrated in **Figure A-4**.

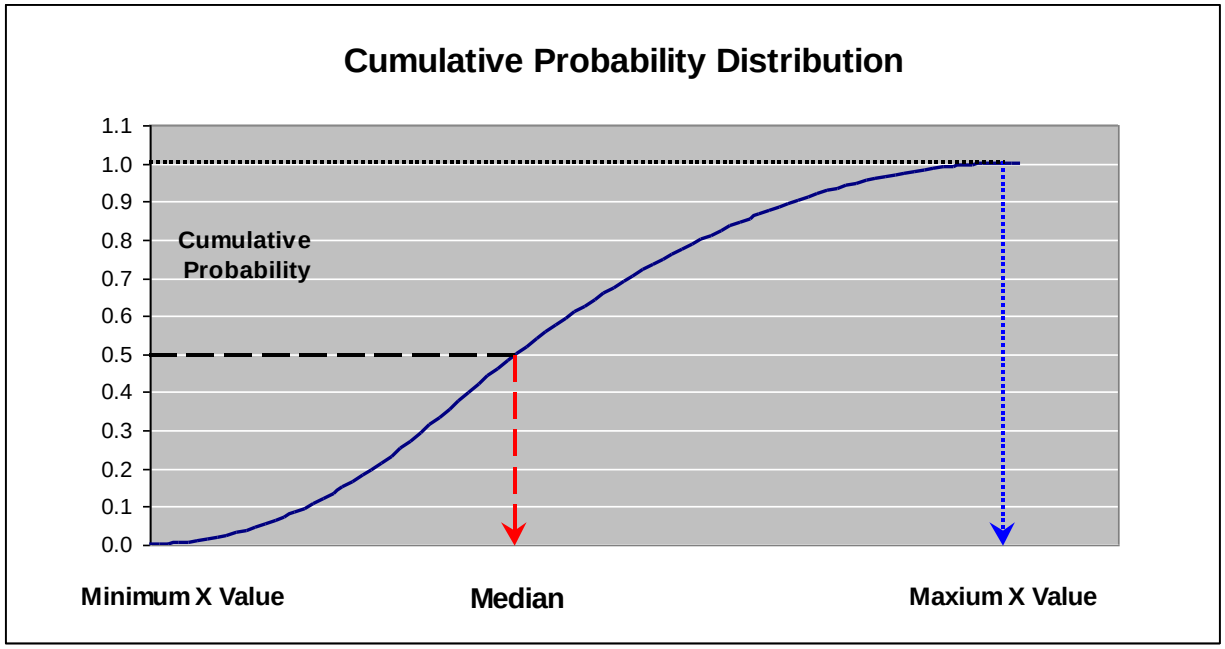


Figure A-4 Cumulative Probability Distribution (CDF)

A.6 PROBABILITY DISTRIBUTIONS

Probability distributions are a central feature of any uncertainty model. They are used to define the potential range of a CER result and/or the variables that drive them. This section describes basic characteristics of common probability distributions used in uncertainty models.

A.6.1 Normal Distribution

The normal distribution is a continuous probability distribution, defined on the entire real line. It has a bell-shaped probability density function, sometimes called a bell curve. A normal distribution is often used as a first approximation to describe real-valued random variables that cluster around a single mean value.

The normal distribution probability density (PDF) function is also known as the Gaussian function or informally as the bell curve. The PDF is given by:

Equation A-9 Normal Probability Density Function (PDF)

$$f(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

Where:

μ is the mean
σ^2 is the variance.
σ is known as the standard deviation.

On a non-truncated normal distribution the mean, median and mode are all the same. The distribution with μ = 0 and σ = 1 is called the standard normal distribution or the unit normal distribution and is usually denoted by Z. **Figure A-5** compares Z with other normal distributions with various standard deviations.

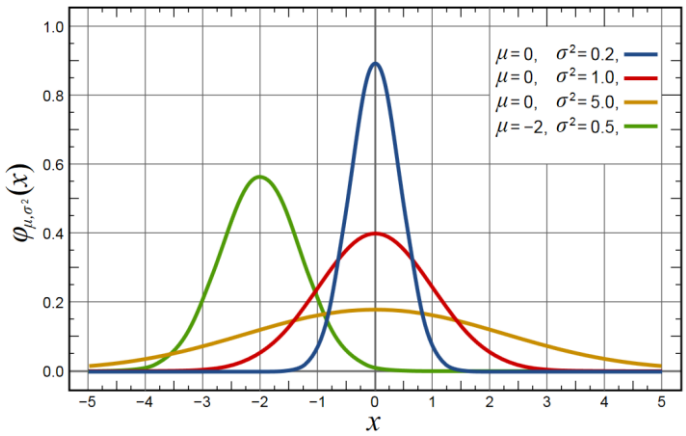


Figure A-5 Normal Distribution Probability Density Function

The cumulative distribution function (CDF) describes probability of a random variable falling in the interval $[-\infty, x]$. The CDF of the standard normal distribution is denoted with the capital Greek letter Φ (phi), and can be computed as an integral of the probability density function:

Equation A-10 Normal Cumulative Distribution Function

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)\right], \qquad x \in \mathbb{R},$$

This integral cannot be expressed in terms of elementary functions, so is simply called a transformation of the error function, or **erf**, a special function. For a generic normal random variable with mean μ and variance $\sigma^2 > 0$ the CDF will be equal to:

Equation A-11 Generic Normal Cumulative Distribution Function

$$F(x;\ \mu, \sigma^2) = \Phi\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{x-\mu}{\sigma\sqrt{2}}\right)\right], \qquad x \in \mathbb{R},$$

where x is defined on the entire real line.

A.6.2 Student's-t

The Student's t-distribution (or simply the t-distribution) is a family of continuous probability distributions that arises when estimating the mean of a normally distributed population in situations where the sample size is small (less than 30) and population standard deviation is unknown.

Student's t-distribution has the probability density function given by:

Equation A-12 Student's-t Probability Density Function

$$f(t) = \frac{\Gamma(\frac{v+1}{2})}{\sqrt{v\pi}\,\Gamma(\frac{v}{2})}\left(1 + \frac{t^2}{v}\right)^{-\frac{v+1}{2}}$$

Where:

ν is the number of degrees-of-freedom
Γ is the Gamma function.

In regression analysis, the degrees-of-freedom is number of sample points minus number of estimated parameters. The student's-t distribution, illustrated in **Figure A-6**, may also be written as:

Equation A-13 Student's-t Probability Density Function Alternative Form

$$f(t) = \frac{1}{\sqrt{v}\,\mathrm{B}(\frac{1}{2},\frac{v}{2})}\left(1+\frac{t^2}{v}\right)^{-\frac{v+1}{2}}$$

where *B* is the beta function.

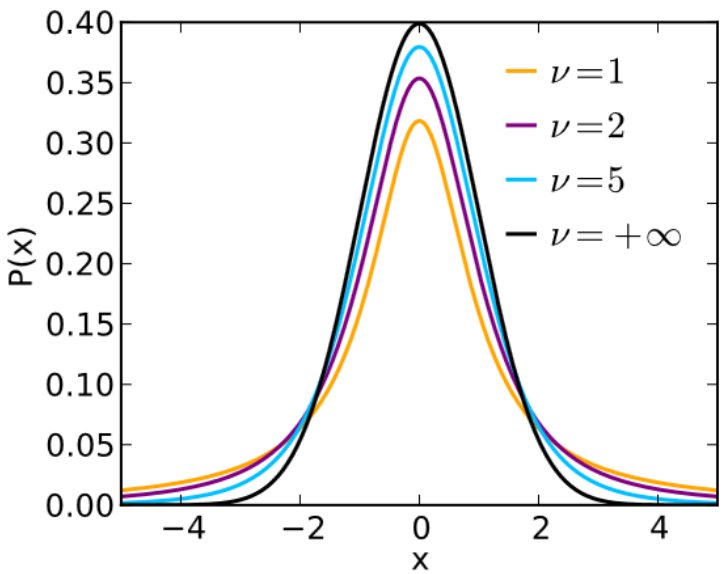


Figure A-6 Student's-t Probability Density Function

The cumulative distribution function can be written in terms of *I*, the regularized incomplete beta function.

Equation A-14 Student's-t Cumulative Distribution Function

$$\mathrm{F}(t) = \int_{-\infty}^{t} f(u)du = 1 - \frac{1}{2}\, I_{x(t)}\left(\frac{v}{2},\frac{1}{2}\right),$$

For $t > 0$

with

$$x(t) = \frac{v}{t^2+v}$$

A.6.3 Lognormal Distribution

In probability theory, a lognormal distribution is a continuous probability distribution of a random variable whose logarithm is normally distributed. If *X* is a random variable with a normal distribution, then *Y* = exp(*X*) has a lognormal distribution; likewise, if *Y* is lognormally distributed, then *X* = log(*Y*) has a normal distribution. A random variable which is lognormally distributed takes on only positive real values.

The probability density function, illustrated in **Figure A-7**, of a lognormal distribution is:

Equation A-15 Lognormal Probability Density Function

$$f_X(x;\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}}\, e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}, \qquad x > 0$$

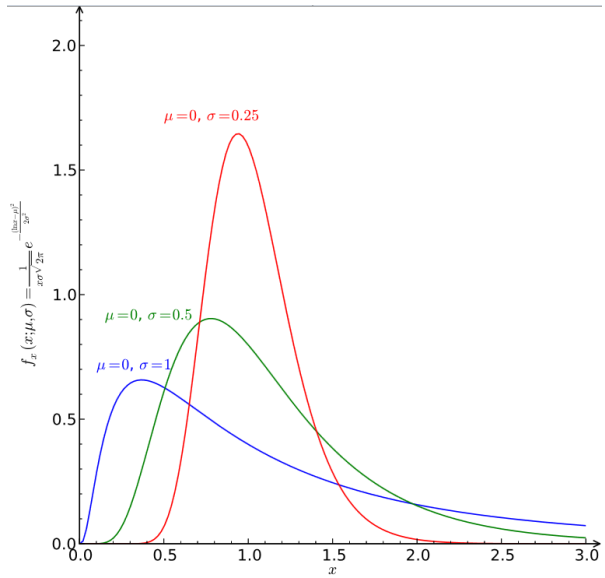


Figure A-7 Lognormal Probability Density Function

Equation A-16 Lognormal Cumulative Distribution Function

$$F_X(x;\ \mu,\sigma\,) = \frac{1}{2}\ \mathrm{erfc}\left[-\frac{\ln x-\mu}{\sigma\sqrt{2}}\right] = \ \Phi\ \left(\frac{\ln x-\ \mu}{\sigma}\right),$$

Where:

erfc is the complementary error function
Φ is the cumulative distribution function of the standard normal distribution

Properties of the lognormal distribution in unit space can be calculated as follows:

Equation A-17 Lognormal Mean

$$e^{\mu+\sigma^2/2}$$

Equation A-18 Lognormal Median

$$e^{\mu}$$

Equation A-19 Lognormal Mode

$$e^{\mu-\sigma^2}$$

Equation A-20 Lognormal Variance

$$e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right)$$

A.6.4 Calculating Alternative Lognormal Parameters

Analysts (and tools) have preferred ways for defining a lognormal distribution. In this section, **Table A-3**, **Table A-4** and **Table A-5** provide some common translations. In these tables, the "85% Bound" is the 85%

value divided by the median. This is provided to support those wishing to define the 85% bound as a percent of the lognormal median which is a common assumption for the location of the point estimate within the lognormal. Additionally, note Standard Error (SE) and Standard Error of the Estimate (SEE) have the same meaning and are used interchangeably in these tables.

Table A-3 Calculate Lognormal Parameters Given Unit Space Mean and Std Dev

	Unit Space Input			Log Space		Unit Space	
WBS Element	**Mean**	**Std Dev**	**Median**	**Mean**	**SEE**	**85% Value**	**85% Bound**
CER Result	100.0000	35.0000	94.39	4.5474	0.3399	134.25	142.24%

Unit Space Mean	From regression report
Unit Space Stdev	From regression report
Unit Space Median	Exp(MeanLogSpace)
Log Space Mean	LN(MeanUnitSpace)-SElogSpace^2/2
Log Space SE	SQRT(LN(StdevUnitSpace^2/(MeanUnitSpace^2)+1))
85% of Median	LOGINV(0.85,MeanLogSpace,SElogSpace)/MedianUnitSpace

Table A-4 Calculate Lognormal Parameters Given Unit Space Median & Std Dev

	Unit Space Input			Log Space		Unit Space	
WBS Element	**Mean**	**Std Dev**	**Median**	**Mean**	**SEE**	**85% Value**	**85% Bound**
CER Result	100.0000	35.0000	94.39	4.5474	0.3399	134.25	142.24%

Unit Space Mean	MedianUnitSpace*EXP((SElogSpace^2)/2)
Unit Space Stdev	From regression report
Unit Space Median	From regression report
Log Space Mean	LN(MedianUnitSpace)
Log Space SE	SQRT(LN((1+SQRT(1+4*(SdevUnitSpace/MedianUnitSpace)^2))/2))
85% of Median	LOGINV(0.85,MeanLogSpace,SElogSpace)/MedianUnitSpace

Table A-5 Calculate Lognormal Parameters Given Unit Space Median & Log Space SEE

	Unit Space Input			Log Space		Unit Space	
WBS Element	**Mean**	**Sdev**	**Median**	**Mean**	**SEE**	**85% Value**	**85% Bound**
CER Result	100.0000	35.0000	94.39	4.5474	0.3399	134.25	142.24%

Unit Space Mean	EXP(MeanLogSpace+SElogSpace^2/2)
Unit Space Stdev	((EXP(SElogSpace^2)-1)*MeanUnitSpace^2)^0.5
Unit Space Median	CER result
Log Space Mean	LN(MedianUnitSpace)
Log Space SE	From regression report
85% of Median	LOGINV(0.85,MeanLogSpace,SElogSpace)/MedianUnitSpace

A.6.5 Log-t Distribution

This distribution is typically used to quantify the uncertainty for a point estimate obtained from a CER that was created from a small sample set and the error term of the CER is assumed to follow a lognormal distribution.

The log-t distribution is derived from a student's-t distribution. If a random variable X follows a student's-t distribution with n degrees of freedom, then **Y = exp(μ + σX)** has a log-t distribution with the location parameter (μ), the scale parameter (σ), and n degrees of freedom. The probability density function of Y is given by the following formula:

Equation A-21 Log-t Probability Density Function

$$f(y) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\sqrt{n\pi}\,\sigma y}\left(1 + \frac{(\mathrm{LN}(y) - \mu)^2}{n\sigma^2}\right)^{-\frac{n+1}{2}}$$

for y > 0

The parameters for the log-t distribution are the (1) location parameter (μ), (2) scale parameter (σ), and (3) degrees of freedom (n). Both the location and scale parameters are evaluated in **log** space.

The CDF of a log-t random variable Y (Y ~ log-t(μ,σ,n)) can be expressed as follows (in terms of its probability density function):

Equation A-22 Log-t Cumulative Distribution Function

$$F_{Lt}(y) = P(Y \le y) = \int_0^y \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)\sqrt{n\pi}\,\sigma t}\left(1 + \frac{(\mathrm{LN}(t) - \mu)^2}{n\sigma^2}\right)^{-\frac{n+1}{2}} dt = F_t\left(\frac{\mathrm{LN}(y) - \mu}{\sigma}\right)$$

Where:

F_{Lt} denotes the CDF of log-t(μ,σ,n)
n is an integer
F_t is the CDF of a t distribution with n degrees of freedom
LN stands for the natural logarithm.

The mean of the log-t distribution does not exist because the right tail is too heavy. Other properties of the log-t distribution in unit space can be calculated as follows:

Equation A-23 Log-t Mode

$$Mode = \exp\left(\mu - 0.5 * ((n+1) - \sqrt{(n+1)^2 - 4n\sigma^2})\right)$$

A.6.6 Triangular Distribution

The triangular distribution is a continuous probability distribution with lower limit a, upper limit b and mode c, where a < b AND a ≤ c ≤ b. The probability density function, illustrated in **Figure A-8**, is given by:

Equation A-24 Triangular Probability Density Function

$$f(x|a,b,c) = \begin{cases} 0 & for\ x < a, \\ \dfrac{2(x-a)}{(b-a)(c-a)} & for\ a \le x \le c, \\ \dfrac{2(b-x)}{(b-a)(b-c)} & for\ c < x \le b, \\ 0 & for\ b < x, \end{cases}$$

whose cases avoid division by zero if c = a or c = b.

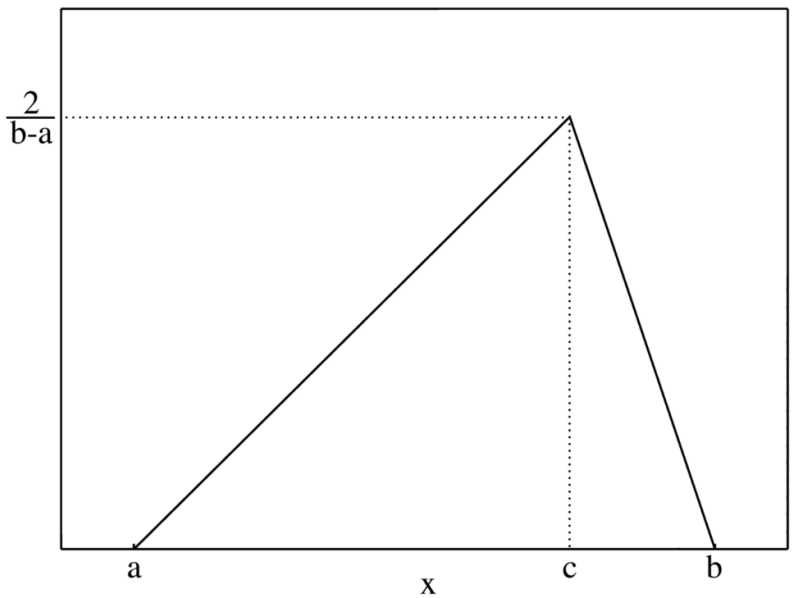


Figure A-8 Triangular Probability Density Function

The cumulative distribution function is given by:

Equation A-25 Triangular Cumulative Distribution Function

$$CDF = \begin{cases} 0 & for\ x < a, \\ \dfrac{(x-a)^2}{(b-a)(c-a)} & for\ a \le x \le c, \\ 1 - \dfrac{(b-x)^2}{(b-a)(b-c)} & for\ c < x \le b, \\ 1 & for\ b < x. \end{cases}$$

Properties of the triangular distribution can be calculated as follows:

Equation A-26 Triangular Mean

$$Mean = \frac{a+b+c}{3}$$

Equation A-27 Triangular Median

$$Median = \begin{cases} a + \frac{\sqrt{(b-a)(c-a)}}{\sqrt{2}} \; for\ c \geq \frac{a+b}{2}, \\ b - \frac{\sqrt{(b-a)(b-c)}}{\sqrt{2}} \; for\ c \leq \frac{a+b}{2} \end{cases}$$

Equation A-28 Triangular Variance

$$Variance = \frac{a^2 + b^2 + c^2 - ab - ac - bc}{18}$$

A.6.7 Beta Distribution

In probability theory and statistics, the beta distribution is a family of continuous probability distributions defined on the interval [0, 1] parameterized by two positive shape parameters, denoted by α and β, that appear as exponents of the random variable and control the shape of the distribution. The beta distribution has been applied to model the behavior of random variables limited to intervals of finite length in a wide variety of disciplines

The probability density function of the beta distribution, for $0 \leq x \leq 1$, and shape parameters $\alpha > 0$ and $\beta > 0$, illustrated in **Figure A-9**, is a power function of the variable x and of its reflection (1−x) as follows:

Equation A-29 Beta Probability Density Function

$$PDF = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\mathrm{B}\,(\alpha, \beta)}$$

The beta function, B, appears as a normalization constant to ensure that the total probability integrates to unity.

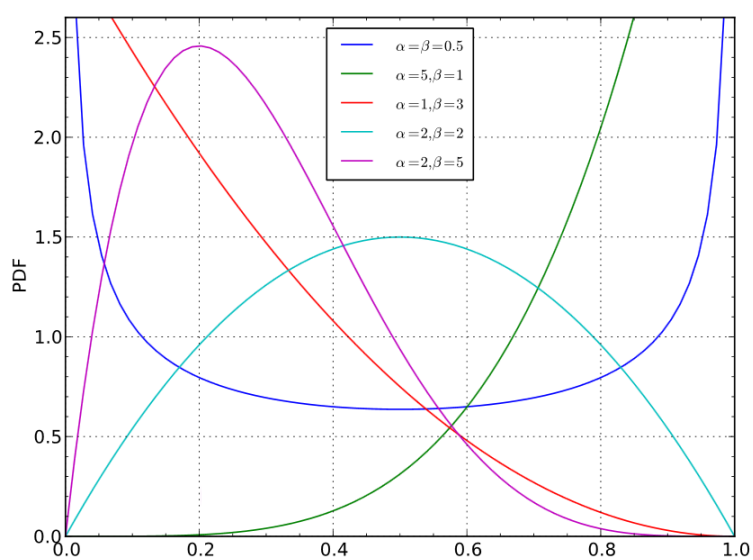


Figure A-9 Beta Probability Density Function

The cumulative distribution function is given by:

Equation A-30 Beta Cumulative Distribution Function

$$F(x; \alpha, \beta) = \frac{\mathrm{B}(x; \alpha, \beta)}{\mathrm{B}(\alpha, \beta)} = \mathrm{I}_x(\alpha, \beta)$$

where $\mathrm{B}(x;\alpha,\beta)$ is the incomplete beta function and $\mathrm{I}_x(\alpha,\beta)$ is the regularized incomplete beta function. The regularized incomplete beta function is the cumulative distribution function of the beta distribution.

Properties of the general beta distribution where the minimum and maximum can be any value (rather than assumed to be 0 and 1) can be calculated as follows:

Equation A-31 Beta Mean

$$Mean = a + \frac{(b-a)\,\alpha}{(\alpha+\beta)}$$

where *a*=minimum value and *b*=maximum value

Equation A-32 Beta Variance

$$Variance = \frac{(b-a)^2\,\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

A.6.8 BetaPERT Distribution

The betaPERT distribution is a variation of the beta distribution, but only requires three parameters: minimum, most-likely and maximum. The fourth parameter can be determined by assuming a relationship between the mode and the minimum and maximum. The most common assumption, used by all popular simulation tools is that the mode is weighted four times as much as the minimum or maximum. With that assumption in mind, α and β of the beta distribution can be calculated as follows (see **Reference 62**):

Equation A-33 BetaPERT Calculating Alpha and Beta for the Beta Distribution

$$\alpha = 1 + 4\frac{(c-a)}{(b-a)}$$

$$\beta = 1 + 4\frac{(b-c)}{(b-a)}$$

Where a= min, b= max and c = the mode

Properties of the betaERT can be calculated as follows:

Equation A-34 BetaPERT Mean

$$Mean = \frac{(a+4\mathrm{c}+b)}{6}$$

For a and b > 1, the mode can be found as follows:

Equation A-35 BetaPERT Mode

$$Mode = a + \frac{(b-a)(\alpha-1)}{\alpha+\beta-2}$$

Equation A-36 BetaPERT Variance

$$Variance = \left(\frac{\alpha}{\alpha+\beta}\right)\left(\frac{\beta}{\alpha+\beta}\right)\left(\frac{(b-a)^2}{\alpha+\beta+1}\right)$$

A.6.9 Uniform Distribution

The continuous uniform distribution or rectangular distribution is a family of probability distributions such that every value is equally probable. Defined by the two parameters, *a* and *b*, which are its minimum and maximum values, the distribution is often abbreviated *U*(*a*,*b*).

The probability density function is given by

Equation A-37 Uniform Probability Density Function

$$f(x) = \begin{cases} \dfrac{1}{b-a} & for\ a \le x \le b, \\ 0 & for\ x < a\ or\ x > b \end{cases}$$

The cumulative distribution function is given by:

Equation A-38 Uniform Cumulative Distribution Function

$$CDF = \begin{cases} 0 & for\ x < a \\ \dfrac{x-a}{b-a} & for\ a \le x < b \\ 1 & for\ x \ge b \end{cases}$$

The uniform distribution does not have a mode. Other properties for the uniform distribution can be calculated as follows:

Equation A-39 Uniform Mean

$$Mean = \frac{1}{2}(a+b)$$

Equation A-40 Uniform Median

$$Median = \frac{1}{2}(a+b)$$

Equation A-41 Uniform Variance

$$Variance = \frac{1}{12}(b-a)^2$$

A.7 REGRESSION METHODS

Regression analysis is a statistical technique used to find relationships between variables for the purpose of predicting future values. In cost estimating, regression analysis is used to develop cost estimating relationships (CERs) between a dependent variable (cost) and one or more independent variables (cost drivers such as weight, power, volume, etc.) from completed project data. By a statistical relationship it is meant that the observed variation of the dependent variable (cost) across similar projects can be explained or predicted by one or more independent variables (technical, performance, programmatic, etc.). The objective is to find the functional relationship that most accurately estimates the cost of a particular element in a project work breakdown structure. Additionally, the regression method should yield an objective assessment of the predictive capability of the CER. This assessment provides an objective basis for characterizing the uncertainty of the CER itself.

There are various techniques available to perform regression analysis. In order to correctly assign the appropriate uncertainty distribution to a given CER, the analyst needs to know how the CER was generated. The objective uncertainty distribution is a function of the method employed. Several of the most popular methods are described in the following sections along with guidance on how to capture their uncertainty in a simulation-based cost risk and uncertainty model.

A.7.1 Ordinary Least Squares (OLS)

Ordinary least squares is one of the most popular methods to employ because it is easy, the theory is well known, the CERs tend to be easy to understand and it yields meaningful descriptive statistics to characterize its significance and predictive power. It is a mathematical optimization technique used to find a "best linear fit" to a set of data. The object is to minimize the sum of the squared errors (SSE) which is the sum of the squared difference between the fitted line (i.e. the predicted values of the CER) and the source data for each data point. The goal of the OLS method is to find the linear equation such that the sum of all the squared deviations is as small as possible. Expressed mathematically, the best-fitting line is derived by solving for the coefficients (i.e., β_0, β_1, …, β_k) in the following equation such that SSE is minimized:

Equation A-42 Ordinary Least Squares

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k + \varepsilon \quad \text{where } \varepsilon \text{ is a random error term}$$

$$\text{Minimize } SSE = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$$

where: n = the total number of data points in the sample

Y_i = the ith observed value of the dependent variable (i.e., cost)

$\hat{Y}_i$ = the ith estimated value

In the context of cost estimating, Y is generally cost, man-hours or some other effort or resource. X is generally some technical or performance characteristic or metric that helps to explain the variation in cost across a number of projects. More than one X parameter might be found to be statistically significant in explaining cost variation.

A.7.2 Log-Error SEE

If the regression model is fit in log space, the SEE measure is given by

Equation A-43 Standard Error of the Estimate for a Log Error

$$\begin{aligned} SEE &= \left(\sum_{i=1}^{n} [\ln(Y_i) - \ln(\hat{Y}_i)]^2 \,/(n-p) \right)^{0.5} \\ &= \sqrt{SSE / dfe} \\ &= \sqrt{MSE} \end{aligned}$$

The Log-Error SEE is not stated with the units (e.g. dollars) of Y. For definitions of SSE, dfe and MSE, see **Appendix A.8.2**.

A.7.3 Other Multiplicative Error Term Regression Techniques

Multiplicative error terms are preferred in the cost analysis field because experience tells us that the error of an individual observation (e.g., cost) is generally proportional to the magnitude of the observation (not a constant). The general specification for a CER with a multiplicative error is stated as:

Equation A-44 Multiplicative Error Term CER Specification

$$Y_i = f(\mathbf{x}_i, \boldsymbol{\beta})\varepsilon_i \qquad \text{for i = 1, …, n}$$

where:

n = sample size

Y_i = observed cost of the ith data point, i = 1 to n

$f(\mathbf{x}_i,\boldsymbol{\beta})$ = the value of the hypothesized equation at the ith data point

$\boldsymbol{\beta}$ = vector of coefficients to be estimated by the regression equation

$\boldsymbol{x}_i$ = vector of cost driver variables at the ith data point

ε_i = error term with mean of 1 and variance σ^2

Minimization algorithms can be explored for modeling CERs with multiplicative errors based upon a generalized error term definition:

Equation A-45 Generalized Error Term

$$e_i = \frac{y_i - f(\mathbf{x}_i, \boldsymbol{\beta})}{f(\mathbf{x}_i, \boldsymbol{\beta})}$$

where e_i is normally distributed with a mean of 0 and variance σ^2.

This error term expresses the error of estimation as a percentage of the estimate. The percentage error represents the percent error of the residual about the regression function and the optimization objective is to find the coefficient vector $\boldsymbol{\beta}$ that minimizes the sum of squared e_is.

A.7.4 Minimum Unbiased Percentage Error (MUPE) Method

A refinement to Minimum Percentage Error (MPE) was proposed in **Reference 15** and **25** to solve for the function in the numerator separately from the function in the denominator through an iterative technique.

Equation A-46 MUPE Minimization Formulation

$$\sum_{i=1}^{n}\left(\frac{y_i - f(\mathbf{x}_i, \boldsymbol{\beta}_k)}{f(\mathbf{x}_i, \boldsymbol{\beta}_{k-1})}\right)^2 = \sum_{i=1}^{n}\left(\frac{y_i - f_k(\mathbf{x}_i)}{f_{k-1}(\mathbf{x}_i)}\right)^2$$

where k is the iteration number and the other terms are as defined previously.

This optimization technique is called the Minimum-Unbiased-Percentage Error (MUPE) method; it is also referred to as Iteratively Reweighted Least Squares (IRLS). As shown in the equation above, the weighting factor of each residual in the current iteration is equal to the reciprocal of the predicted value from the previous iteration. The final solution is derived when the change in the estimated coefficients ($\boldsymbol{\beta}$ vector) between the current iteration and the last iteration is within the analyst-specified tolerance limit. No transformation or adjustment (to correct the bias in unit space) is needed to fit a MUPE CER. Goodness-of-fit measures (or asymptotic goodness-of-fit measures) can be applied to judge the quality of the model under the “normality” assumption (i.e., $\varepsilon_i \sim N(1,\sigma^2)$). The MUPE CER has no "positive" sample bias; it has zero proportional error for all points in the database. It is an unbiased estimator of the model mean if the function is linear. Also, it produces consistent estimates of the parameters.

A.7.5 ZPB/MPE Method (or ZMPE Method)

There is another alternative method (see **Reference 10, 18, 50**, **83**) to reduce the positive bias for MPE CERs and yet maintain the same objective function. Mathematically, it is stated as follows:

Equation A-47 ZMPE Minimization Function

$$\text{Minimize} \sum_{i=1}^{n} \left(\frac{y_i - f(\mathbf{x}_i, \boldsymbol{\beta})}{f(\mathbf{x}_i, \boldsymbol{\beta})} \right)^2 = \sum_{i=1}^{n} e_i^{\,2}$$

$$\text{Subject to} \sum_{i=1}^{n} e_i = 0$$

This alternative method is called the "Constrained Minimum Percent Error" solution. It is also referred to as the MPE method under the Zero-Percentage Bias constraint, i.e., the ZPB/MPE or ZMPE method by Book and Lao, 1997 (see **Reference 18**).

A.7.6 Iterative Regression Techniques

The Gauss–Newton algorithm is a method used to solve non-linear least squares problems. It has been observed to have convergence problems on many datasets. For this reason there has been considerable interest in other methods. There are several different non-linear optimization techniques that might be used to fit non-linear functional forms to data. Well known techniques include Quasi-Newton, Conjugate Gradient, Downhill Simplex, and Marquart's methods. Non-linear regression techniques do not typically result in the normal types of statistics expected when performing OLS or MUPE. For example, rather than reporting traditional t-statistics, a non-linear regression will report approximate prediction ranges for each coefficient.

A.7.7 Error Term Summary

Regardless what method is used to generate the CER, it is very important that the user of the CER is aware of the CER result meaning and how the error should be modeled. Generally, lognormal distributions should be used as a default approach. If normal distributions for cost or effort are selected, they should be supported by appropriate supporting evidence.

For relatively small data sets, all CER best-fit methods tend to underestimate the level of underlying dispersion (as measured by the standard error) in the population as a whole. The result is a regression equation that is likely to be substantially different from the underlying relationships between the cost and its driver variables and (on the average) an underestimate of dispersion. Although there is a statistical bias (underestimating the underlying dispersion), in any given case, the CER dispersion can also be greater than the underlying dispersion.

The best way to assess the likelihood of an abnormally low (or high) SE is by comparing many CERs for similar products. Specifically, if the CERs have different size data sets (and degrees of freedom), the SE value for each CER should be plotted against its degrees of freedom (DF). CERs with low DF that are far away from the average should be used carefully. If they represent costly products and hence play a significant role in determining the overall system cost variance, then remedial action may be appropriate in the form of subjective, expert opinion increases (or decreases) to the CER SE.

A.8 ESTIMATING REGRESSION METHOD ACCURACY

A.8.1 The Use of Student's-t and Log-t Distributions

There are two types of error terms for a CER, additive or multiplicative, and they are given by:

Equation A-48 Additive Error Term

$$Y = f(x) + \varepsilon$$

Equation A-49 Multiplicative Error Term

$$Y = f(x)*\varepsilon$$

For an additive error term, the student's-t is an appropriate error distribution while the log-t is appropriate for the multiplicative error term when less than 30 data points are available. In either case, three parameters are required to fully define the distribution. Available parameters in ACE and Crystal Ball for the student's-t include:

- Two points on the curve (such as low, high, median) along with their associated percentiles
- Degrees of freedom
- Scale, populated by the adjusted SEE.

Note: you can either specify two points on the curve or the scale parameter of the distribution, but you must enter the degrees of freedom, which is a required input. Although the uncertainty distribution for an additive-error model is generally assumed to be normal (as in OLS), the student's t distribution is chosen to adjust for the degrees of freedom for small samples. When the sample size n is sufficiently large, say n > 50, you can select normal distribution instead. Here, the PE is assumed to be the mean of the normal distribution, which is the same as the median for a student's t distribution.

@RISK is somewhat different. It only requires the degrees of freedom because it models the standardized student's-t distribution of which the median is 0 and the variance is given by the following formula:

Equation A-50 Calculate Student-t Variance Given Degrees of Freedom (Mean is Zero and Scale is One)

$$\frac{\boldsymbol{v}}{\boldsymbol{v}-\mathbf{2}}$$

For degrees of freedom $v > 2$

It is necessary to scale the @RISK distribution based on the parameters of your random variable. To scale the @RISK result, multiply by one of the following equations:

Equation A-51 Formula to Scale the Standardized Student's-t Given a Value Above the Median

$$Scale = \frac{High - Median}{t^{-1}(p, df)} = \frac{High - Median}{TINV(2*(1-p), df)}$$

Equation A-52 Formula to Scale the Standardized Student's-t Given a Value Below the Median

$$Scale = \frac{Low - Median}{t^{-1}(p, df)} = \frac{Low - Median}{-TINV(2*p, df)} = \frac{Median - Low}{TINV(2*p, df)}$$

where:

$t^{-1}(p,\mathrm{df})$ (commonly denoted by $t_{1-p,\mathrm{df}}$) is the $(100p)^{th}$ percentile of the student's-t distribution with mean of zero, scale 1, and a specific degrees of freedom (denoted by df)

TINV	is an Excel function, which calculates the inverse of a cumulative t distribution at a specified probability level (see the help of the TINV function in Excel for details)
p	is used to denote the probability level of the given percentile

For the multiplicative error term, it is common to assume it follows a lognormal distribution, especially when f(x) = a*x^b (i.e., an OLS log-linear CER). The Log-T distribution is used to capture the effect of low degrees of freedom. When the sample size is sufficiently large (>30), the lognormal is appropriate.

For a MUPE CER select a student's-t distribution when the CER's SPE is moderately tight (say < 0.4 or 0.5).

A.8.2 Additive Error SEE

For CERs with additive error terms (for example, linear OLS), the standard deviation of the dependent variable (Y) is assumed to be the same across the entire range of the data, regardless of the value of the independent variables. Mathematically, it is equal to the square root of the mean squared error (MSE). If the regression model is fit in unit space, the SEE measure is given by:

Equation A-53 Standard Error of the Estimate

$$SEE = \left(\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2 / (n-p) \right)^{0.5}$$
$$= \sqrt{SSE / dfe}$$
$$= \sqrt{MSE}$$

Where dfe is the number of degrees of freedom of the sum of squares error (SSE). Dfe is the number of observations (n) minus the total number of estimated coefficients in the equation (p). The SEE measure is typically stated in absolute terms (i.e., if Y is in dollars then the SEE is in dollars as well).

A.8.3 Defining a Prediction Interval From SEE

A proper measure of the quality of the regression estimate for a future observation is the Prediction Interval. The prediction interval accounts for the sample size and the location of the estimate within the data used to create the regression. The further from the center of the regression data, the greater the uncertainty. SEE is an average measure. The prediction interval adjusts the SEE to account for location.

The regression equation estimates a value based upon the independent variable(s) for the project in question. But the point estimate is almost surely not exactly right. A prediction interval provides a range of values in which the actual value can be expected to fall with a certain degree of confidence. For example, "There is a 90% probability that the warhead cost will be between $48K and $64K." A prediction interval of a parametric CER is calculated from the:

- standard error of the CER
- CER sample size (i.e., the number of data points used to derive the CER)
- desired confidence level
- distance from the center of the CER's independent variables to the location of the independent variable of the point being estimated

A prediction interval can be thought of as a range defined by the point estimate plus or minus some number of adjusted standard errors (standard errors adjusted for prediction). This adjusted standard error is a function of the standard error of the regression, the size of the database used in the CER development, and the "distance" of the estimating point from the center of the database as exemplified by the means of all the

independent (driver) variables. In the simple case of one independent variable [X], the adjusted standard error is specifically defined by the following:

Equation A-54 Adjusted Standard Error

$$Adj.\ SE = SE * \sqrt{1 + \frac{1}{n} + \frac{\left(\frac{Xe - \bar{x}}{S_X}\right)^2}{n}}$$

Where:

Adj. SE	=	adjusted standard error for prediction interval
SE	=	CER standard error (data must be normalized for base year, adjustments, quantity)
Xe	=	the value of the independent variable used in calculating the estimate
$\bar{x}$	=	the mean of the independent variable in database
S_X	=	uncorrected sample standard deviation of the independent variable
n	=	the number of data points

In a simple linear CER where $Y = \beta_0 + \beta_1 X + \varepsilon$, a 95% prediction interval when $X = x_o$ is given by

Equation A-55 Linear CER 95% Prediction Interval

$$\hat{y}_0 \pm t_{0.025,(n-2)} * SE * \sqrt{1 + \frac{1}{n} + \frac{(x_0 - \bar{x})^2}{SS_{xx}}}$$

Where:

$$SS_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$\hat{y}_0$ is the estimated cost from the CER when X = xo

$t_{0.025,(n-2)}$ is the upper 2.5% cut-off point of student's-t distribution with (n-2) degrees of freedom.

As one moves away from the mean of the CER dataset, the *adjusted standard error* is always greater than the SEE. Thus, using the SEE as a quantifier for uncertainty underestimates the true error unless the point of interest is at the mean of the data. For estimating points not near the dataset mean, the difference can be significant. This is especially true when the CER is used beyond the range of the data used in developing the CER.

If a simple linear CER is fit in log space, assuming the multiplicative error term:

Equation A-56 Linear CER Fit in Log Space

$$Y_i = (\beta_0 X_i^{\beta 1})\ \varepsilon_i$$

$$\text{for } i = 1, \ldots, n$$

The (1-α)100% prediction interval (PI) for a future observation Y, when $X = x_o$ is given by

Equation A-57 Prediction Interval for a Linear CER Fit in Log Space

$$Exp\left(\hat{y}_0 \pm (t_{\alpha/2,n-2})\, SE \sqrt{1 + \frac{1}{n} + \frac{(LN(x_0) - \overline{LN(x)})^2}{SS_{xx}}}\right)$$
$$= Exp\left(\hat{y}_0 \pm (t_{\alpha/2,n-2}) * (Adj.\ SE)\right)$$

where:

$\hat{y}_0$	= the estimated value in **log space** when X = x_0,
x_0	= the value of the independent variable used in calculating the estimate
$\overline{LN(x)}$	= the average value of the independent variable evaluated in **log** space,
SE	= the standard error of estimate in **log** space
Adj. SE	= the adjusted standard error for PI in log space
SS_{xx}	= the sum of squares of the independent variable about its mean (in log space)
"LN"	= the natural logarithm function

The above PI equations can be extended to CERs with multiple drivers. For example, if there are several predictors in a linear CER, namely, $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots \beta_k X_k + \varepsilon$, the PI can be computed using matrix operations. A (1-α)100% PI for a future observation Y at a given driver vector $\underline{x}_o$ is then given below:

Equation A-58 Prediction Interval For CERs with Multiple Independent Variables

$$\hat{y}_0 \pm (t_{\alpha/2,\,n-p}) * SE * \sqrt{1 + (\underline{\mathbf{x}}_{\mathbf{0}})(\mathbf{X}'\mathbf{X})^{-1}(\underline{\mathbf{x}}_{\mathbf{0}})'} = \hat{y}_0 \pm (t_{\alpha/2,\,n-p}) * (Adj.SE)$$

where:

$\hat{y}_0$	= the estimated value of Y from the CER when $\underline{x} = \underline{x}_o$
$\underline{x}_o$	= (1, x_{1o}, ..., x_{ko}), a row vector of given driver values and 1 is for the intercept
p	= the total number of estimated parameters, including the intercept (p = k+1)
n	= the number of data points
$t_{(\alpha/2,n-p)}$	= the upper α/2 cut-off point of student's-t distribution with (n-p) DF
SE	= CER's standard error of estimate
Adj. SE	= the adjusted standard error for PI
X	= the design matrix of the independent variables
	(The apostrophe superscript denotes the transpose of a vector or a matrix.)

If not all statistics of the CER are available, the Adjusted Standard Error can be calculated with the following equation based on a distance assessment of the primary independent variable.

Equation A-59 Simplified Adjusted Standard Error

$$Adj.\,SE = SE * \sqrt{1 + \frac{1}{n} + \frac{\left(\frac{\text{distance}}{sample\,std}\right)^2}{n}}$$

where:

distance	= distance between the point estimate independent variable value and the center of the independent variable data used to generate the CER
sample std	= uncorrected sample standard deviation of the primary independent variable

Note that this distance assessment need only be characterized in terms of a number of standard deviations from the center. For example, if the distance is assessed as approximately 2 sample standard deviations of the driver variable, then the ratio (of "distance" to "sample std") becomes 2. For simplicity, the following default values address the assessment of this ratio based upon the similarities between the systems:

$$\frac{distance}{sample\ std} = \begin{cases} 0.00 & Very\ Similar \\ 0.75 & Similar \\ 1.50 & Dissimilar \\ 2.25 & Different \\ 3.00 & Very\ Different \end{cases}$$

For example, if the system being estimated is deemed very similar to the database from which the CER was developed, this qualitative assessment might translate into a quantitative assessment of the ratio with a value of 0.25. Similarly, if the system being estimated is deemed very different from the database from which the CER was developed, this qualitative assessment might translate into a quantitative assessment of the ratio with a value of 3.0. Using these default values the adjusted standard error can then be calculated.

In addition, if no statistical information is available, then the Adjusted Standard Error can be chosen subjectively based on a subjective distance assessment of the primary independent variable and the relative sample size. **Table A-6** provides a list of multipliers that can be used to estimate the Adjusted SE.

Table A-6 Standard Error Adjustment Factors

SE Multiplier		Number of Data Points in Sample						
		5	10	15	20	25	30	
Distance / StdDev for Independent Variable	**0.00**	1.095	1.049	1.033	1.025	1.020	1.017	Very Similar
	0.25	1.101	1.052	1.035	1.026	1.021	1.018	
	0.50	1.118	1.061	1.041	1.031	1.025	1.021	
	0.75	1.146	1.075	1.051	1.038	1.031	1.026	Similar
	1.00	1.183	1.095	1.065	1.049	1.039	1.033	
	1.25	1.230	1.121	1.082	1.062	1.050	1.042	
	1.50	1.285	1.151	1.103	1.078	1.063	1.053	Dissimilar
	1.75	1.346	1.186	1.127	1.097	1.078	1.066	
	2.00	1.414	1.225	1.155	1.118	1.095	1.080	
	2.25	1.487	1.267	1.185	1.142	1.115	1.096	Different
	2.50	1.565	1.313	1.218	1.167	1.136	1.114	
	2.75	1.647	1.362	1.253	1.195	1.159	1.134	
	3.00	1.732	1.414	1.291	1.225	1.183	1.155	Very Different

Figure A-10 illustrates the use of the standard error adjustment factor. The correct prediction interval high value is calculated to be (A) 123.01% of the point estimate at the 90% interpretation. (For guidance on how to calculate a prediction interval, see **Appendix A.8.5** and **A.8.8**). This is compared with the lognormal based upon the reported SEE in log space (B) 0.1413. The regression SEE underestimates the standard deviation (C) and 95th percentile (D). If, however, the SEE multiplied by the adjustment 1.267, which is consistent with 10 data points and an assessment that the source data is "different" for the project to be estimated, the adjusted SEE is 0.1790 (E) and the results are more conservative than those obtained using SEE alone. The adjustment factor approach tends to over compensate, but is a reasonable, conservative and simple approach when statistical tools and/or source data is unavailable for a more precise estimate.

Alternative Ways to Define a Lognormal Distribution

Uncertainty Estimating Method	PE	Eq / Thruput	Dist	PE Pos	DF	Adj SE	High %	High Interpretation
Propulsion First Unit Cost (PI)	$ 78.565 (50%)	1.618 * 290 ^ 0.6848	Log-t	Median	8		123.01 A	90.00
Propulsion First Unit Cost (Adj SE)	$ 78.565 (50%)	1.618 * 290 ^ 0.6848	Log-t	Median	8	0.1790	E	
Propulsion First Unit Cost (SE)	$ 78.565 (50%)	1.618 * 290 ^ 0.6848	Log-t	Median	8	0.1413	B	

Alternative Results

Uncertainty Estimating Method	Point Estimate	Mean	C Std Dev	CV	5.0% Level	25.0% Level	50.0% Level	75.0% Level	D 95.0% Level
Propulsion First Unit Cost (PI)	$ 78.565 (50%)	$79.733	$13.947	0.175	$59.634	$70.753	$78.566	$87.238	$103.513
Propulsion First Unit Cost (Adj SE)	$ 78.565 (50%)	$79.843	$14.630	0.183	$58.894	$70.416	$78.564	$87.659	$104.808
Propulsion First Unit Cost (SE)	$ 78.565 (50%)	$79.358	$11.382	0.143	$62.578	$72.060	$78.564	$85.659	$98.646

Figure A-10 Compare Calculated Prediction Interval with Estimated Approach

A.8.4 OLS Unit Space Interpretation and Error

The result of a linear OLS derived CER is interpreted to be the mean of a normal distribution (that is, the error is assumed to be normally distributed about the CER line). Since a normal distribution is symmetrical, the CER result can also be characterized as the median or the mode.

In OLS, the cost (Y) variation is assumed to be independent of the magnitude of the cost. In other words, the error distribution is assumed to be identical throughout the data range and the error term is additive to the equation. It is given by

Equation A-60 OLS Error Distribution

$$\varepsilon_i = Y_i - f(\mathbf{x}_i, \boldsymbol{\beta})$$

This may be mathematically correct, but it is not reasonable in cost estimating and is a big reason why many analysts will not use the OLS method. For instance, if the CER predicts cost as a function of weight and the valid cost range is $500 to $1500, a fixed $100 (a result consistent with OLS) average error over that range is not appropriate. It may be reasonable at the mid-range of the cost, but not at the low or high end. In cost estimating, it may generally be assumed that the error is proportional rather than fixed. In this case, an average error of 10% (rather than $100) would be used by most analysts to model the error of the CER. A common rule of thumb is that the errors are believed to be proportional to the magnitude of the result (the dependent variable) if it ranges over more than one order of magnitude. It is appropriate in such a case to assume the percentage errors of the item estimates will be identically distributed throughout the range of the data set. This is what a multiplicative error is supposed to represent.

The uncertainty of a linear OLS derived CER may be modeled as a normal distribution where the CER result is always the mean of the distribution. If the dataset is small (less than 30), then the student's-t distribution should be used rather than the normal distribution.

A.8.5 Calculating the Prediction Interval for Linear OLS CERs

If the CER has the form: a + b*Var1 + c*Var2 + etc and it was derived using OLS, then it is acceptable to assume the CER produces the mean and the uncertainty distribution shape is the student t or normal. To estimate the bounds of the distribution, calculate the prediction interval based upon the point estimate value for the input variables.

Many statistical packages will calculate a lower and upper bound for the OLS generated CER based upon a specified value for the independent variable(s). As illustrated in **Table A-7**, entering 25 lbs as the value for the point estimate weight and choosing to calculate the 80% prediction interval, the statistical package calculates the upper and lower bound. In this case, these bounds are associated with the 10 and 90 percent probability levels.

Table A-7 Statistic Package Prediction Interval for a Linear OLS CER

UC1 = 30.15 + 1.049 * WarheadWt		
WarheadWt	25.00	
Confidence Level (%)	80.00%	
Statistical Tool	**Result**	**PE Multiplier**
Low (10 percentile)	48.61	0.8624
Estimate (mean, median)	56.36	1.0000
High (90 percentile)	64.12	1.1376

If the statistical package is unavailable, you can perform the prediction interval calculation manually as illustrated in **Table A-8**.

Table A-8 Manual Calculation of Prediction Interval For A Linear OLS CER

Observations	UC1	Warhead Wt
System #1	31.00	6.00
System #2	46.00	8.00
System #3	36.00	9.00
System #4	48.00	13.00
System #5	40.00	14.00
System #6	50.00	17.00
System #7	55.00	20.00
System #8	54.00	27.00
System #9	58.00	30.00
System #10	67.00	31.00

Element	Range Name	Value	Formula
Confidence Level (%)	ConfLvl	80%	Arbitrary, but 80% is consistent with calculating the 10/90 bounds.
Degrees of Freedom	DegOfFreedom	8	Number of observations minus number of coefficients estimated.
Student t	StudentT	**1.397**	TINV(1-ConfLvl,DegOfFreedom)
Std Error (SE)	StdErr	5.126	See Appendix
# of Observations	NumObs	10	
TBE Warhead Wgt	TBEwgt	25.00	User Input
Ave Warhead Wgt	AveWgt	17.50	AVERAGE(WarheadWtObservations)
Warhead Wgt Stdev	Sx	8.73	STDEVP(WarheadWtObservations)
Delta to Bound	Delta	7.76	StdErr*StudentT*SQRT(1+1/NumObs+((TBEwgt-AveWgt)/Sx)^2/NumObs)
Lower Bound		48.61	TBE-Delta
Point Estimate	TBE	56.36	UC1 = 30.15 + 1.049 * WarheadWt
Upper Bound		64.12	TBE+Delta

If the statistical package and/or the data to perform the calculation manually are unavailable, the analyst may estimate the bounds based upon the CER result, the Standard Error for the CER and an adjustment factor from **Table A-6**. This simple procedure is illustrated in **Table A-9**. As the results for the bounds show in **Table A-9**, the estimate without the adjustment underestimates the prediction interval range while the result that includes the adjustment slightly over estimates the range. It is appropriate that a less accurate approach (using an adjustment factor) tends to overestimate the uncertainty.

Table A-9 Manual ROM Estimate of a Prediction Interval for a Linear OLS CER

Element	Range Name	No Adjustment	With Adjustment	Formula
Standard Error	SEE	5.126	5.126	From Statistical Package
Adjust for Sample Size & Relevance	SEEAdjust	1.000	**1.267**	10 datapoints, source data different than project
Adjusted Standard Error	SEEUnitSPace	5.126	6.495	SEE * SEEAdjust
Mean	Mean	56.364	56.364	
Lower Bound (10% level)		49.79	48.04	NORMINV(0.10,Mean,SEEUnitSpace)
Upper Bound (90% level)		62.93	64.69	NORMINV(0.90,Mean,SEEUnitSpace)
Lower Bound as % of PE		88.34%	85.23%	Actual = 86.24
Upper Bound as % of PE		111.66%	114.77%	Actual = 113.76

If information about the CER is so sparse that even the standard error is unavailable then the analyst must resort to a subjective assessment of the CER uncertainty bounds. The table of last resort subjective uncertainties (see **Section 2.5.5**) are considerably larger than the examples in this section.

All simulation packages will allow a normal distribution to be defined with the mean and some other probability level. Since the normal is symmetrical, only one of the bounds plus the mean (the CER result) is required

A.8.6 OLS Log Space Interpretation and Error

CERs of the form Cost = a*Var1^b*Var2^c…*ε can be transformed into linear forms in log space. In unit space, the CER result is closer to the median (not the mean) of a lognormal distribution. A key result of this approach is that the error term is multiplicative, that is, it is proportional to the result of the CER. A log-error model is used when the error term is believed to follow a lognormal distribution. This is a very common and intuitive assumption because the error in cost is usually skewed upward and bounded below by zero. Log-linear models are in a very common and distinct class of non-linear relationships that are rendered linear when transformed to log-space.

If the error term (ε_i) is assumed to follow a normal distribution with a mean of 0 and variance σ^2 in log space (i.e., $\varepsilon_i \sim LN(0,\sigma^2)$), then the error can be measured by the following:

Equation A-61 Log Space OLS Error Distribution

$$\ln(\varepsilon_i) = \ln(Y_i) - \ln(f(\mathbf{x}_i, \boldsymbol{\beta}))$$

where "ln" stands for natural logarithmic function. The objective is then to minimize the sum of squared e_is (i.e., $(\Sigma(\ln(\varepsilon_i))^2)$ in log space. In this case, the errors are assumed to be normally distributed around the CER (a straight line) in log space. When transformed back to unit space, the mathematics show that:

The unit space CER error term follows a lognormal distribution

The CER result, while being the mean of a normal distribution in log space, is closer to the median in unit space. If adjusting the CER result to be closer to the mean of the lognormal distribution is desired, please see the **Appendix A.8.7**.

In summary, if the transformed function is linear in log space, then OLS can be applied in log space to derive a solution for $\boldsymbol{\beta}$. If this function cannot be linearized in log space, then apply a non-linear regression technique to derive a solution. See the pros and cons of log-error models in **Reference 44**.

A.8.7 Adjusting a Log Space OLS CER Result to reflect the Unit Space Mean

Although a least squares optimization in log space produces an unbiased estimate in log space, the estimate is no longer unbiased when transformed back to unit space (see **Reference 1**). By unbiased, we mean the result is the mean of the uncertainty distribution. The unit space CER by direct translation tends to underestimate the mean value of the original population. Two distinct methods for adjusting the OLS log-linear CER result to reflect the mean are presented. As stated in **Reference 15**, neither of these adjustments is necessary to model the lognormal uncertainty.

The most accurate approach is to make use of a correction factor based upon statistics of the CER. A theoretical correction factor was first introduced by Neyman and Scott in 1960 and again by Goldberger in 1968 to adjust the CER result to reflect the mean in unit space for the log-linear CERs (see **Reference 1 and 3**). A simplified form, commonly known as the PING factor (PF) is given by:

Equation A-62 Factor to Adjust a Log Space CER Result to Reflect the Mean

$$PF = \exp((1 - \frac{p}{n})\frac{s^2}{2})$$

Where:

p = the total number of coefficients to be estimated
s = standard error of estimate in log space
n = the sample size

This simplified form can be applied to log-linear CERs to correct the downward bias in unit space. For a more accurate form of the factor and its detailed derivations, see **Reference 44.**

By way of an example, consider the following CER to estimate propulsion costs:

Y($K) = 1.618*MotorWt^0.6848

Where

n = 10 and SE = 0.25 and two coefficients (the intercept 1.618 and exponent 0.6848)

the factor is:

$$\exp((1-\frac{2}{10})\frac{0.25^2}{2})$$
$$=1.025$$

If, for example, the motor weight is 200 lbs, the CER yields $60.91 K. This is the median of the lognormal distribution. If the mean is desired, then the CER result must be multiplied by 1.025. The mean of the lognormal is therefore approximately $62.43 K.

As indicated by **Equation A-62**, there are two terms involved in the adjustment: the first one is for adjusting the downward bias between the mean and the median (a transformation bias); the other is used to adjust the upward bias for estimating the median (a sampling bias). It can be concluded that:

- At a given sample size, the adjustment increases with standard error.
- At a given standard error, the adjustment increases with the sample size.
- The adjustment is always greater than 1 for p < n and standard error > 0.

Table A-10 provides typical ranges for PING values illustrating that the adjustment increases with greater dispersion or larger sample size and decreases with more CER coefficients.

Table A-10 Typical Log Space CER Adjustment Factor Ranges To Obtain the Mean

	Factor to Convert Loglinear CER Median to Mean											
	2 Coefficient Loglinear CER				3 Coefficient Loglinear CER				4 Coefficient Loglinear CER			
Dispersion --> (Adjusted SE) -->	Low (0.15)	Med (0.25)	High (0.35)	E High (0.45)	Low (0.15)	Med (0.25)	High (0.35)	E High (0.45)	Low (0.15)	Med (0.25)	High (0.35)	E High (0.45)
Small Sample (5)	1.007	1.019	1.037	1.063	1.005	1.013	1.025	1.041	1.002	1.006	1.012	1.020
Medium Sample (10)	1.009	1.025	1.050	1.092	1.008	1.022	1.044	1.084	1.007	1.019	1.037	1.077
Large Sample (25)	1.010	1.029	1.058	1.098	1.010	1.028	1.055	1.093	1.009	1.027	1.053	1.089

A.8.8 Calculating the Prediction Interval for Log Linear OLS CERs

A very common mistake is to assume OLS log-linear CERs produce the "mean" of the uncertainty distribution. This is not true. The back-transformed unit-space CER produces a value that is closer to the median. The mathematics involved in adjusting the result to reflect the mean, while not complicated, does add a further layer of unnecessary calculations. The details of how to calculate this adjustment are contained

in **Appendix A.8.7**. However, this is not necessary to accurately define the distribution. A lognormal distribution based upon the median and one other point will be identical to one modeled based upon the mean of the same distribution and one other point. The simulation result will be identical.

If the CER has the form: a*Var1^b * Var2^c * ε and was derived using OLS in log space (see **Appendix A.7.2** for details), then the CER produces the median and the uncertainty distribution shape is lognormal. To estimate the bounds of the lognormal distribution, calculate the prediction interval based upon the point estimate for the input variables.

Many statistical packages will calculate a lower and upper bound for the OLS generated CER based upon a specified value for the independent variable(s). As illustrated in **Table A-11**, entering 290 lbs as the value for the point estimate and choosing 80%, the statistical package calculates the upper and lower bound. In this case, these bounds are associated with the 10 and 90 percent probability levels.

Table A-11 Statistic Package Prediction Interval For A Log Linear OLS CER

UC1 = 1.618 * MotorWt ^ 0.6848		
MotorWt	290.00	
Confidence Level	80.00%	
Statisitcal Tool	**Result**	**PE Multiplier**
Lower Bound	63.31	80.58%
Estimate	78.56	Median
Upper Bound	97.48	124.08%

To perform this calculation manually in Excel, follow the steps in **Table A-12**. Note that the assessment of fit occurs in fit space, in this case log space. Consequently, the CER result and the 10/90 results from fit space need to be converted to unit space using the exponential function.

Table A-12 Manual Calculation of Prediction Interval for a Log Linear OLS CER

	LN(MotorWt)	MotorWt
System #1	4.499809670	90
System #2	4.718498871	112
System #3	4.867534450	130
System #4	5.135798437	170
System #5	5.272999559	195
System #6	5.347107531	210
System #7	5.416100402	225
System #8	5.669880923	290
System #9	5.768320996	320
System #10	5.828945618	340

Element	Range Name	Value	Formula
Confidence Level (%)	ConfLvl	80%	Arbitrary, but 80% is consistent with calculating the 10/90 bounds.
Degrees of Freedom	DegOfFreedom	8	Number of observations minus number of coefficients estimated.
Student t	StudentT	**1.397**	TINV(1-ConfLvl,DegOfFreedom)
Std Error (SE)	StdErr	0.1413	See Appendix A
# of Observations	NumObs	10	
TBE Motor Wgt	TBEwgt	5.67	User Input LN(290)
Ave Motor Wgt	AveWgt	5.25	AVERAGE(MotorWtObservations)
Motor Wgt Stdev	Sx	0.43	STDEVP(MotorWtObservations)
Delta to Bound	Delta	0.22	StdErr*StudentT*SQRT(1+1/NumObs+((TBEwgt-AveWgt)/Sx)^2/NumObs)
Lower Bound	LowerBound	4.15	TBE-Delta
Point Estimate	TBELogSpace	4.36	UC1 = 1.618 * MotorWt ^ 0.6848
Upper Bound	UpperBound	4.58	TBE+Delta

Lower Bound		63.31	EXP(LowerBound)
Point Estimate	TBE	78.56	EXP(TBELogSpace)
Upper Bound		97.48	EXP(UpperBound)

If the number of observations (and degrees of freedom) is not known, the upper and lower bound can be estimated using the normal distribution similar to the example shown in **Table A-13**. Note that the upper and lower bounds are not symmetrical about the mean (CER result). Lognormal rather than normal is a more appropriate distribution to use in this case.

As the results for the bounds show in **Table A-13**, the ROM estimate without the adjustment underestimates the prediction interval range while the ROM approach that includes the adjustment slightly overestimates the range. It is appropriate that a less accurate approach tends to overestimate the uncertainty.

Table A-13 Manual ROM Estimate of a Prediction Interval For A Linear OLS CER

Element	Range Name	No Adjustment	With Adjustment	Formula
Standard Error (Fit Space)	SEEFitSpace	0.141	0.141	From Statistical Package
Adjust for Sample Size & Relevance	SEEAdjust	1.000	**1.267**	10 datapoints, source data different than project
Adjusted Standard Error (Fit Space)	SEEFitSpaceAdj	0.141	0.179	SEE * SEEAdjust
Mean (Fit Space)	MeanFitSpace	4.364	4.364	
Mean (Unit Space)	Mean	78.558	78.558	
Lower Bound (10% level)		65.55	62.45	EXP(NORMINV(0.10,MeanFitSpace,SEEFitSpaceAdj))
Upper Bound (90% level)		94.15	98.82	EXP(NORMINV(0.90,MeanFitSpace,SEEFitSpaceAdj))
Lower Bound as % of PE		83.44%	79.50%	Actual = 80.59
Upper Bound as % of PE		119.85%	125.79%	Actual = 124.08

If even the standard error is unavailable then the analyst must resort to a subjective assessment of the CER uncertainty bounds (however, lognormal should be used). Last resort subjective uncertainties (see **Section 2.5.5**) are considerably larger than the example in this section.

All simulation packages will allow a lognormal distribution to be defined with the mean and some other probability level. A lognormal can be uniquely defined from two points such as one of the bounds plus the median (the CER result).

A.8.9 MUPE Interpretation and Error

The result of a MUPE derived CER is interpreted to be the mean of the error distribution regardless of the functional form of the CER. Since MUPE CERs estimates the mean in unit space, the roll-up elements in an all-MUPE cost estimate will be the expected value of the sum of the elements below. This is not true of CERs developed using uncorrected (see **Appendix A.7.2**) OLS in log space (and may or may not be true for other regression types).

Note that the standard error of the estimate (also commonly termed multiplicative error) can be used as an estimate of the standard deviation (σ) of the error term. For instance, if the SEE is 20% for a MUPE CER, it can be interpreted that the CER has plus/minus 20% estimating error (for one standard deviation) at the center of the database.

A more rigorous estimate of the MUPE prediction interval is possible for linear MUPE CERs (**Reference 51**). The general specification for a MUPE CER is stated as:

Equation A-63 MUPE Prediction Interval Specification

$$Y_i = f(\mathbf{X}_i, \boldsymbol{\beta})\varepsilon_i \qquad \text{for i = 1, …, n}$$

where:

- n = sample size
- Y_i = observed cost of the ith data point, i = 1 to n
- $f(\mathbf{X}_i,\boldsymbol{\beta})$ = the value of the hypothesized equation at the ith data point
- $\boldsymbol{\beta}$ = vector of coefficients to be estimated by the regression equation
- $\mathbf{X}_i$ = vector of cost driver variables at the ith data point
- ε_i = error term with mean of 1 and variance σ^2

Statistical inferences can be made for the regression equation if the normality assumption is further applied to the error term (ε). If the hypothesized equation is a simple linear function:

$f(\mathbf{X}_i,\boldsymbol{\beta}) = \alpha + \beta X_i$ for i = 1, …, n

The (1-α)% prediction interval (PI) for a future observation Y when X is at x_o (i.e., $\hat{y}_0$) is then given by

Equation A-64 MUPE Prediction Interval For a Future Observation

$$\hat{y}(when\ X = x_0) \pm t_{(\alpha/2,\, df)} * SE * \left(\hat{y}_0^2 + \frac{1}{\sum w_i} + \frac{(x_0 - \bar{x}_w)^2}{SS_{wxx}} \right)^{0.5}$$

$$= \hat{y}_0 \pm t_{(\alpha/2,\, df)} * SE * \left(\hat{y}_0^2 + \frac{1}{\sum_{i=1}^{n} w_i} + \frac{(x_0 - \bar{x}_w)^2}{(\sum_{i=1}^{n} w_i) S_{wx}^2} \right)^{0.5}$$

$$= \hat{y}_0 \pm t_{(\alpha/2,\, df)} * (Adj.\ SE)$$

where:

w_i is the weighting factor for the ith data point and $w_i = 1/(f_i * f_i)$
f_i denotes the predicted value of the ith data point
SE is CER's standard error of estimate

$$\bar{x}_w = \sum_{i=1}^{n} w_i (x_i) \Big/ \sum_{i=1}^{n} w_i$$

$$SS_{wxx} = \sum_{i=1}^{n} w_i (x_i - \bar{x}_w)^2$$

$$S_{wx} = \sqrt{\sum_{i=1}^{n} w_i (x_i - \bar{x}_w)^2 / \sum_{i=1}^{n} w_i}$$

df is the degrees of freedom
$t_{(\alpha/2,df)}$ is the upper $\alpha/2$ cut-off point for a t-distribution with "df" degrees of freedom
Adj. SE is the adjusted standard error for PI
S_{wx} is the weighted sample standard deviation of the independent variable x. It is the sample standard deviation of the x variable evaluated in the fit space.

If the hypothesized equation is a simple factor equation: $f(\mathbf{X}_i, \boldsymbol{\beta}) = \beta X_i$ for i = 1, …, n

The (1-α)% prediction interval (PI) for a future observation Y when X is at x_o (i.e., $\hat{y}_0$) is then simplified to the following formula:

Equation A-65 MUPE Prediction Interval For a Future Observation For a Factor CER

$$\hat{y}_0 \pm t_{(\alpha/2,\, n-1)} * SE * \sqrt{\frac{1}{w_0} + \frac{(x_0)^2}{SS_{wxx}}}$$

$$= \hat{y}_0 \pm t_{(\alpha/2,\, n-1)} * SE * \sqrt{(1 + \frac{1}{n})\, \hat{y}_0^2}$$

$$= \hat{y}_0 \left(1 \pm t_{(\alpha/2,\, n-1)} * SE * \sqrt{1 + \frac{1}{n}} \right)$$

$$= \hat{y}_0 (1 \pm t_{(\alpha/2,\, n-1)} * Adj.\, SE)$$

A.8.10 MUPE SEE

For MUPE equations, the SEE measure is typically stated in percentage terms and is provided below.

Equation A-66 Standard Error of the Estimate for MUPE CERs

$$SEE = \left(\frac{1}{n-p} \sum_{i=1}^{n} \{(Y_i - \hat{Y}_i) / \hat{Y}_i\}^2 \right)^{0.5}$$

$$= \sqrt{SSE / dfe}$$

$$= \sqrt{MSE}$$

Note: Since SEE measures the average amount of variation between the sample data and the regression equation, the smaller the value of SEE, the tighter the equation (i.e., the more precise the prediction). MUPE's SEE is commonly termed "standard percent error" (SPE).

A.8.11 Calculating the Prediction Interval for MUPE CERs

Regardless of the CER form, if it was derived using the MUPE method, it can be assumed that the CER produces the mean. To estimate the bounds of the normal distribution for a linear MUPE CER, calculate the prediction interval based upon the Technical Baseline point estimate for the input variables. Details on a closed form solution are provided in **Appendix A.8.9**.

A.8.12 ZPB/MPE (or ZMPE Method) Interpretation and Error

ZMPE also delivers zero sample bias, i.e., the average of the proportional errors is zero. The ZMPE technique does not yield the traditional statistical properties suitable for characterizing the meaning of the CER result and its error that are customary with OLS and LOLS regressions. Also, the PI for the ZMPE equation has not been validated yet. For these reasons, many organizations prefer OLS or MUPE over the ZMPE process (**Reference 57**). However, ZMPE has been selected as a technique of choice by several organizations.

ZMPE makes no assumptions about the nature of the error distribution, so any type of distribution can be used. Some analysts prefer using lognormal to model the CER error distribution. To add a little rigor to the process, one could perform a distribution fitting process to identify the appropriate curve. ZMPE process is blind to the shape of the distribution, and simply seeks fit parameters that minimize the sum of the squared percent errors between the data points and the CER, with the added constraint that the bias be zero.

A.8.13 Iterative Regression Interpretation and Error

There have been various attempts to generate suitable statistics from iterative regression analysis. The Bootstrap method (see **Reference 50**) is one approach. However, the reference did not provide an error assessment of the Bootstrap prediction intervals. The only claim is that the Bootstrap PIs were “close” to the PIs generated by the OLS method for a simple linear CER with 10 data points.

A.9 GOODNESS OF FIT STATISTICS

A.9.1 Overview

Goodness of fit statistics measure how well the fitted distribution compares to the input data. What most analysts forget is that there are a variety of ways to define the input data. Determining how the cumulative probability will be assigned to the input data will have a profound impact on the fit results when only a few data points are available. **Section 2.4.3.2** covers this topic in detail.

When fitting distributions to the input data, it is not enough to find the best fit that minimizes the error of some statistical metric to identify the best fit. The best fit may not be a statistically significant fit. So in addition to finding the best fit, there is a need to perform a goodness-of-fit test. Guidance on what combination of statistics to use is provided in **Section 2.4.3.5**.

A.9.2 Kolmogorov-Smirnov (K-S)

The K-S test compares the sample CDF with the fitted CDF and computes the maximum vertical distance between them. An attractive feature of this test is that the distribution of the K-S test statistic itself does not depend on the underlying cumulative distribution function being tested. Another advantage is that it is an exact test (the chi-squared goodness-of-fit test depends on an adequate sample size for the approximations to be valid). Despite these advantages, the K-S test has several important limitations:

- It only applies to continuous distributions
- It tends to be more sensitive near the center of the distribution than at the tails

Due to the first two limitations, many analysts prefer to use the Anderson-Darling goodness-of-fit test.

A.9.3 Anderson-Darling (A-D)

The A-D test measures total area between the sample and the fit CDF. Weightings can be used to focus on the fit in the tails. The Anderson-Darling test makes use of the specific distribution in calculating critical values. This has the advantage of allowing a more sensitive test and the disadvantage that critical values must be calculated for each distribution. Currently, tables of critical values are available for the normal, lognormal, exponential, Weibull, extreme value type I, and logistic distributions.

The basic test statistic for a given sample $\{X_1, X_2, \ldots, X_n\}$ (in ascending order) is listed below:

Equation A-67 Anderson Darling Goodness of Fit Test

$$\text{Area between the curves} = A^2 = -n - S$$

where:

$$S = \sum_{k=1}^{n} \frac{2k-1}{n}\left(\ln(F(X_k) + \ln(1 - F(X_{n+1-k}))\right),$$

F is the cumulative distribution function of the fitted distribution,
X_i's are the ordered data, and
n is the sample size

Like the K-S statistic, the lower the resulting statistic, the better the fit.

A.9.4 K-S and A-D Test for Statistical Significance

One of the more confusing aspects of the K-S and A-D tests are their applicability to distribution fitting. If either the form of the distribution or the parameters of a distribution are ***determined from the data***, then the basic formulas for calculating a p-value (confidence level that the fit is statistically significant) need to be adjusted. In such cases, Monte Carlo or other methods (@RISK uses a bootstrap method) may be required. Tables have been prepared for some cases. Details for the required modifications to the test statistic and for the critical values for normal, lognormal, exponential, extreme-value, Weibull, gamma, logistic, cauchy, and von Mises distributions can be found in **Reference 8**. Neither beta nor triangular is addressed in **Reference 8** and neither Crystal Ball nor @RISK report a p-value for triangular, beta or other distributions.

The Crystal Ball Help manual states: P-values are also displayed for the following continuous distributions when the Anderson-Darling or Kolmogorov-Smirnov methods are used: normal, exponential, logistic, maximum extreme, minimum extreme, uniform, gamma, Weibull, and lognormal. P-values for the other distributions are under development. Since p-values for Anderson-Darling and Kolmogorov-Smirnov statistics are influenced by the number of data points being fitted, an adjustment formula is used to arrive at the asymptotic Anderson-Darling and Kolmogorov-Smirnov statistic for a given sample size. The quality of fitted parameters and the calculated p-value deteriorates as the sample size decreases. Currently, Crystal Ball needs at least 15 data points for fitting all the distributions.

@RISK uses a different method (bootstrap, see the @RISK user's manual for details) to estimate p-values for the K-S and A-D test. Just like with Crystal Ball, the quality of fitted parameters and the calculated p-value deteriorates as the sample size decreases. ACEIT (CO$TAT) does not currently report the K-S or A-D statistic, but will in a future release.

What does this mean? It means that despite the usefulness of K-S and A-D as goodness-of-fit tests their p-values have limited utility when *comparing* statistical significance *between* distribution types. Furthermore, both Crystal Ball and @RISK warn the user that the quality of the K-S and A-D results deteriorates as the sample size gets smaller. That does not deter a large segment of our community from using these tests.

A.9.5 Chi-squared Test

For the chi-squared test, the sample is divided into equal probability bins based upon the fitted curve. It counts the number of sample points that fall in the bins and compares the deviation between the number of observations in each bin and the curve's prediction of how many observations should land in each bin. The greater the deviation, the more likely we are correct in rejecting the fitted curve for prediction purposes.

The chi-squared test is based upon dividing the sample into a number of bins. **Table A-14** presents five popular options for determining the number of bins to use. For each bin, the following formula is evaluated and then summed to calculate the chi-squared statistic:

Equation A-68 Chi-squared Goodness of Fit Statistic

$$\sum_{i=1}^{k} (SampleFreq(i) - ExpectedFreq(i))^2 / ExpectedFreq(i)$$

where:

k is the number of bins,
SampleFreq(i) is the actual number of observations in the ith bin (i = 1,…,k), and
ExpectedFreq(i) is the expected number of observations falling in the ith bin according to the fitted distribution.

The following sequence is used to evaluate the formula:

- Select the number of equal probability bins
- Calculate the cumulative probability level for each bin
- Estimate the bin boundary values using the fitted distribution
- Count the number of data points (SampleFreq) falling in each bin
- Calculate the expected number of observations (ExpectedFreq) per bin (i.e., number of sample observations/number of bins)
- Calculate the chi-squared statistic
- Calculate the p-value for the Chi^ test (a one-tailed test) and compare to the level of significance to determine the statistical significance of the fit

Table A-14 identifies five popular methods for estimating the optimum bin count. Excel formulations are also provided. The Mann-Wald method divided by 2 (discussed in detail in **Reference 8**) is used by Crystal Ball and CO$TAT to determine the number of equal probability bins for the chi-squared goodness of fit test (see **Section 2.4.3.5)**. The general form and the 5% significance level form is provided.

Table A-14 Formulas for Choosing Histogram Bin Count

Mann-Wald/2 and 5% significant level version	$2\left(\frac{2n^2}{\left(\Phi^{-1}(\alpha)\right)^2}\right)^{0.2}$ $\quad 5\%siglevel = 1.88n^{0.4}$ ROUND(4*(2*ObsCount^2/(NORMSINV(ChiSigLvl))^2)^0.2,0)
Sturges (performs poorly for n<30)	$1+\log_2(n) \cong 1+3.322\log_{10}(n) \cong 1+1.443\ln(n)$
Scott's Choice	$\frac{\sqrt[3]{n}(\max-\min)}{3.5s}$ ROUNDUP((*n*^(1/3)*SampleRange)/(3.5*Stdev(Sample)),0)
Freedman-Diaconis	$\frac{\sqrt[3]{n}(\max-\min)}{2IQR}$ ROUNDUP((n^(1/3)*SampleRange)/(2*SampleInnerQuad),0)
Square Root choice	$\sqrt{n}$

where,

n is the sample size

Φ $^{-1}$ is the inverse standard normal distribution

α is the level of confidence for the chi-squared test

s is the sample standard deviation

IQR is the Inter-Quartile Range of the sample set Y

A.9.6 Akaike Information Criterion (AIC)

The AIC is used to measure the relative goodness-of-fit for a statistical model. AIC is founded in information theory, offering a relative measure of the information lost when a given model is used to describe reality. Some information will almost always be lost due to using a candidate model to represent the "true" model. Among the candidate models, we want to select the one that minimizes the information loss.

This criterion was developed by Hirotsugu Akaike, under the name of "an information criterion" (AIC). And it was first published by Akaike in 1974 (see **Reference 4**). In the general case, the AIC is given by:

Equation A-69 Akaike Information Criterion (AIC)

$$-2\ln(L) + 2p = n*\ln(SSE/n) + 2p$$

Where,

p is the number of parameters estimated in the statistical model
n is the sample size
SSE is the sum of squared deviations between the actual observations and the predicted values from an estimating model
L is the maximized value of the likelihood function for the estimated model.

The selection criterion is as follows: given a set of candidate models, the model which has the smallest AIC value, is the preferred model.

When fitting models to a data set, we can increase the likelihood by adding more parameters, but doing so may result in over-fitting. Based upon the formula above, the AIC is driven by the goodness-of-fit measure; meanwhile, it also penalizes over-fitting. In other words, the more estimated parameters there are in the model, the larger the AIC becomes at a given goodness-of-fit measure. For example, if both a lognormal distribution and a beta distribution are equally good candidates for fitting a particular data set, the lognormal distribution is preferable because it has only two estimated parameters while the beta has four.

In the actual implementation, we first select a set of candidate models for a given data set. We then compute these models' respective AIC values and we find the model with the smallest AIC. Additionally, we can use the statistic below to indicate the relative probability that the ith model minimizes the (estimated) information loss:

Equation A-70 AIC Statistic to Indicate Relative Probability That ith Model Minimizes Info Loss

$$\exp((AICmin - AICi)/2)$$

$$\text{for } i = 1, 2, \ldots, m$$

where,

m is the total number of candidate models
AIC1, AIC2, …, AICm are used to denote their respective AIC values
AICmin represents the minimum of these values.

These numbers (i.e., exp((AICmin−AICi)/2)) can be used as the weighting factors for developing a weighted model if they are relatively large. Let us use the following example to explain the process. If there are three candidate models for a particular data set, with AIC values 50, 52, and 60. The first model is the best one because its AIC value is the smallest among the three. The second model is exp((50−52)/2) = 0.368 times as probable as the first model to minimize the information loss, while the third model is exp((50−60)/2) = 0.007 times as probable as the first model to minimize the information loss. In this case, we would eliminate the third model from further consideration. We could also take a weighted average of the first two models, with weights 1 and 0.368, respectively, and then do statistical inference based on the weighted model. (See **Reference 61** for details.) Alternatively, we could gather more data to distinguish between the first two models.

Note that AIC values provide a means for model selection, but AIC does not tell how well a model fits the data in an absolute sense. If all the candidate models fit poorly, AIC will not give any warning of that.

A.9.7 Bayesian information criterion (BIC)

The BIC or Schwarz criterion (also SBC, SBIC) was developed by Gideon E. Schwarz in 1978 (see **Reference** 6). The BIC is also used for model selection among a finite set of models and is closely related to the AIC. Mathematically, it is given by

Equation A-71 Bayesian information criterion (BIC)

$$-2\ln(L) + p*\ln(n) = n*\ln(SSE/n) + p*\ln(n)$$

Just like the AIC, the BIC statistic is also derived from the log-likelihood function[14] as shown by the formula above. Both statistics take into account the number of estimated parameters of the fitted distribution. However, the BIC penalizes more strongly than AIC for the number of estimated parameters.

[14] Log-likelihood function: A likelihood function, by definition, is the joint probability density function of a random sample.

Note that Akaike was so impressed with Schwarz's Bayesian formula that he developed his own Bayesian formula, which is now referred to as “Akaike's Bayesian Information Criterion” (ABIC). (See **Reference 7** for details.)

The general recommendation of @RISK Version 6 is to use the AIC or BIC for selecting a fit result unless there is a specific reason for not doing so.

A.10 RISK SIMULATION SAMPLING METHODS

A.10.1 Monte Carlo

The Monte Carlo method has been successfully used in scientific applications for at least 65 years. It is a problem solving technique used to approximate the probability of certain outcomes by running multiple trial runs, called simulations, using random variables. Credit for inventing the Monte Carlo method often goes to Stanislaw Ulam, a Polish born mathematician who worked for John von Neumann on the United States’ Manhattan Project during World War II. Ulam is primarily known for designing the hydrogen bomb with Edward Teller in 1951. He invented the Monte Carlo method in 1946 while pondering the probabilities of winning a card game of solitaire. Ulam and Metropolis published the first paper on the Monte Carlo method in 1949 (**Reference** 1).

A.10.2 Latin Hypercube

The Latin Hypercube technique is a modification of the Monte Carlo sampling process. The concept was developed to ensure that the entire range of each variable is sampled. Latin Hypercube sampling (also known as stratified sampling) has been shown to require fewer model iterations to approximate the desired variable distribution to the same level of accuracy as the Monte Carlo method. It works as follows:

- The distribution is divided into segments of equal probability
- Segments are randomly selected for sampling
- All segments are sampled before a segment is sampled again
- Each sample is drawn from its segment by uniform random sampling

In **Figure A-11**, the triangular cumulative distribution function has been divided into 10 intervals of equal probability (i.e. the area of each interval is the same) and a sample is randomly selected from each interval. Once a sample is taken from a particular interval, this interval is not sampled again until all segments have been sampled. The triangular distribution to the left illustrates how Crystal Ball made 10 random selections. Note that the tails of the distribution are not sampled. But in the triangle to the right, 10 random samples using the Latin Hypercube method does sample more of the distribution. Tools like @RISK and ACE automatically set the number of segments to equal the number of samples (also called trials or iterations). Tools like Crystal Ball allow the user to specify the number of segments independent of the number of samples taken. This handbook recommends that segments be set to the same number as set for trials.

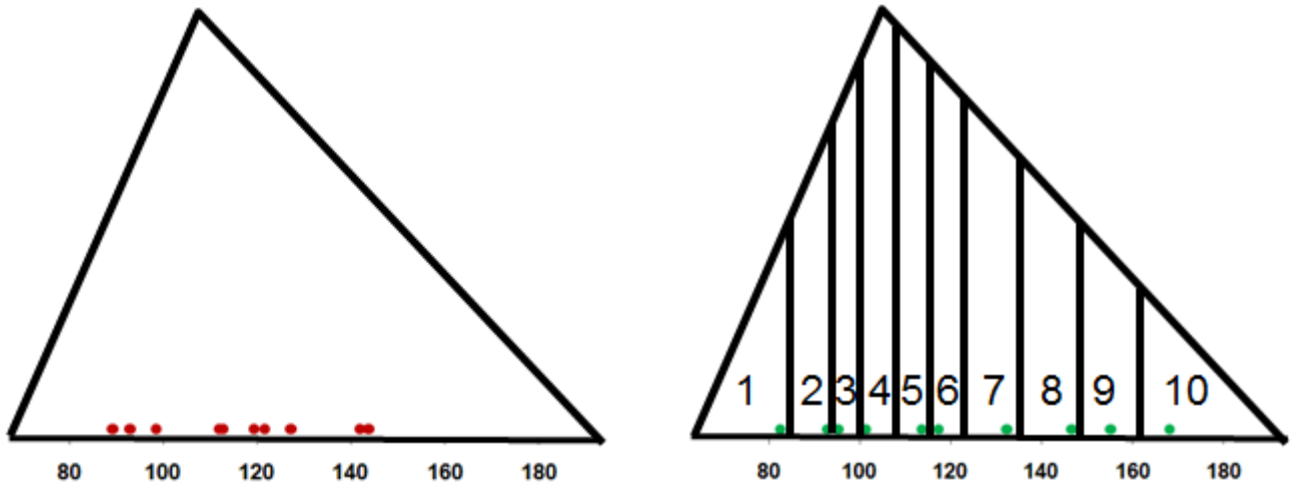


Figure A-11 Compare Monte Carlo to Latin Hypercube

A.10.3 Correlated Random Samples

Drawing random samples from a random number generator is one thing, generating correlation across two or more random variables is another. There is a lot of literature on the topic, including **Reference 20**. Several tools (e.g., Crystal Ball, @RISK) apply a user defined correlation matrix as rank order correlation. Others (e.g., ACE) induce Pearson product moment (PPM) correlation. **Reference 20** discusses how to apply a PPM correlation matrix in detail. A quick synopsis of what is involved would go something like this: Let us assume a total cost is the sum of many uncertain throughputs defined by different uncertainty distributions. Let's further assume that the distributions need to be correlated. The first step is to apply something called the "Cholesky's decomposition method" to a set of independent standard normal random variables to construct correlated standard normal distributions. These correlated normal random variables are then transformed into correlated uniform random variables based upon their respective ranks (not to be confused with rank correlation. The inverse of the user-defined distribution are then applied to invert these uniform draws into draws resembling the user-defined distributions. The final step is to iterate on the Cholesky's factors to achieve the desired correlations among the WBS elements. This way, both the user-defined distributions and the correlation structure are preserved. The details of this process are beyond the scope of this handbook.

A.11 CORRELATION

A.11.1 Overview

An important consideration in risk and uncertainty analysis is to adequately account for the relationships between the cost elements during a risk simulation. This interrelationship between the WBS elements is commonly known as "dependency" or "correlation." For example, data from a number of similar projects, or expert opinion may indicate that as the cost of WBS element A increases, the cost of WBS element B also tends to increase (positive correlation); and perhaps the cost of WBS element F is expected to decrease (negative correlation). Often, it is easier to defend the application of correlation between uncertain input variables that are driving the results of one or more WBS elements. It is also important to note that correlation between any two random variables does not prove or disapprove a cause-and-effect relationship between them.

Correlation is applied at the lowest levels of the WBS, where CER uncertainty is defined. It is also applied across input variables. Correlation does not impact the distributions it is applied to; it impacts the parent level of the WBS or anywhere two or more correlated variables are combined. The specification of correlation within an uncertainty assessment will magnify the uncertainty impact at the aggregate level as

child WBS elements are forced to move together. If the child element uncertainty distributions are left to be sampled independently of one another, then the high sample on one distribution can be canceled by a simultaneous low on another element. A common reason for very steep S-curves at the parent levels in estimates with many elements is a lack of correlation. Positive correlation causes elements to move in the same direction (tending to magnify the uncertainty effect), while negative correlation causes elements to move opposite to each other (tending to cancel each other). Care should be taken when a random variable occurs in the denominator of an equation as opposed to the numerator. In such cases movement of the variable has the opposite impact than in the cases of the variable is in the numerator. As the denominator variable gets larger, the overall value of the equation get smaller rather than larger. As such, the movement of a variable in the denominator positively correlated to one in the numerator will tend have the opposite effect of the movement of the numerator. This results in the combined effect of the movement of the two variables tending to cancel each other out. That is, if the denominator proportionality increases in line with the increased value of the numerator, the overall ratio (equation value) remains constant.

Implementing positive correlation across the risk and uncertainty distributions will result in broader dispersion (increased variance) of the uncertainty result at the aggregate or parent levels in the WBS. The impact can be significant. There have been many papers on the topic, such as **Reference 20, 21, 22, 26, 27** and **31. Reference 32** provides reasons why the correlation between weight variables should not be calculated from the same data set used to develop a CER.

A.11.2 Functional Correlation

Correlation of the risk distributions in a cost model often will be already captured through the mathematical relationships within the cost model. For instance, if both the costs of Data and SEPM are modeled by using certain factors times the cost of the Prime Mission Product (PMP), then Data and SEPM will be positively correlated in the simulation. In this situation, as PMP changes in the risk simulation, the costs of Data and SEPM will change in the same direction accordingly; therefore, they are positively correlated. Beware of unintended correlation. For example, if the same uncertainty labor rate is used across a number of elements, those elements will be highly, positively correlated. To control this situation, separate uncertain labor rates should be created and the correlation between them controlled.

A.11.3 Applied Correlation

Applied correlations are those specified by the user and implemented within a model. Before specifying any additional correlation among the WBS elements, it is recommended that the user measure the correlations already present in the cost risk model. Correlations (or dependencies) between the uncertainties of WBS CER uncertainties are generally determined subjectively. There have been many studies attempting to find objective evidence for correlations. However, these correlations should not be estimated by the cost-vs.-cost correlations from the same historical database from which the CERs are derived. In other words, strong correlations between cost elements in a database should not be mistaken as evidence that residuals or percentage errors of the CERs derived from the same database are correlated. See **Reference 26, 27** and **32** for details.

A.11.4 Measuring Correlation

The two primary methods for measuring correlation are Pearson's Product Moment and Spearman Rank order. The appropriateness of these two types in the context of cost estimating is discussed in **References 20, 21** and **22**. Crystal Ball and @RISK employ Spearman Rank and ACE employs a variation on the Pearson Product Moment method recommended in **Reference 31.** In practice, there is little or no impact on the total cost results when the only difference is the correlation type applied. The results of the example model in this handbook demonstrate this.

A.11.5 Pearson's Correlation Coefficient

Pearson's correlation coefficient between two sets of numbers is a measure of the linear association between these two sets. It measures the degree to which two sets of data move together in a linear manner. A high positive correlation indicates a strong direct linear movement and a high negative correlation represents a strong inverse relationship. The correlation ranges between –1.0 and +1.0, where -1 indicates a perfect inverse relationship, 0 indicates no correlation, and +1 indicates a perfect positive relationship. In probability theory and statistics, Pearson's correlation coefficient indicates both strength and direction of the linear relationship between two variables. Pearson's correlation coefficient is important in cost-risk analysis because it appears explicitly in the formula for the total-cost standard deviation and therefore impacts the spread of the total-cost distribution.

By definition, Pearson's correlation coefficient (Pearson's r) calculated between two sets of numbers $\{x_i\}$ and $\{y_i\}$ is given by

Equation A-72 Pearson's Correlation Coefficient

$$r_{xy} = \frac{\sum_{i=1}^{n}(y_i - \bar{y})(x_i - \bar{x})}{\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}}$$

Where $\bar{x}$ and $\bar{y}$ are the means of $\{x_i\}$ and $\{y_i\}$, respectively, and n is the sample size.

Pearson Product Moment Correlation coefficient (PPMC) is the most widely used method for determining correlation. It is a measurement of the linear relationship between two related variables. **Figure A-12** demonstrates a different aspect of the PPMC, specifically:

- First row demonstrates how the strength relates to the dispersion
- Second row demonstrates how the strength does not represent the slope of the relationship
- Third row demonstrates how the coefficient alone could be an insufficient measurement of the relationship

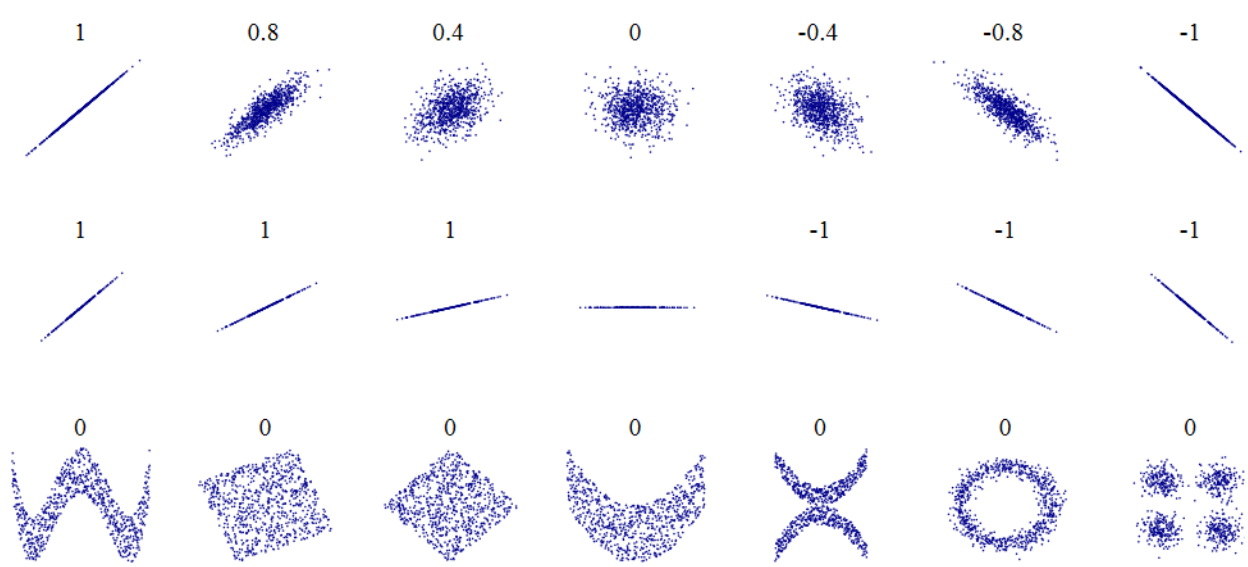


Figure A-12 PPMC Result For Various Patterns of Data[15]

[15] Image was put into the public domain by Wikimedia Commons.

A.11.6 Spearman's Rank Correlation Coefficient

Spearman's rank order correlation is used in nonparametric inferences to determine if two random variables are independently distributed. Therefore, no assumptions are made about the underlying distributions. When Spearman's correlation coefficient is significantly different from zero, it can be interpreted as an association between two variables, just like the ordinary Pearson product-moment correlation. However, the Spearman's rank correlation coefficients do not appear explicitly in the formula for the total-cost standard deviation. Therefore, their impact upon the spread of the total-cost distribution is not generally understood.

The Spearman Rank Order correlation is computed from the ranking of the elements in the ordered pairs, as opposed to the actual values. If some of the ranks are identical, all of the ties are assigned the average of the ranks that they would have had if their values had been slightly different. In this situation, some of the ranks could be partial integers. In all cases the sum of all assigned ranks will be the same as the sum of the integers from 1 to n where n represents the number of elements, namely n(n + 1)/2.

To calculate the Spearman Rank correlation coefficient, let R_i be the rank of x_i among the other x's, let S_i be the rank of y_i among the other y's, and let ties be assigned the appropriate average as described above. Then the rank-order correlation coefficient is defined to be the linear correlation coefficient of the **ranks**, and is given below:

Equation A-73 Spearman Rank Correlation Coefficient

$$r_s = \frac{\sum_{i=1}^{n}(R_i - \overline{R})(S_i - \overline{S})}{\sqrt{\sum_{i=1}^{n}(R_i - \overline{R})^2} * \sqrt{\sum_{i=1}^{n}(S_i - \overline{S})^2}}$$

If there are no ties in the ranking, then the equation can be reduced to:

Equation A-74 Spearman Rank Correlation Coefficient When No Ties in the Ranking

$$r_s = 1 - \frac{6 * \sum_{i=1}^{n} d^2}{n * (n^2 - 1)}$$

Where:

d = difference between the paired ranks
n = number of paired ranks

A.11.7 Comparing Spearman Rank to Pearson Product Moment

Crystal Ball and @RISK employ "Spearman Rank" correlation. Excel's CORREL measures Pearson Product-Moment. While there have been several papers denouncing rank order correlation as inappropriate for cost analysis (see **References 20** and **21**), it is rare to see notable differences in results between the two approaches (see **References 31, 36, 38, 39, 42**). As an example, the resulting correlation matrices from the missile example as generated by Crystal Ball (which uses Spearman Rank) and ACE (which uses Pearson Product-Moment) are almost identical as illustrated in **Table A-15**. The shaded cells are the only ones where the correlation coefficient difference by more than 0.02 but less than 0.03.

Table A-15 Measured Pearson Product Moment Correlations

	Missile Sys	EMD	AV EMD	DesignDevEMD	Proto EMD	SW EMD	SysEng EMD	PM EMD	STE EMD	Trg EMD	Data_EMD	PSE_EMD
	CB Resultant Pearson Product Moment Correlation											
Missile Sys	1.00	0.90	0.75	0.72	0.75	0.73	0.46	0.48	0.61	0.74	0.73	0.61
EMD		1.00	0.82	0.80	0.57	0.81	0.56	0.58	0.49	0.81	0.81	0.48
AV EMD			1.00	1.00	0.48	0.42	0.30	0.34	0.37	0.97	0.97	0.37
DesignDevEMD				1.00	0.42	0.40	0.29	0.33	0.33	0.97	0.97	0.32
Proto EMD					1.00	0.45	0.21	0.23	0.78	0.47	0.47	0.78
SW EMD						1.00	0.29	0.33	0.35	0.40	0.40	0.34
SysEng EMD							1.00	0.67	0.29	0.33	0.33	0.29
PM EMD								1.00	0.29	0.37	0.36	0.29
STE EMD									1.00	0.40	0.40	0.72
Trg EMD										1.00	0.96	0.40
Data_EMD											1.00	0.39
PSE_EMD												1.00

	Missile Sys	EMD	AV EMD	DesignDevEMD	Proto EMD	SW EMD	SysEng EMD	PM EMD	STE EMD	Trg EMD	Data_EMD	PSE_EMD
	ACE Resultant Pearson Product Moment Correlation											
Missile Sys	1.00	0.91	0.75	0.72	0.75	0.72	0.45	0.47	0.61	0.74	0.74	0.61
EMD		1.00	0.82	0.80	0.56	0.80	0.55	0.57	0.48	0.81	0.81	0.47
AV EMD			1.00	1.00	0.47	0.40	0.30	0.33	0.36	0.97	0.97	0.36
DesignDevEMD				1.00	0.41	0.38	0.29	0.32	0.31	0.97	0.97	0.31
Proto EMD					1.00	0.44	0.20	0.22	0.77	0.46	0.46	0.78
SW EMD						1.00	0.27	0.32	0.34	0.38	0.39	0.34
SysEng EMD							1.00	0.65	0.28	0.34	0.34	0.28
PM EMD								1.00	0.28	0.36	0.36	0.28
STE EMD									1.00	0.39	0.39	0.72
Trg EMD										1.00	0.96	0.38
Data_EMD											1.00	0.39
PSE_EMD												1.00

A.12 ALTERNATIVE ALLOCATION METHODS

A.12.1 Adjusting Standard Deviation for Correlation

Section 3.6.3 described a very simple allocation method where standard deviation was used as the basis for allocating the difference between a parent's result at a specific probability to the sum of its children at the same probability (adjusting the percentile, not the point estimate). An alternative is to use correlation adjusted standard deviations. Lower-level variances (standard deviation squared) will not sum to the parent level variance unless the impact of correlation is captured. To obtain the correct variance sum, the element variances need to be adjusted for correlation using **Equation 3-1**. The process to perform this adjustment is illustrated in **Figure A-13**. The Excel function to make this calculation in cell K9 is I9*MMULT(Q9:Y9,$I$9:$I$17).

Need^2 Adjusted for Correlation = Need*MMULT(CorrRow, EMD Need)

Calculate and Adjust Std Dev For Correlation	Std Dev Reported	Std Dev Adj For Corr	Variation Adj For Corr
Missile System	$69,883		
Engineering and Manufacturing D	$46,974	$46,974	$2,206,594,963
Air Vehicle	$20,493	$27,862	$776,302,671
Design & Development	$19,800	$27,134	$736,251,907
Prototypes	$1,520	$6,329	$40,050,764
Software	$23,532	$29,723	$883,431,894
System Engineering	$8,324	$14,611	$213,482,727
Program Management	$5,480	$12,132	$147,180,502
System Test and Evaluation	$1,684	$6,231	$38,829,821
Training	$1,503	$7,518	$56,527,069
Data	$1,997	$8,673	$75,220,490
Peculiar Support Equipment	$693	$3,952	$15,619,790

	EMD									
EMD	AV	Des & Dev	Proto	SW	Sys Eng	PM	STE	Trg	Data	PSE
1.000	0.806	0.792	0.561	0.799	0.546	0.572	0.491	0.801	0.802	0.480
0.806	1.000	0.998	0.485	0.385	0.273	0.323	0.378	0.970	0.970	0.363
0.792	0.998	1.000	0.425	0.365	0.269	0.318	0.331	0.967	0.968	0.316
0.561	0.485	0.425	1.000	0.437	0.178	0.207	0.774	0.474	0.472	0.771
0.799	0.385	0.365	0.437	1.000	0.272	0.314	0.339	0.379	0.378	0.345
0.546	0.273	0.269	0.178	0.272	1.000	0.660	0.268	0.306	0.308	0.267
0.572	0.323	0.318	0.207	0.314	0.660	1.000	0.274	0.348	0.352	0.274
0.491	0.378	0.331	0.774	0.339	0.268	0.274	1.000	0.407	0.409	0.717
0.801	0.970	0.967	0.474	0.379	0.306	0.348	0.407	1.000	0.959	0.388
0.802	0.970	0.968	0.472	0.378	0.308	0.352	0.409	0.959	1.000	0.391
0.480	0.363	0.316	0.771	0.345	0.267	0.274	0.717	0.388	0.391	1.000

Figure A-13 Adjusting Standard Deviation for Correlation

The correlation matrix is obtained from extracting the trial data and measuring the correlation between elements. The process defined in **Section 3.6.3** is the same, but using the adjusted standard deviations.

Alternative approaches to risk dollar allocation are discussed in detail in **Reference 56**.

A.12.2 Needs-Based Allocation

The Needs-Based Allocation method is an alternative to the standard deviation based allocation method described in **Section 3.6.** Details of this approach can be found in **Reference 54** and they are summarized here. Key assumptions and goals of this method include:

- Allocation of risk dollars[16] to project elements must put risk dollars where they are "needed"
- Need of any WBS element is defined as the probability it will overrun its point estimate
- Need is calculated as the difference between the selected percentile and the point estimate
- An element that has preponderance of probability below its point estimate (such as a left skewed distribution) has little or no need
- Need is adjusted by inter element correlation in the same way as **Equation 3-1**, but instead of variation, the correlation adjustment is applied to need as follows:
    - Risk dollars at the parent level are prorated (allocated) to the child elements based on need
    - WBS elements having more cost uncertainty shall be allocated more risk dollars
    - Element uncertainty used to allocate risk dollars shall capture the impact of inter-element correlation
    - Risk-dollar allocation shall not result in an element's estimate being reduced below its point estimate

Reference 54 discusses a variation of the method identified in in **Section 3.6**. Instead of adjusting the selected percentile (as defined in **Section 3.6**), the author discusses the issues arising when prorating risk dollars relative to the point estimate. The key complaint is that the method in **Section 3.6** will blindly reduce an element's cost below the point estimate. This is a valid point, but others may view this as a strength rather than a weakness if the intent is to fund all elements to roughly the same percentile. Business rules such as these need to be considered and adopted or rejected as agencies deem fit.

[16] Note that we have retained the term "risk dollars" rather than replace it with probability adjustment (PA) dollars in order to remain consistent with the language Dr. Book used in the many publications he authored on the Needs Method.

The needs method is summarized as follows:

- Compute the need for each WBS element, $Need_j$ (minimum need is zero)
- Compute correlation between risk dollar requirements of elements i and j, $Corr_{ij}$
- Compute Total Need Base

Equation A-75 Need Based Calculation

$$\text{Total Need Base} = \sum_{i=1}^{n} \sum_{j=1}^{n} Corr_{ij}\, Need_i\, Need_j$$

- Compute the Correlation Adjusted Need For Each Element

Equation A-76 Correlation Adjusted $Need_j$

$$\sum_{i=1}^{n} Corr_{ij}\, Need_i\, Need_j$$

- Risk dollars are prorated to child elements based upon the correlated adjusted need per element divided by the total base need. Need is used instead of standard deviation to prorate the risk dollars.

While the reference does not address it, the algorithm has to account for child elements where the point estimate is greater than the desired probability level. Presumably, the difference between the point estimate and the desired probability level (a negative number in this case) is subtracted from the parent risk dollars and the remainder is allocated to those elements where the desired probability level is in fact higher than the point estimate. This is necessary if the child elements are to sum to the parent correctly. It is unclear what should happen if the amount to be subtracted from the parent level risk dollars is higher than the parent level risk dollars.

The standard deviation method in **Section 3.6** performs the allocation to immediate children and from there to the next lower level and so on. In our example for the needs approach, the process is applied to the lowest level WBS elements all at the same time. The first step, however, is to develop the need adjusted for correlation to be used as the basis for the allocation. The data required to perform the calculation is the point estimate, the simulation results for every WBS element at the probability of interest and the correlation matrix. We can obtain the correlation matrix by extracting the simulation data and measuring the correlation between elements using the Excel CORREL function.

The necessary data for the EMD phase of our example model is illustrated in **Figure A-14**. It also shows how to apply **Equation A-76** in Excel to generate the correlation adjusted need. The formula in cell F9 is E9*MMULT(Q9:Y9,$E$9:$E$17). This formula is written to facilitate copy/pasting it to cells F10:F17.

Need^2 Adjusted for Correlation = Need*MMULT(CorrRow, EMD Need)

Calculate and Adjust Need For Correlation	Point Estimate	85% Result	Need Unadjusted	Need^2 Adj For Corr
Missile System	$246,836	$393,263		
Engineering and Manufacturing D	$83,539	$175,643	$92,104	$5,478,432,524
Air Vehicle	$14,944	$44,271	$29,327	$1,649,681,403
Design & Development	$12,000	$39,226	$27,226	$1,530,361,494
Prototypes	$2,944	$5,822	$2,877	$119,319,909
Software	$31,500	$66,192	$34,692	$2,010,072,404
System Engineering	$17,500	$35,481	$17,981	$818,016,531
Program Management	$15,000	$26,045	$11,045	$511,171,578
System Test and Evaluation	$1,767	$5,281	$3,514	$132,107,590
Training	$897	$3,145	$2,249	$128,973,713
Data	$1,196	$4,231	$3,035	$174,402,151
Peculiar Support Equipment	$736	$2,208	$1,472	$54,007,154

	EMD									
EMD	AV	Des & Dev	Proto	SW	Sys Eng	PM	STE	Trg	Data	PSE
1.000	0.806	0.792	0.561	0.799	0.546	0.572	0.491	0.801	0.802	0.480
0.806	1.000	0.998	0.485	0.385	0.273	0.323	0.378	0.970	0.970	0.363
0.792	0.99	1.000	0.425	0.365	0.269	0.318	0.331	0.967	0.968	0.316
0.561	0.485	0.425	1.000	0.437	0.178	0.207	0.774	0.474	0.472	0.771
0.799	0.385	0.365	0.437	1.000	0.272	0.314	0.339	0.379	0.378	0.345
0.546	0.273	0.269	0.178	0.272	1.000	0.660	0.268	0.306	0.308	0.267
0.572	0.323	0.318	0.207	0.314	0.660	1.000	0.274	0.348	0.352	0.274
0.491	0.378	0.331	0.774	0.339	0.268	0.274	1.000	0.407	0.409	0.717
0.801	0.970	0.967	0.474	0.379	0.306	0.348	0.407	1.000	0.959	0.388
0.802	0.970	0.968	0.472	0.378	0.308	0.352	0.409	0.959	1.000	0.391
0.480	0.363	0.316	0.771	0.345	0.267	0.274	0.717	0.388	0.391	1.000

Figure A-14 Calculating Need and Correlation Adjusted Need

The need allocation process is illustrated in **Figure A-15**.

"NEEDS" ALLOCATION METHOD		Column	1	2	3	4	5	6	7
85% Allocated from Second Level BY 2014 $K			Point Estimate	Need Required (6-1)	Need Allocation Basis (x 0.001)	Need Allocation (3/(3 Bold))	Child Adjustment (2 * 4)	Allocated Result (1 + 5)	PA Dollars (6-1)
	Missile System	MissileSys	$246,836					$401,374	$154,538
1	**Engineering and Manufacturing**	EMD	$83,539	$91,955	**7,518,677**			**$175,494**	$91,955
1.1	Air Vehicle	AV_EMD	$14,944					$43,682	$28,738
1.1.1	Design & Development	DesignDev_EMD	$12,000		2,112,669	28.1%	$25,838	$37,838	$25,838
1.1.2	Prototypes	Proto_EMD	$2,944		237,089	3.2%	$2,900	$5,844	$2,900
1.2	Software	SW_EMD	$31,500		2,770,902	36.9%	$33,889	$65,389	$33,889
1.3	System Engineering	SysEng_EMD	$17,500		984,158	13.1%	$12,036	$29,536	$12,036
1.4	Program Management	PM_EMD	$15,000		636,340	8.5%	$7,783	$22,783	$7,783
1.5	System Test and Evaluation	STE_EMD	$1,767		247,254	3.3%	$3,024	$4,791	$3,024
1.6	Training	Trg_EMD	$897		185,600	2.5%	$2,270	$3,167	$2,270
1.7	Data	Data_EMD	$1,196		244,700	3.3%	$2,993	$4,188	$2,993
1.8	Peculiar Support Equipment	PSE_EMD	$736		99,966	1.3%	$1,223	$1,959	$1,223
2	**Production & Deployment**	Production	$163,297	$62,582	**4,811,902**			**$225,880**	$62,582
2.1	Air Vehicle	ProdAV	$104,826					$151,615	$46,790
2.1.1	Airframe	Airframe	$21,651		670,080	13.9%	$8,715	$30,366	$8,715
2.1.2	Propulsion	Propulsion	$21,849		350,401	7.3%	$4,557.24	$26,406	$4,557
2.1.3	Guidance	Guidance	$27,810		1,193,852	24.8%	$15,526.96	$43,337	$15,527
2.1.4	Payload	Payload	$17,246		389,213	8.1%	$5,062.01	$22,308	$5,062
2.1.5	Air Vehicle IAT&C	IAT&C	$16,269		994,076	20.7%	$12,928.71	$29,198	$12,929
2.2	System Engineering	SysEng	$12,000		18,018	0.4%	$234.33	$12,234	$234
2.3	Program Management	PM	$10,000		14,013	0.3%	$182.25	$10,182	$182
2.4	System Test and Evaluation	STE	$5,000		6,785	0.1%	$88.25	$5,088	$88
2.5	Training	Trg	$4,193		288,088	6.0%	$3,746.81	$7,940	$3,747
2.6	Data	Data	$4,193		258,152	5.4%	$3,357.46	$7,550	$3,357
2.7	Peculiar Support Equipment	PSE	$7,634		17,043	0.4%	$221.66	$7,856	$222
2.8	Initial Spares and Repair Parts	InitalSpares	$15,451		612,181	12.7%	$7,961.89	$23,413	$7,962

Figure A-15 The Need Allocation Process

A.12.3 Comparing Three Allocation Methods

Figure A-16 compares three methods: Allocate by standard deviation, Allocate by standard deviation adjusted for correlation and the Needs method. **Figure A-16** is evidence that for this model, any of the approaches are reasonable. Consequently, the simplest (**Section 3.6**) to implement is the recommended approach.

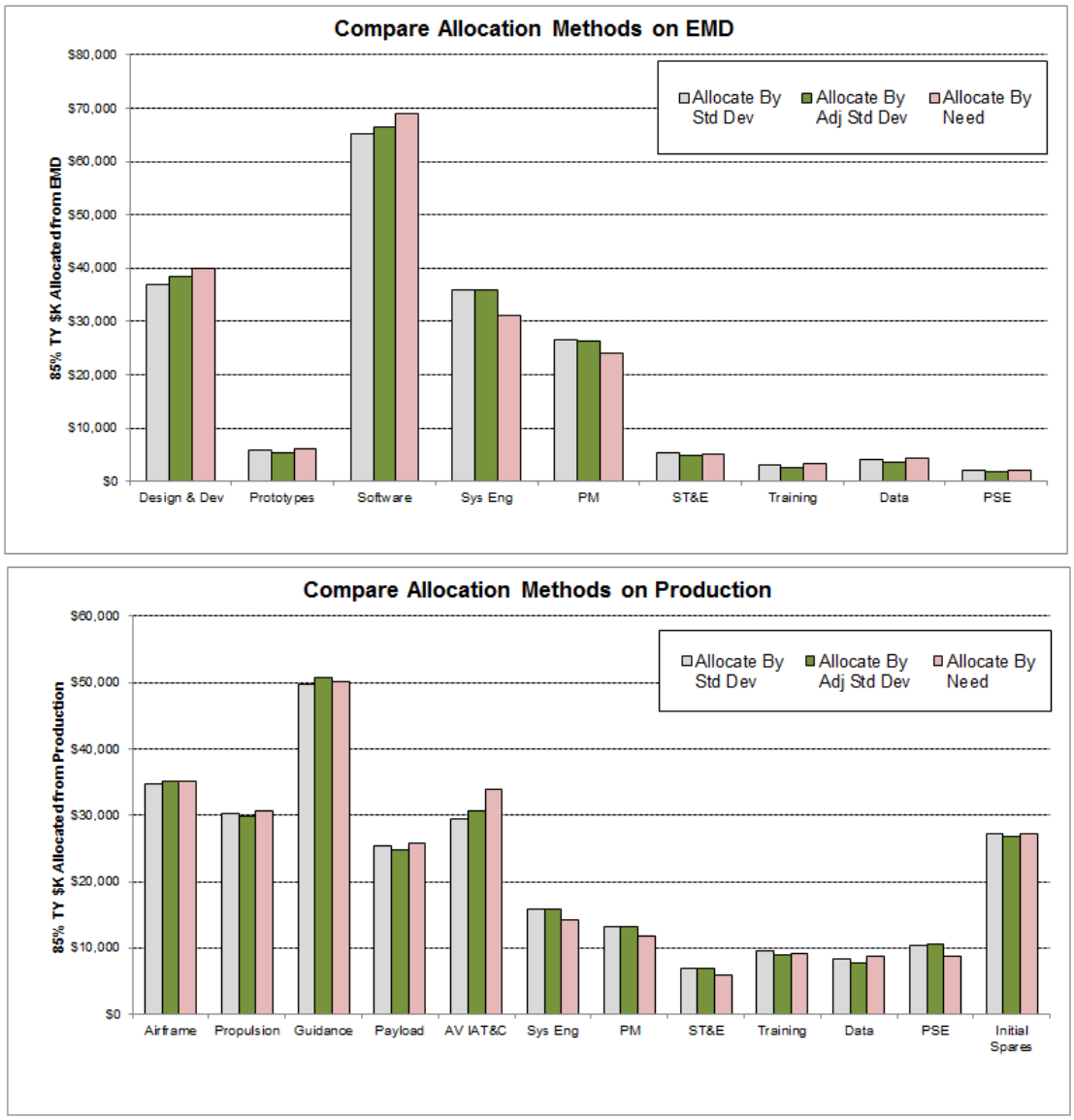


Figure A-16 Comparing Three Allocation Methods On 85% TY $K

A.13 ENHANCED SCENARIO-BASED METHOD

A.13.1 Overview, Approach and Assumptions

eSBM is an alternative to the CISM approach. Rather than building up risk and uncertainty element by element as in Monte Carlo simulation, eSBM instead shifts attention to the identification and quantification of what can go right and what can go wrong with an acquisition program from a high-level management perspective point of view. For a detailed explanation of eSBM, see **Reference 86**.

The scenario-based method is centered on articulating and costing a program's risk scenarios. Risk scenarios are coherent stories or narratives about potential events that, if they occur, increase program cost beyond what was planned. eSBM is implemented in accordance to the steps in **Figure A-17**.

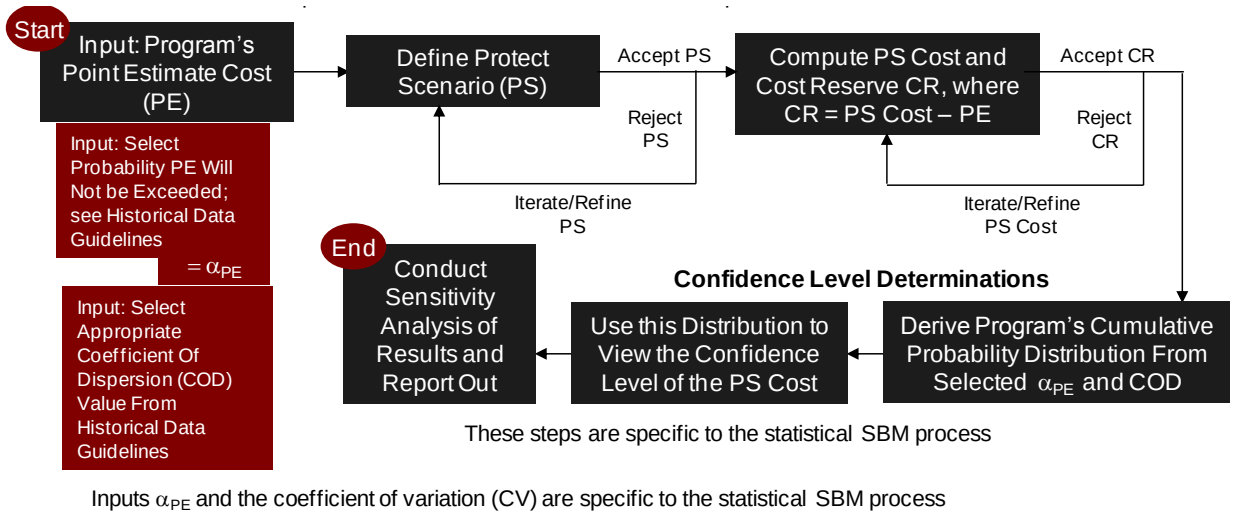


Figure A-17 eSBM Process and Implementation Steps

The steps to eSBM are illustrated in **Figure A-17** as follows:

- **Step 1: Develop eSBM Inputs**: The Start Step is the input to the process. It is the program's point estimate cost (PE). The point estimate cost is the cost that does not include allowances for reserve. The PE is the sum of the cost-element costs across the program's work breakdown structure without adjustments for uncertainty. The PE is often developed from the program's cost analysis requirements description (CARD). In the Start Step, eSBM also requires inputs on the probability α that PE will not be exceeded and the coefficient of variation (CV).

  The probability a program's point estimate cost PE will not be exceeded is the value α such that $P(Cost \leq PE) = \alpha$. It is anecdotally well understood a program's PE usually faces higher, not lower, probabilities of being exceeded – especially in the early life cycle phases. The interval $0.10 \leq \alpha \leq 0.50$ expresses this anecdotal experience. It implies a program's PE will very probably experience growth instead of reduction. Unless there are special circumstances, a value for α from this interval should be selected for SBM/eSBM and a justification written for the choice. Yet, the question remains what value of α should be chosen? With eSBM, ways to guide that choice are available from the analysis of program cost growth histories presented in **Reference 86**.

  The CV is the ratio of a probability distribution's standard deviation to its mean. The CV is a way to examine the variability of any distribution at plus or minus one standard deviation around its mean. Select an appropriate realistic, historically based, coefficient of variation for use in generating a probability distribution (**Reference 86)**. If future values of inflation are regarded as stochastic, then select then-year dollar CVs. Otherwise; select base-year dollar CVs. Likewise, select quantity-adjusted or non-quantity-adjusted CVs according to whether or not potential changes in the numbers of baseline developmental and production units of a weapon system are regarded as falling within the purview of the cost estimate. "Quantity-adjusted" is the usual selection in the context of executing risk and uncertainty analysis, and in producing S-curves.

  With values assessed for α and CV, the program's cumulative cost probability distribution can then be derived. This distribution is used to view the probability level associated with the protect scenario cost PS, as well as probability levels associated with any other cost outcome along this distribution.

- **STEP 2: DEFINE PROTECT SCENARIO**: The next step in **Figure A-17** is defining a protect scenario. A protect scenario captures the cost impacts of major known risks to the program – those events the program must monitor and guard against occurring. The protect scenario is not arbitrary, nor should it reflect extreme worst-case events. It should reflect a possible program cost that, in the

judgment of the program, has an acceptable chance of not being exceeded. In practice, it is envisioned that management will converge on an "official" protect scenario after deliberations on the one initially defined. This part of the process ensures all parties reach a consensus understanding of the program's risks and how they are best described by the protect scenario.

- **STEP 3: COST THE PROTECT SCENARIO**: Once the protect scenario is established its cost is then estimated. Denote this cost by PS. The amount of cost reserve dollars (CR) needed to protect program cost can be computed as the difference between the PS and the PE. Shown in **Figure A-17**, there may be additional refinements to the cost estimated for the protect scenario, based on management reviews and other considerations. The process may be iterated until the reasonableness of the magnitude of the cost reserve dollars is accepted by management.

  The final step in **Figure A-17** is a sensitivity analysis to identify critical drivers associated with the protect scenario and the program's point estimate cost. It is recommended that the sensitivity of the amount of reserve dollars, computed in the preceding step, be assessed with respect to variations in the parameters associated with these drivers.

- **STEPS 4, 5: DERIVE THE eSBM S-CURVE AND DETERMINE PROBABILITY LEVELS**: With values assessed for α and CV, the program's cumulative cost probability distribution can then be derived. This distribution is used to view the probability level associated with the protect scenario cost PS, as well as probability levels associated with any other cost outcome along this distribution.

- **STEP 6: COST THE PROTECT SCENARIO**: The final step is sensitivity analyses on key assumptions or conditions expressed in the protect scenario(s), as well as uncertainties in values chosen for α and CV. This allows a broad assessment of probability level variability, which includes determining a range of possible program cost outcomes for any specified probability level.

A.13.2 eSBM Equations

The following are the core equations for producing an eSBM cost risk analysis. The following equations derive from the assumption that a program's total cost is normally distributed and the point estimate (PE) falls within this distribution. Given PE, α, and CV, then the mean and standard deviation of the program's cost are given by the following:

Equation A-77 eSBM Mean and Standard Deviation if the Program Cost Distribution is Normal

$$\mu = PE - z\frac{(CV)PE}{1+z(CV)}$$

$$\sigma = \frac{(CV)PE}{1+z(CV)}$$

Equation A-78 eSBM Mean and Standard Deviation if the Program Cost Distribution is Lognormal

$$\mu = e^{a+\frac{1}{2}b^2}$$

$$\sigma = \sqrt{e^{2a+b^2}(e^{b^2}-1)} = \mu\sqrt{(e^{b^2}-1)}$$

where,

$$a = \ln PE - z\sqrt{\ln(1+(CV)^2)}$$

$$b = \sqrt{\ln(1+(CV)^2)}$$

where CV is the coefficient of variation, PE is the program's point estimate cost, and z is the value such that P(Z≤z)=α where Z is the standard normal random variable. Values for z are available in look-up tables for the standard normal or from the Excel function z = Norm.S.Inv(percentile); e.g., z = 0.525 = Norm.S.Inv(0.70).

With the values computed from the above equations, the normal or lognormal distribution function of the total cost is fully specified, along with the probability that the cost may take any particular outcome, such as the protect scenario cost PS. Reference 82 presents a numerical example that illustrates implementing the above eSBM lognormal equations

A.13.3 eSBM Numerical Example

Suppose the distribution function a program's total cost is lognormal. Suppose the program's point estimate cost is $100M and this was assessed to fall at the 25th percentile. Suppose the type and life cycle phase of the program is such that 30 percent variability in cost around the mean has been historically seen. Suppose the protect scenario was defined and determined to cost $145M.

To compute the mean and standard deviation of the total cost distribution, use **Equation A-78** as follows:

$$a = \ln PE - z\sqrt{\ln(1+(CV)^2)} = \ln(100) - (-0.6745)\sqrt{\ln(1+(0.30)^2)} = 4.80317$$

$$b = \sqrt{\ln(1+(CV)^2)} = \sqrt{\ln(1+(0.30)^2)} = 0.29356$$

$$\mu = e^{a+\frac{1}{2}b^2} = e^{4.80317+\frac{1}{2}(0.29356)^2} \approx 127.3 \text{ (\$M)}$$

$$\sigma = \sqrt{e^{2a+b^2}(e^{b^2}-1)} = \mu\sqrt{(e^{b^2}-1)} = 127.3\sqrt{(e^{(0.29356)^2}-1)} \approx 38.2 \text{ (\$M)}$$

To determine the probability level of the protect scenario ($145 $M) is equivalent to solving for z_{ps} in the equation a + z_{ps}(b)=ln PS, giving:

$$z_{PS} = \frac{\ln PS - a}{b}$$

$$z_{PS} = \frac{\ln PS - a}{b} = \frac{\ln 145 - 4.80317}{0.29356} = 0.59123$$

Thus, we want α such that P(Z ≤ z_{ps}=0.59123)= α. Values for α are available in Excel as follows. With , enter into Excel: NORM.S.DIST(0.59123,TRUE); that is

NORM.S.DIST(0.59123,TRUE) = 0.723

Therefore, the $145M protect scenario cost falls at the 72nd percentile of the distribution.

A.13.4 eSBM Summary

eSBM encourages and emphasizes a careful and deliberative approach to cost-risk analysis. It requires the development of scenarios that represent the program's "risk story" rather than a debate about what percentile to select. Time is best spent building the case arguments for how a confluence of risk events that form a risk scenario might drive the program to a particular percentile. This is where the debate and the analysis take focus. In summary, eSBM

- Provides an analytic argument for deriving the amount of cost reserve needed to guard against well-defined "scenarios"

- Brings the discussion of "scenarios" and their credibility to the decision-makers; this may be a meaningful topic to many decision makers
- Allows percentiles (probability measures) to be designed into the approach with a minimum set of statistical assumptions
- Allows percentiles (as well as the mean, median, variance, etc.) to be calculated algebraically in near-real time within a simple spreadsheet environment (see worked example in **Appendix A.13.3**).
- Avoids the requirement to develop probability distributions for all the uncertain variables in a cost model
- Supports traceability and focuses attention on key risk events in the written scenarios that have the potential to drive cost higher than expected

The strength of the eSBM approach will depend on the completeness of the risk identification process, the basis for the protect scenario point estimate, the basis for the assumed CV and a defendable way to identify where the point estimate falls in the distribution.

A.14 OBLIGATIONS VS. EXPENDITURES

Building ships, aircraft, or missiles is expensive and takes place over a considerable amount of time. Multiyear funds grant authority to spend money appropriated in one year over a period of years. This is the origin of outlay rates, which provide a profile of how money appropriated (obligated) for a program may be spent over time according to the type of program. Some appropriations, such as Military Pay, Civilian Pay, and Fuel, are assumed to be spent 100% within the year of appropriation. Other categories, such as ship acquisition, are allowed to be spent (pay an invoice) over a period of up to seven years. In this context, outlays rates are used to estimate the total dollars required to account for inflation. Since budgets are expressed in obligations, cost estimates to support the budgeting process must be in terms of obligations.

Further complicating the landscape is that:

- Appropriated funds may be available to be obligated (put on contract) for several years. Thus appropriations are often referred to as one, two or three (or more) year money
- Contractors collect costs as they are accrued (as work is performed). This forms the basis for contract cost reporting to the DoD. These accruals represent actual or estimated unpaid liabilities for the DoD. However, the accruals may be quite different from the contractor vouchers submitted to and the funds (outlays) paid by the DoD.
- Incremental payments (expenditures) may or may not match the actual work performed and will often lag the reported work, especially if there is a dispute over what work has been performed

The Comptroller of the Office of the Secretary of Defense (OSD) releases inflation guidance (**Reference 90**) between January and March of each year. This guidance includes the outlay rates (profile) for each DoD appropriation. **Table A-16** contains some of the outlay rates published in February 2012. The Navy RDT&E and Weapons rates are used in the example model for this handbook.

Table A-16 FY13 President's Budget Outlay Rates

	FIRST YEAR	SECOND YEAR	THIRD YEAR	FOURTH YEAR	FIFTH YEAR	SIXTH YEAR	SEVENTH YEAR
RDT&E							
Army	14.55	56.53	22.35	3.68	1.97	0.92	
Navy	35.96	52.03	9.34	1.33	1.34		
Air Force	52.90	36.64	5.23	3.14	1.05	1.04	
Defense Wide	43.36	43.25	7.21	4.12	1.03	1.03	
Dir OT&E	55.00	36.20	6.00	1.80	1.00		
Procurement - Navy							
Navy Aircraft	13.00	35.00	26.50	14.00	7.00	3.00	1.50
Navy Weapons	20.00	32.00	27.00	12.00	5.50	2.00	1.50
Navy Shipbuilding	8.00	28.50	22.00	15.00	12.00	9.00	5.50
Navy Ammunition	9.00	39.00	27.50	12.00	7.00	4.00	1.50
Navy Other	23.00	35.00	17.00	10.00	8.00	5.00	2.00
Procurement MC	12.00	35.00	25.00	13.00	7.00	8.00	

The outlay rates can be used to develop an implied expenditure profile from the estimated obligation profile. **Table A-17** demonstrates this by applying the Navy Weapons outlay rates to each year of the example estimate Then Year (TY) results. Note that the dollars of the implied expenditures are in the year in which they were expended (same year dollars, SY) and they sum identically to the TY result. To obtain a constant year total, each year of the expenditure would be adjusted by the raw inflation (not the weighted) to convert from each year to a specific base year.

The Army performs this calculation different from the Navy and Air Force. The method the Army uses assumes that the outlay rates are applied then the obligation profile is expressed in constant dollar terms. The method the Navy and Air Force use assumes that the outlays are applied to the then year obligation profile.

Table A-17 Converting an Obligation Profile to an Expenditure Profile

	Total	FY 2019	FY 2020	FY 2021	FY 2022	FY 2023
Prod Obligations TY$	$179,872	$39,472	$44,411	$43,924	$43,936	$8,129

Navy WPN Outlay SY$	Total	FY 2019	FY 2020	FY 2021	FY 2022	FY 2023	FY 2024	FY 2025	FY 2026	FY 2027	FY 2028	FY 2029
Implied Expenditures	$179,872	$7,894	$21,513	$33,654	$39,570	$35,045	$22,967	$11,363	$4,937	$1,985	$822	$122
2019 Obligation Outlay		$7,894	$12,631	$10,657	$4,737	$2,171	$789	$592				
2020 Obligation Outlay			$8,882	$14,212	$11,991	$5,329	$2,443	$888	$666			
2021 Obligation Outlay				$8,785	$14,056	$11,859	$5,271	$2,416	$878	$659		
2022 Obligation Outlay					$8,787	$14,060	$11,863	$5,272	$2,416	$879	$659	
2023 Obligation Outlay						$1,626	$2,601	$2,195	$975	$447	$163	$122

The **Table A-17** example should not be confused with the contractor plan to perform the work. The example is an estimate of how the government may pay (outlay) for the project, not how the work will be performed.

A.15 CISM BEST PRACTICE CHECKLIST

The methods and processes in this handbook are designed to be tool independent. The best practice checklist provided here will include some tool specific recommendations based on the behaviors of the latest versions of the tools at the time.

- Data
  - Uncertainty parameters should be data-driven rather than subjective
  - Distribution shapes should be selected based on data rather than subjective

- Simulation Preferences
  - Simulation settings such as type (Monte Carlo vs. Latin Hypercube), sampling, random seed, correlation on/off and others may not be saved with the model file. These and similar settings will have an impact on the simulation results. Establish a standard for these settings and verify they are correct in the model.
  - Ensure correlation is enabled
  - Agencies are encouraged to publish recommended settings for each tool that is used
- Defined Distributions
  - Try to define all uncertainty distributions in a single location within the model. This will simplify review and the application of correlation.
  - Create logical, concise names for all defined distributions to facilitate building of correlation matrices and understanding reports Consider standardizing naming conventions such as: Prefix to identify phase, UpperLowerCase to make names easier to read, suffix to identify type (Rate, Wgt, Factor, Etc). Examples: EMDeSLOC, ProdAirFrameWgt, OSLabRate.
  - In the absence of better information, use lognormal where the point estimate is the median and the upper bound is the 85% as the default. If the distribution is known to be left skewed, use betaPERT as the default. Refer to **Table 2-2** for guidance on when to use other distributions
  - Treat subjective bounds as low and high at 15/85% unless there is evidence to do otherwise. Adjust for skew
  - Truncate distributions at zero unless there is evidence to do otherwise
  - Apply uncertainty to the CIC relationship as a whole rather than its parts (T1 and slope)
  - Use lognormal with the median as the point estimate and 85% value to define 3rd party tool uncertainty results in your own model if more precise methods are unavailable
  - Distribution parameter values should be visible on the spreadsheet and linked to the distribution definition
  - Use Crystal Ball Cell preferences to ensure assumption comments are enabled
  - Ensure Crystal Ball (cell preferences) and @RISK (RiskStatic) leave cell contents alone, rather than replacing them with the mean or median
- Correlation
  - Measure correlation present in the model first then determine what should be applied
  - Create a few large correlation matrices rather than many small ones
  - Use a minimum of 0.3 as the default
  - Consider negative correlations where it makes the most sense
- CISM Model Features
  - Be able to easily switch off the schedule uncertainty, risk register and correlation in order to investigate the impacts of these key aspects of the model. Additionally, in order to move cost and uncertainty data to a FICSM model, it will be necessary to generate CISM results with schedule and risk register contributions removed.

- Forecast cells
  - Follow the same naming conventions as used to name uncertainty distributions
  - Report the simulation results (trials, mean, standard deviation, coefficient of variation, probabilities every 5%) on the same sheet, ideally just to the right of the forecast
  - Perform a convergence analysis to verify the number of trials required to develop a stable result
- Reports
  - Use multiple methods (Pareto, Tornado, Sensitivity, etc.) to identify cost, duration and uncertainty drivers. These reports are powerful for exploring the model within the estimating team, but should only be in backup to support a brief description of the drivers to decision makers.
  - Tornado charts should be prepared based on TY$
  - Sensitivity charts based on measuring the correlation between the target total output and input distributions should be run with correlation enabled and again with correlation disabled
  - S-curves should be in TY$ and report CV, 50%, mean and a high probability value. X-axis range should be fixed when comparing two or more S-curves. When comparing dissimilar costs, x-axis should be conditioned to a specific CV (0.3 by default) with the mean in the center and bounds +/- a given number of standard deviations (3 by default).
- Terminology
  - Uncertainty distributions not risk distributions
  - SMEs provide low/high not min/max
  - We are applying risk AND uncertainty (where risk is the risk register)
  - Risk register contains both risks AND opportunities
  - Probability is read off the S-curve, not confidence level
  - Probability adjustment not risk dollars
  - Cost contributors (WBS elements) vs cost drivers (input variables)

A.16 GLOSSARY

ACAT	Acquisition Category
ACE	Automated Cost Estimator
ACEIT	Automated Cost Estimating Integrated Tools
AFCAA	Air Force Cost Analysis Agency
BY	Base Year
CARD	Cost Analysis Requirements Description
CB	Crystal Ball
CDF	Cumulative Distribution Function
CER	Cost Estimating Relationship
CGF	Cost Growth Factor
CGPF	Cost Growth Potential Factor
CIC	Cost Improvement Curve (also known as a “learning curve”)
CISM	Cost Informed by Schedule Method
COCOMO	Constructive Cost Model
CR	Cost Reserve
CRUH	Cost Risk and Uncertainty Handbook
CSRUH	Cost/Schedule Risk and Uncertainty Handbook
CV	Coefficient of Variation
df	Degrees of freedom
ECO	Engineering Change Order
eSBM	Enhanced Scenario Based Method
EVM	Earned Value Management
FY	Fiscal Year
GERM	Generalized Error Regression Model
IC CAIG	Intelligence Community Cost Analysis Improvement Group
IRLS	Iteratively Reweighted Least Squares
LOLS	Ordinary Least Squares, Log Space
MDA	Missile Defense Agency
MPE	Minimum Percentage Error
MSE	Mean Squared Error
MUPE	Minimum Unbiased Percentage Error

OLS	Ordinary Least Squares
NCCA	Naval Center for Cost Analysis
PDF	Probability Density Function
PE	Point Estimate
PF	Ping Factor
PI	Prediction Interval
PS	Protect Scenario
QAIV	Quantity as an Independent Variable
SBM	Scenario-Based Method
SE	Standard Error
SEE	Standard Error of the Estimate
SEPM	Systems Engineering and Program Management
SME	Subject Matter Expert
SRA	Schedule Risk Analysis
Std Dev	Standard Deviation
SW	Software
TY	Then Year
WBS	Work Breakdown Structure
ZMPE	Zero Bias Minimum Percent Error

APPENDIX B FULLY INTEGRATED COST AND SCHEDULE METHOD

B.1 OVERVIEW

Cost, schedule and risk assessments traditionally have been performed by separate teams of professionals. In recent years, it has become more common for the cost analyst to report a risk-adjusted result as a budget recommendation, rather than a point estimate. However, it appears that cost uncertainty models routinely fail to account for two key components of an integrated estimate:

- Associate a risk-adjusted cost result with an associated schedule (that is, the risk-adjusted cost result is too often forced into point estimate schedule)
- Capture the impact of the risk register

A fully integrated cost and schedule method (FICSM) is a disciplined, systematic and repeatable process to integrate three critical pieces of information: cost uncertainty, schedule uncertainty, and the risk register. For a variation on the approach described here, see **Reference 76**.

B.2 SCHEDULE NETWORK AND DEFINITIONS

Before embarking on the process of building a FICSM model, we will review some key aspects of a schedule model. Cost analysts work with models where the total of interest is the sum of all the subordinate tasks. The units are dollars, a tangible commodity that can be spent or held back. A schedule is not a list of activities where the durations sum. It is a network of activities arranged in sequence and parallel where the commodity is time. Time cannot be saved and redistributed later. A parent task duration is not the sum of its sub-tasks, but equal to the longest of the various paths that define its completion.

A task, project, or program schedule is constructed from a number of specific individual activities, interrelated among themselves in complicated ways and is called a network. In general, each activity in the schedule will (or should) have:

- **Predecessors**: A task whose start or finish date determines the start or finish date of its successor
- **Successors**: A task whose start or finish date is driven by its predecessor task(s)

A schedule is a listing of activities, each with predecessors, a duration and successors. Other common terminology includes:

- **Baseline**: At a specific point in time, saving all the task start and finish dates to serve as the reference point for tracking progress and as the basis for calculating EVM metrics. Tools generally provide a means for saving multiple baselines as the project progresses.
- **Critical Path**: Longest slack-less path to completion
  - Microsoft Project lists every task in the schedule that has zero free float. Some of these tasks can slip without affecting the overall project duration.
  - Primavera Risk Analysis only reports those tasks with zero free float on the longest path to the end of the project (there is an option to report the same way that MS Project does)
- **Free Slack (float)**: Where a task has more than one immediate predecessor, free slack (also called free float) is the amount of time an activity can slip without impacting its successor start
- **Hammock:** A special type of task within a schedule that is dependent on external dates for both its start and finish dates and, ultimately, its duration

- **Hard Constraint**: A constraint that prevents the calculated schedule from being pushed beyond a specific date for the task. Examples of hard constraints include: Must-Finish-On, Must-Start-On, Start-No-Later-Than, and Finish-No-Later-Than. These should be avoided and for the simulation, hard constraints should be removed.
- **Integrated Master Plan (IMP)**: An event-based, top level plan consisting of a hierarchy of program events, with each event being supported by specific accomplishments, and each accomplishment associated with specific acceptance criteria to be satisfied for its completion
- **Integrated Master Schedule (IMS):** Flows directly from the IMP, creating the task and calendar-based schedule that illustrates the interrelationships among events, accomplishments, criteria, and tasks
- **Lead/Lag:** A lead will cause the successor to start or finish earlier and lag causes the successor to start or finish later than it otherwise could. It is a discouraged practice because it tends to be misused. Where a lag is required, the better practice is to create a task called "margin" to replicate the intent of the lag.
- **Merge Bias**: This occurs when two or more uncertain, parallel paths of activities merge at one milestone. The effect is that the expected completion date for the milestone, in this situation, will be later than the expected completion of the individual paths leading up to it. The number of merging parallel paths and the level of overlap between, and degree of correlation between them (lower correlation between uncertain durations produces greater merge bias) produce an increasing merge bias impact to the schedule.
- **Milestone**: An event, phase gate or key accomplishment with zero day duration
- **Relationships**: These identify how tasks or activities are linked together: Finish to Start is by far the most common and the best practice. Start to Start is also commonly used, but it tends to make it harder to understand the logic. Finish to Finish is known to be useful, but is discouraged because predecessor logic can become hidden.
- **Summary**: A higher level roll-up of related detailed tasks
- **Total Slack (float)**: Amount of time an activity can slip without impacting the project end date
- **Soft Constraint**: A constraint that does not prevent the schedule from being changed based upon its dependencies. Examples of soft constraints include: As-Soon-As-Possible, Start-No-Earler-Than, and Finish-No-Earlier-Than. As-Soon-As-Possible (ASAP) is the default in most scheduling applications.
- **Planning Package**: A higher level representation of the WBS activities for cost account visibility
- **Task**: An activity with duration that is commonly used to represent work items
- **Time Independent (TI)**[17]: elements whose costs are not impacted by the duration of the task they are loaded against (e.g., material cost)
- **Time Dependent (TD)**: elements whose costs are a function of the duration of the task which they are loaded against (e.g., staff cost)
- **Work Package**: Lowest level of the WBS where both the cost and the duration can be reliably estimated

[17] TI and TD are also known as "time-fixed" and "time-variable" costs respectively.

B.3 OVERVIEW OF THE FICSM MODELING PROCESS

FICSM begins with a model of the schedule logic. This serves as the backbone for the integrated analysis. Cost, risks and uncertainty are mapped into the schedule network logic to assess impacts. In general, using the project/program IMS may be too challenging. An IMS may be too big and complex. Additionally, there may be network logic or constraints in an IMS that are unsuitable for the uncertainty simulation to run properly. For example "Must finish on" constraints may be used to generate a suitable IMS, but must be removed to perform a simulation. The general "unhealthiness" of the IMS may make it challenging to use it for a simulation model.

A FICSM appropriate schedule must be created from available data (including the program IMS) and this is typically referred to as an "analysis schedule". Key elements of an analysis schedule provide the following:

- Captures the major work-flows of the project IMS
- Provides insight into major cross-dependencies within or across management responsibility boundaries
- Creates a solid framework to capture cost / schedule uncertainties and discrete risk events
- Structured around management/ budget responsibility
- Aligns the effort and schedule to the cost/budget
- Identifies key tasks that support major deliverables/ tracking items
- Detailed IMS step-by-step work items and task flows are combined while maintaining critical path logic
- Has traceability and transparency to the more detailed IMS and passes all schedule modeling best practice and health checks

Figure B-1 is an example of a simple analysis schedule where uncertainty is applied to the task durations. While triangular distributions are shown, the analyst may use any type of distribution suitable to model the duration uncertainty.

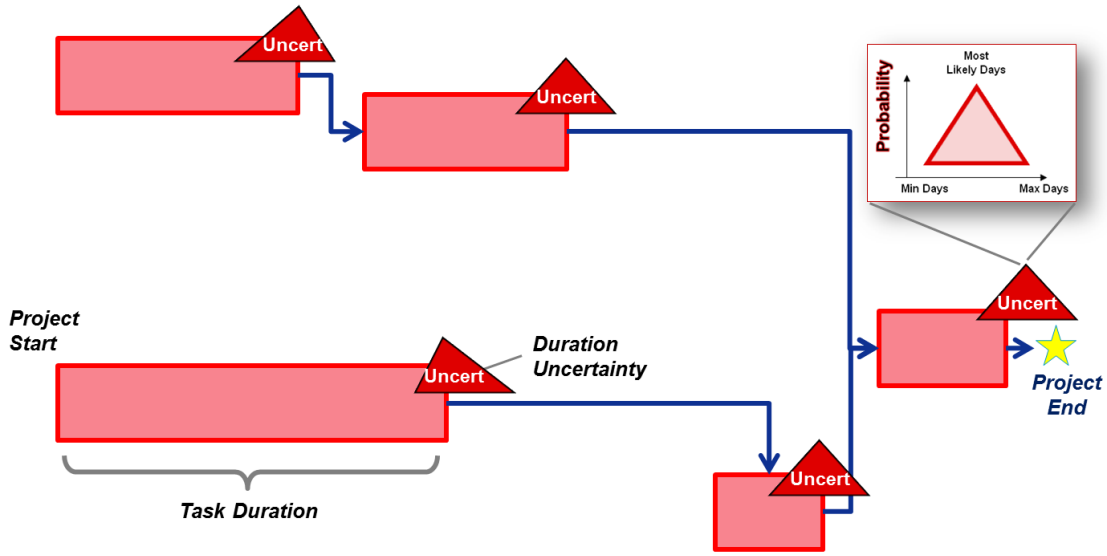


Figure B-1 Simple Schedule Network With Uncertain Task Durations

The next step is to assign time independent (TI) and time dependent (TD) costs to each of the activities. **Figure B-2** illustrates mapping of uncertain TI and TD costs to each activity. The final activity only has a TI cost associated with it. The previous activities have both TI and TD cost components.

It is a common practice to combine similar activities together to assign the TD cost. This is known as a summary hammock task. For instance, a schedule network may contain several specific activities that together are related to a single WBS element cost estimate. The hammock task is tied to the beginning of the first activity in the series and the end of the last. There could be a large complicated network of sequential and parallel activities in between. The TD cost needs to be expressed in dollars per unit time so, as the activity duration changes, so does the total cost for the activity. There are several considerations to keep in mind when building uncertainty into TD costs:

- Hammock tasks need to be added to the schedule, without changing the behavior of the schedule, in order to assign the TD cost
- The basis for the TD cost uncertainty, transferred to the FICSM model, may have already captured some degree of duration uncertainty

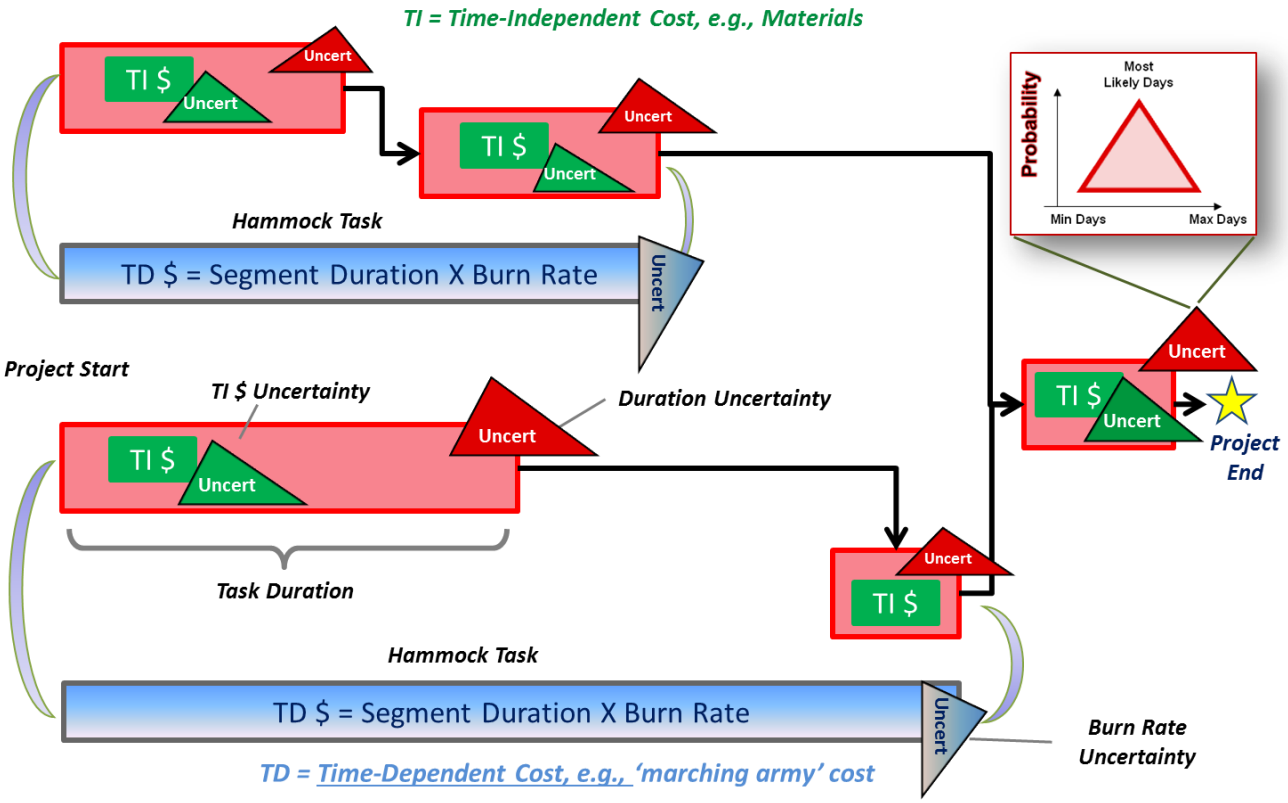


Figure B-2 Simple Schedule Network With Mapped TI and TD Costs

Finally, it is necessary to map risk register events into the network. These events will have a cost and/or schedule impact on one or more tasks if the risk is realized. **Figure B-3** illustrates how a risk register item is embedded into the network. Note that, as constructed in **Figure B-3** (there are multiple ways to do so):

- The event will have no impact on the schedule or cost if it does not occur
- The probability of occurrence defines how often in the simulation this event occurs
- An additional, uncertain TI cost is realized if this event occurs
- Since it is captured within the TD hammock, when the event occurs, the duration that it adds to this sequence of events will result in higher total TD cost
- This particular implementation does not affect the already-defined TD burn rate, only the duration over which the TD rate is applied

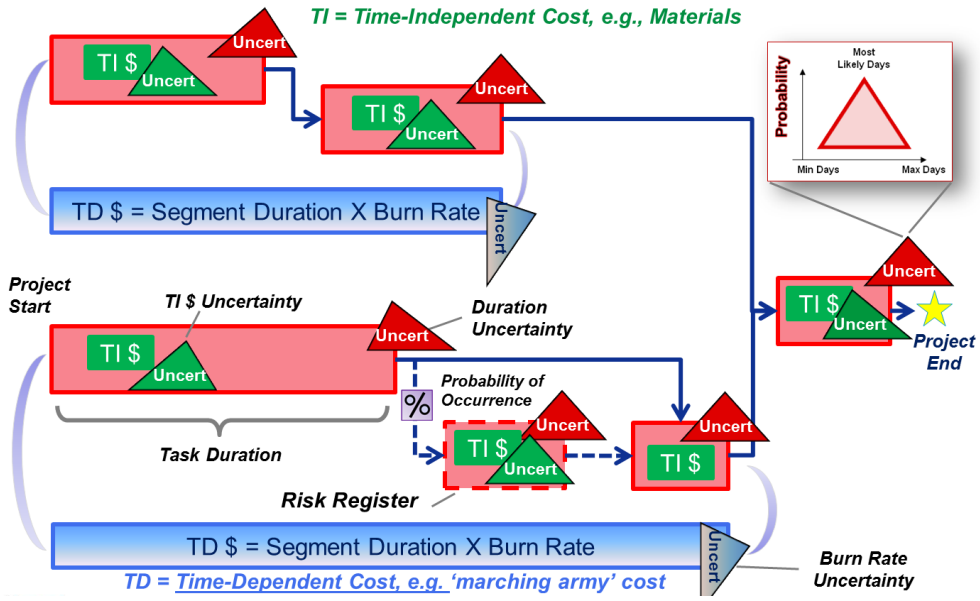


Figure B-3 Simple Schedule Network With A Risk Register Event Included

It is generally easier to map cost model results to a schedule model rather than replicating schedule network logic in a cost model. However, schedule models (tools) are generally populated with throughputs, that is they don't allow equations to estimate the cost of one task based on the cost of another task or the technical characteristics of the task. Consequently the CISM model is still necessary as it can be used as the source for the cost and cost uncertainties that must be mapped to the schedule. Mapping cost estimate results to a schedule model is simplified by:

- Unifying cost (often product-based) and schedule (often task-based) work breakdown structures
- Specifying Time Dependent and Time Independent costs and their uncertainty separately
- Defining how the TI or TD cost is phased over the task duration

Uncertainty should be applied in a consistent manner across the entire model and then correlation must be considered.

B.4 BUILDING A FICSM MODEL

B.4.1 A Consolidated View of the Process

The following steps provide a high-level process for incorporating cost uncertainty, schedule uncertainty and the risk register into a fully integrated model. **Figure B-4** illustrates a tool independent overview of the FICSM process. There are many tools available to perform this modeling and analysis. Unlike the tools to perform cost simulation, they are not as mature and there are many subtle but highly significant differences in how these tools implement the process. We selected one of them to demonstrate the process. While it may be evident from the images used which tool was selected, this handbook does not directly identify it and leaves it to the analyst to make a selection.

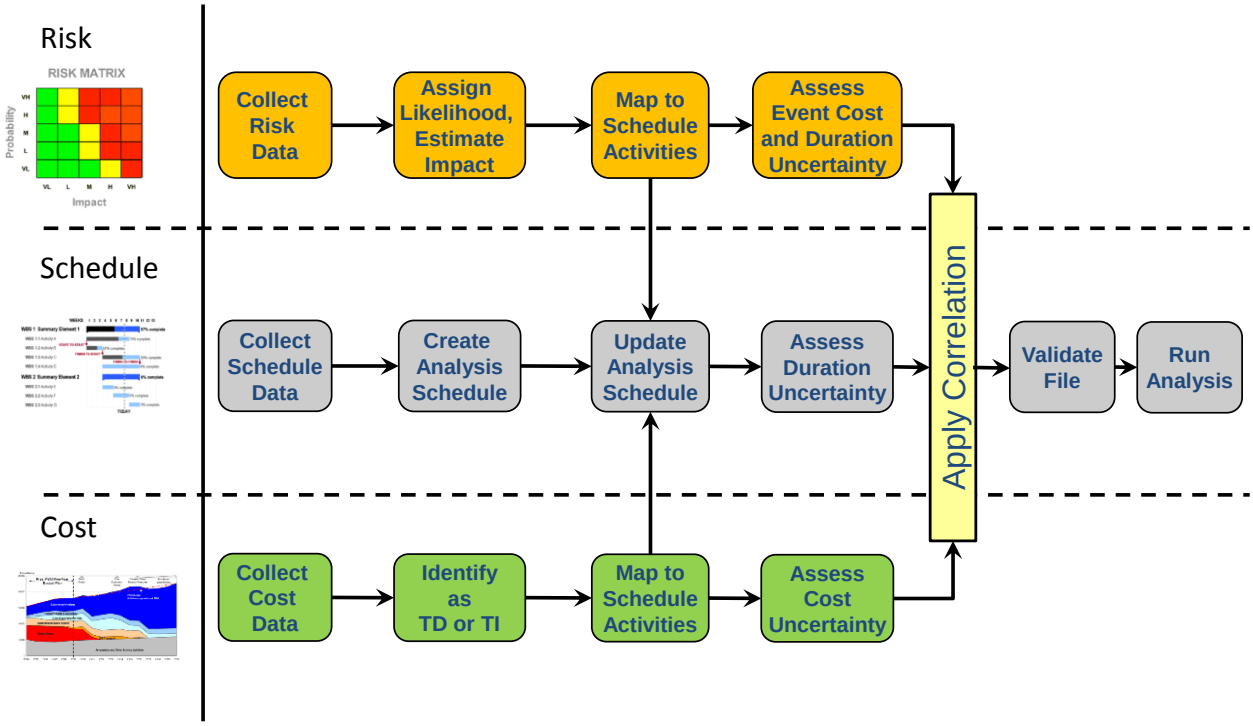


Figure B-4 The FICSM Process

B.4.2 Starting the Process: Collect Schedule Data

Whether building a schedule from scratch or condensing an IMS into a useable analysis schedule, it is important to first gather key pieces of information that will aid in the overall understanding of the project. Many times when building a new schedule the first instinct is to open Microsoft Project, or other scheduling tool, and start entering data. This is rarely a good idea, and instead leads to non-integrated pieces of data that don't tell the whole cohesive story of the project from beginning to end. **Table B-1** identifies some key pieces of information for the example model.

Table B-1 Typical High Level Schedule Data

Item	Point Estimate	Range
WBS Structure	MIL-STD-881C	
EMD		
Possible Project Start Date	01 Oct 2013	
Project Start Date Delay	2 months	0 - 18
Modeled Project Start Date	01 Dec 2013	
EMD Duration	60 months	54 - 72
EMD Prototypes	5	
EMD LOE tasks	Design, PM, & SE	
EMD End Date	01 Dec 2018	
Production		
Production Start Date – is	EMD End date	
Production Qty	600 Units	
First Year Production Qty	Driven by EMD End Date	
Steady State Production Qty	150 per year	105 - 165
Final Year Production Qty	Steady State Rate - First Year Production	
Production LOE tasks	PM & SE	
Production Duration	Total Qty/Steady State Rate * 12 Months	
Production End Date	EMD End date + Production Duration	

B.4.3 Creating an Analysis Schedule Suitable for the FICSM Process

Ideally, FICSM will be based on the IMS for the project. Many analysts choosing to build a FICSM model will quickly find out that neither the IMP nor IMS has been developed. The schedule for the project may be nothing more than a few bullets on a single slide. On the other end of the spectrum will be projects in execution that have fully developed IMP and an ever changing IMS with a huge number of interrelated activities rendering it problematic for use in the FICSM process. In both situations, the FICSM analyst may have no choice but to build a schedule that adequately captures the anticipated workflow and activity interrelationships. This step begins by defining the key elements of the overall plan as discussed in the previous section. Further discussion of the IMP and IMS can be found in **Appendix A.1.10**. Also, **Reference 93** documents lessons learned from building an analysis schedule in the NASA environment.

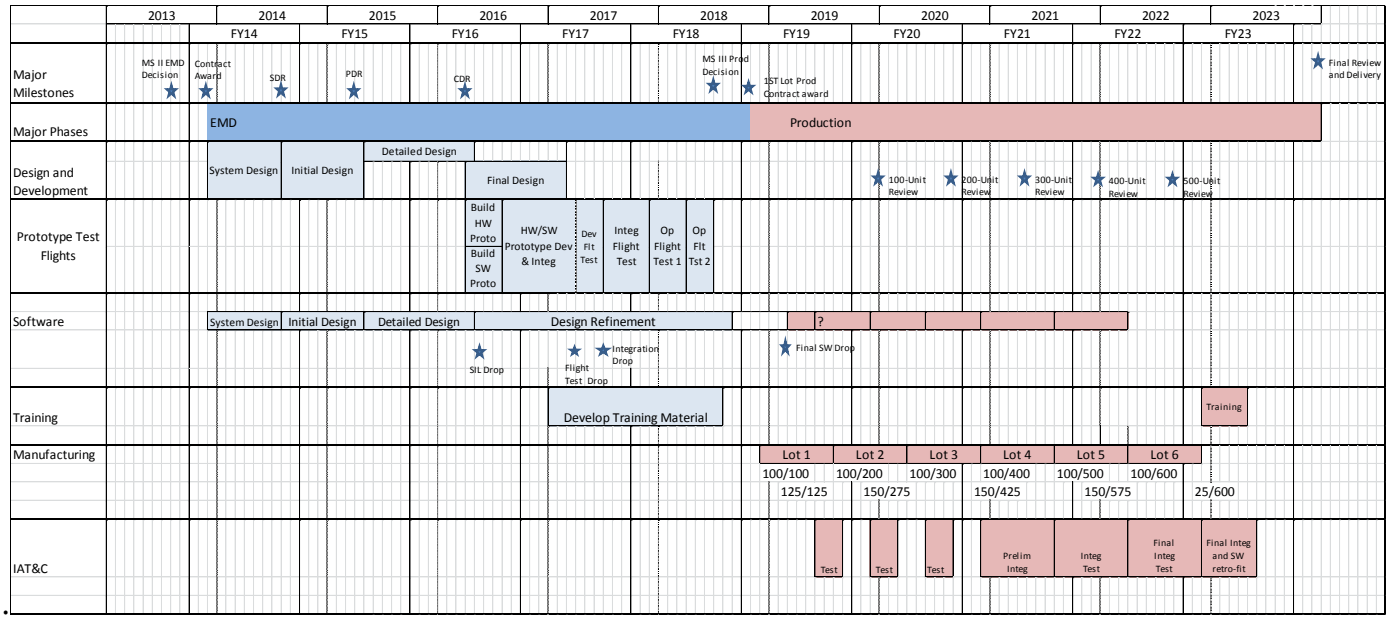


Figure B-5 Example Model Integrated Master Plan

After a high-level understanding of the project and a project plan from start to finish are achieved (should look something like **Figure B-5**), there are a few different approaches to take in building an analysis schedule. If the WBS elements are self-contained, meaning they do not rely on activities from other WBS elements to be predecessors to activities within their process, then building the schedule by WBS may be appropriate. If the activities within the WBS elements are interconnected with activities from others, it may be better to build the analysis schedule organized around activities. However, when building an analysis schedule from a cost model, it is easiest to use the WBS as the basis for building the schedule.

Having identified the key activities for the schedule, the next step is to develop the logical relationships. Logical relationships between activities identify whether they are to be accomplished in *sequence* or in *parallel*. A *sequence* of activities is a serial path along which one activity is completed after another. Activities can also be accomplished in *parallel*. A logic relationship linking a predecessor and successor activity can take one of three forms: finish-to-start (F-S), start-to-start (S-S), and finish-to-finish (F-F). Additionally, there can be leads or lags built into these relationships.

F-S relationships are preferred because it is intuitive, it is common for work to be accomplished serially, it is easier to trace and they clearly indicate to management the work flow. S-S and F-F relationships, in contrast, imply parallel or concurrent work. S–S and F–F relationships represent a valid technique for modeling overlapping activities and may be more predominant in schedules that have not yet evolved to a detailed level.

In summary, an analysis schedule:

- is not an IMS or a detailed schedule
- is not a detailed step by step description of the detailed activity flow
- has traceability and transparency to more detailed IMS and other schedule products
- has the objective to enable an integrated cost and schedule modeling and simulation environment that is structured around management/ budget responsibility
- displays major work-flows of a project
- identifies work required to support major deliverable / tracking items
- allows linkage of budgeted work effort to schedule scope
- facilitates alignment with EVM generated data
- provides insight into major cross dependencies within or across management responsibility boundaries
- creates a framework for incorporating cost / schedule uncertainties and risk register events

Figure B-6 illustrates how the example CISM model compares to an analysis schedule developed in MS Project to support a FICSM analysis. Starting from the cost WBS, the analysis schedule differs by:

- adding the ability to make the start date uncertain
- including milestone dates
- consolidating level of effort tasks
- rearranging to make more sense in the schedule linkages
- requiring a lot more detail as we will see in the following examples with arrows

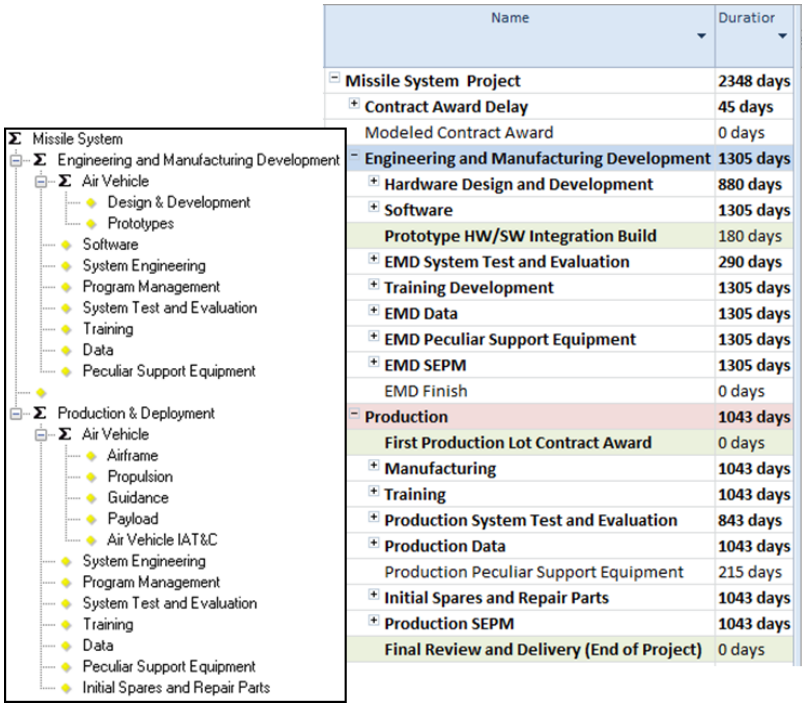


Figure B-6 Compare the Example Cost WBS to the Analysis Schedule

B.4.4 Estimating Activity Durations

Estimating the duration for the PE_S will set the stage for the duration uncertainties to be entered later. There are several approaches for estimating the activity duration, and each should be based upon data analysis whenever possible:

- **Mean Duration**: A common approach is to examine historical tasks and use the average as the point estimate
- **Three Point Estimate**: Another common approach is to identify the minimum, most-likely and maximum duration for the activity and use the most-likely as the point estimate. When obtained subjectively, the adjustment for skew as discussed in **Section 2.5.4** should be considered.

Other terms are used like "the most realistic" duration, which presents problems when determining how to apply uncertainty. Most realistic is not the same as the mode or the mean. It is some other interpretation adding another level of subjectivity.

The duration estimate should take into account the nature of the work and the resources needed to complete the activity. The AACEI Recommended Practice 32-04: Determining Activity Durations suggest the following guidelines for estimating activity durations:

- Duration = Quantity of work / work units per time period. Quantity of work is a function of the definition or scope of the activity. Work units per time period are commonly referred to as the production rate.
- Duration is typically specified in rounded continuous working time periods in the unit common to the activity, e.g. hours, days, weeks, months, years, etc. Generally, durations are rounded up to the next whole unit, even when the estimated duration is less than ½ a work unit (e.g., if estimated duration calculates to 60.25 work days, it is rounded up to 61 work days).
- There are some activities which need to be measured in calendar time periods rather than working time periods
- Durations should be estimated using an analytical and systematic method
- The basis of activity duration estimates should be identified as a part of the overall schedule documentation

B.4.5 Capturing the Impact of the "Standing Army" Using Hammock Tasks

Level of effort (LOE) tasks incur a steady expenditure rate that is expected to be in operation over a period time. If the task ends early, it will cost less. If the task runs longer, it will cost more. Project management and systems engineering were modeled in this manner given that it is reasonable to expect the need for these services while the project is in execution. This gives rise to the concept of "standing armies" which are resources that cannot be terminated due to delays in other areas of the project. **Figure B-7** illustrates how these tasks are implemented in the analysis schedule. They each have a subordinate milestone start and finish that are linked to the associated activities that define the start and the end of the EMD phase. A similar construct is created in production. By doing this, the cost/day for these resources can be assigned to the hammock task and produce a total task cost that is sensitive to the duration of all those tasks that will define the EMD critical path.

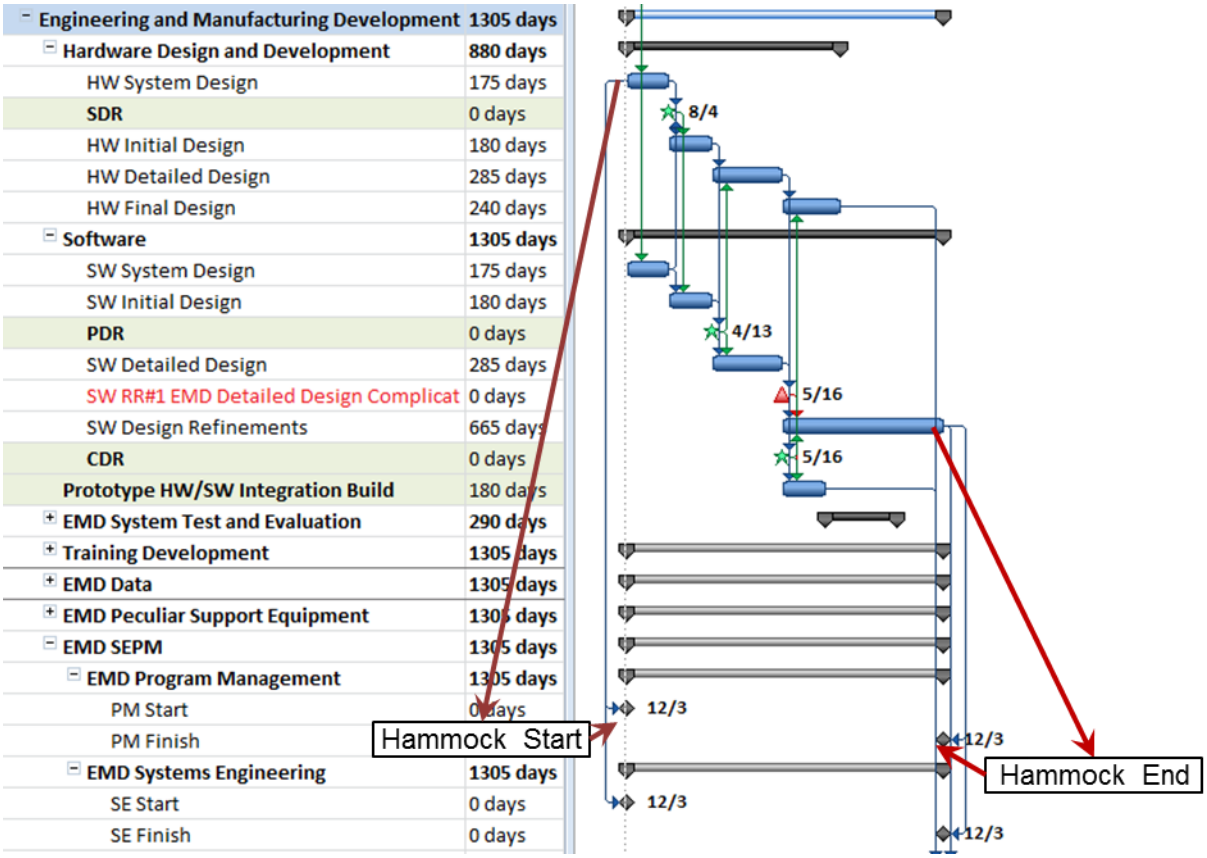


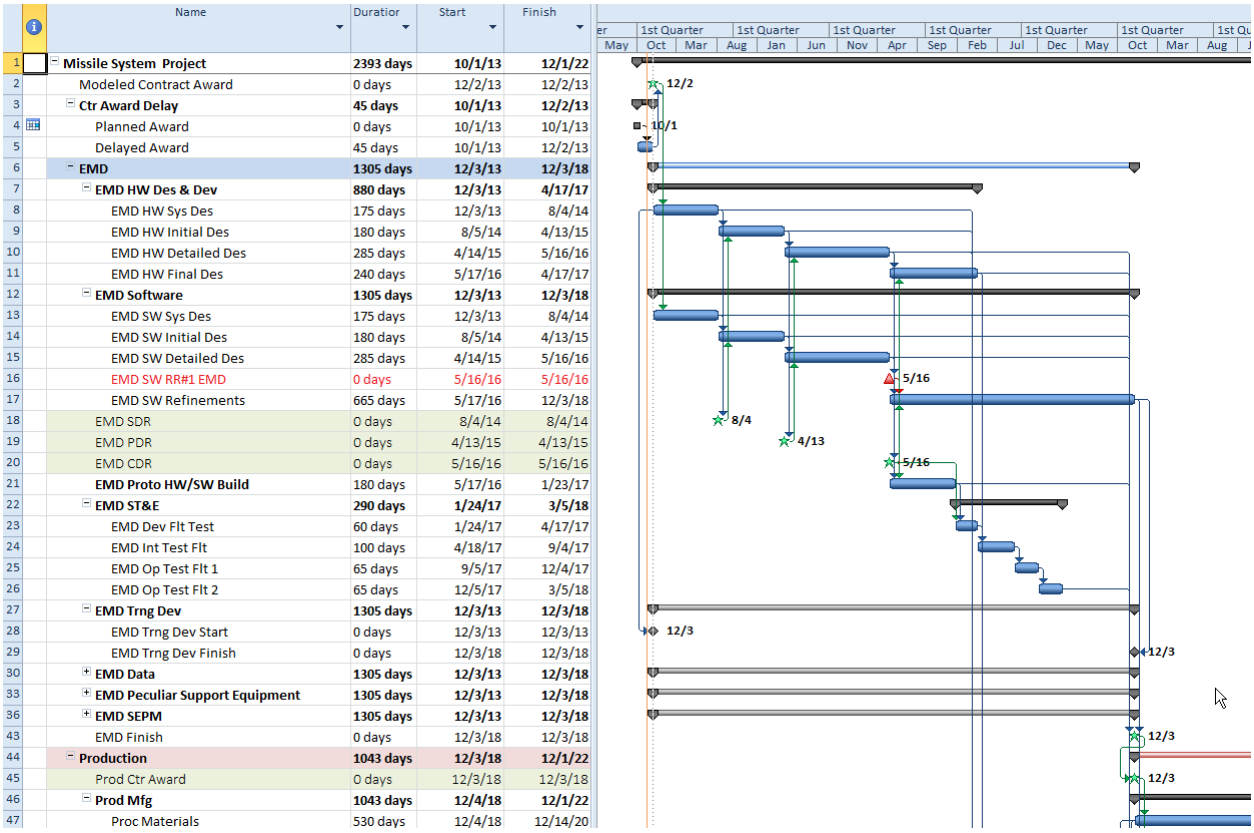
Figure B-7 Implementing Hammock Tasks in the Example Model

B.4.6 Build the Schedule and Perform a Health Check

After identifying the logic relationships and the durations, it is time to build the schedule in the scheduling software. There will be choices to make when organizing the tasks. Some analysts like all the milestones to be prominent at the top of the schedule so they are easy to find. However, this makes it hard to visualize the linkages. Embedding the milestones directly after its last predecessor makes the linkage easier to read (see PDR and CDR in **Figure B-7**) but this could create confusion when looking at the list. A third option is to group the milestones as shown in **Figure B-8**. **Figure B-8** also illustrates the kind of tasks and the linkages that need to be built into the model.

Once the schedule is built, it should be checked against the Defense Contract Management Agency (DCMA) 14-point check. A number of statistics must be calculated before starting the check. These statistics are:

- **Total Tasks**: All the tasks except tasks that represent summary, subproject, level of efforts, zero duration, or milestones
- **Complete Tasks**: Tasks among the "Total Tasks" that have 100% completion and with an actual finish date before the status date
- **Incomplete Tasks**: Tasks among the "Total Tasks" that do not have 100% completion and with an actual finish date before the status date
- **Baseline Count**: Tasks among the "Total Tasks" that should have been completed before the status date in the original baseline schedule



	Name	Duration	Start	Finish
1	**Missile System Project**	**2393 days**	**10/1/13**	**12/1/22**
2	Modeled Contract Award	0 days	12/2/13	12/2/13
3	**Ctr Award Delay**	**45 days**	**10/1/13**	**12/2/13**
4	Planned Award	0 days	10/1/13	10/1/13
5	Delayed Award	45 days	10/1/13	12/2/13
6	**EMD**	**1305 days**	**12/3/13**	**12/3/18**
7	**EMD HW Des & Dev**	**880 days**	**12/3/13**	**4/17/17**
8	EMD HW Sys Des	175 days	12/3/13	8/4/14
9	EMD HW Initial Des	180 days	8/5/14	4/13/15
10	EMD HW Detailed Des	285 days	4/14/15	5/16/16
11	EMD HW Final Des	240 days	5/17/16	4/17/17
12	**EMD Software**	**1305 days**	**12/3/13**	**12/3/18**
13	EMD SW Sys Des	175 days	12/3/13	8/4/14
14	EMD SW Initial Des	180 days	8/5/14	4/13/15
15	EMD SW Detailed Des	285 days	4/14/15	5/16/16
16	EMD SW RR#1 EMD	0 days	5/16/16	5/16/16
17	EMD SW Refinements	665 days	5/17/16	12/3/18
18	EMD SDR	0 days	8/4/14	8/4/14
19	EMD PDR	0 days	4/13/15	4/13/15
20	EMD CDR	0 days	5/16/16	5/16/16
21	**EMD Proto HW/SW Build**	180 days	5/17/16	1/23/17
22	**EMD ST&E**	**290 days**	**1/24/17**	**3/5/18**
23	EMD Dev Flt Test	60 days	1/24/17	4/17/17
24	EMD Int Test Flt	100 days	4/18/17	9/4/17
25	EMD Op Test Flt 1	65 days	9/5/17	12/4/17
26	EMD Op Test Flt 2	65 days	12/5/17	3/5/18
27	**EMD Trng Dev**	**1305 days**	**12/3/13**	**12/3/18**
28	EMD Trng Dev Start	0 days	12/3/13	12/3/13
29	EMD Trng Dev Finish	0 days	12/3/18	12/3/18
30	**EMD Data**	**1305 days**	**12/3/13**	**12/3/18**
33	**EMD Peculiar Support Equipment**	**1305 days**	**12/3/13**	**12/3/18**
36	**EMD SEPM**	**1305 days**	**12/3/13**	**12/3/18**
43	EMD Finish	0 days	12/3/18	12/3/18
44	**Production**	**1043 days**	**12/3/18**	**12/1/22**
45	Prod Ctr Award	0 days	12/3/18	12/3/18
46	**Prod Mfg**	**1043 days**	**12/4/18**	**12/1/22**
47	Proc Materials	530 days	12/4/18	12/14/20

Figure B-8 Example Analysis Schedule

After identifying and calculating the previous statistics the checks can be performed. There are third party tools that specialize in performing these types of checks. A brief summary of the 14-point checks is:

- **Logic**: less than 5% incomplete tasks with missing logic links
- **Leads**: no incomplete tasks with a lead (negative lag) in predecessor relationships
- **Lags**: less than 5% incomplete tasks with lags in predecessor logic relationships
- **Relationship Types**: incomplete tasks containing each type of logic link. At least 90% should be Finish-Start. Start-Finish should be as close to zero as possible.
- **Hard Constraints**: less than 5% of incomplete tasks contain hard constraints in use (such as Must-Finish-On, Must-Start-On, Start-No-Later-Than, and Finish-No-Later-Than)
- **High Float**: less than 5% of incomplete tasks with total float greater than 44 working days (2 months)
- **Negative Float**: no incomplete task with total float less than 0 working days
- **High Duration**: less than 5% of incomplete tasks with a baseline duration greater than 44 working days (2 months)
- **Invalid Dates**: no incomplete tasks that have a forecast start/finish date prior to the IMS status date, or have an actual start/finish date beyond the IMS status date
- **Resources**: all tasks with durations greater than zero have dollars or hours assigned
- **Missed Tasks**: Less than 5% of activities that had a scheduled finish date before the status date but did not finish or are forecast to finish after the baseline finish date. This check is important because it shows how the updated schedule is in compliance with the baseline schedule.

- **Critical Path Test**: Tests the integrity of the overall network logic and, in particular, the critical path. It is one of the two "trip wires" that are required by the OSD (Office of Secretary of Defense). The "critical path test" is performed by adding an intentional delay (600 working days) to the remaining duration of a critical task and then checking to see if the project completion date is delayed by a proportional duration (600 working days). By adding such a delay, any missing predecessor or successor will lead to a mismatch between the project overall delay and the intentional one. The "critical path test" will be passed if there is a match between the project completion delay and the intentional added duration.
- **Critical Path Length Index (CPLI)**: Used to assess if the project finish date will be real or not. It is one of the two "trip wires" that are required by the OSD. The CPLI is calculated by adding the length of the critical path to the total float of the latest activity and dividing the sum by the length of the critical path. For the CPLI to be acceptable, its value should be more than 100%
- **Baseline Execution Index (BEI)**: An IMS-based metric that calculates the efficiency with which tasks have been accomplished when measured against the baseline tasks. If the contractor completes more tasks than planned, then the BEI will be higher than 1.00 reflecting a higher task throughput than planned. A BEI less than 0.95 should be considered a flag and requires additional investigation. This test computes the ratio of all of the tasks that have been completed versus the tasks that should have been completed in the period between the Baseline Schedule and the current schedule.

There are other sources for schedule health check guidance. For instance Appendix III of the GAO Schedule Assessment Guide (**Reference 82**) provides five pages of standard quantitative measurements for assessing schedule health. The NASA Schedule Management Handbook (**Reference 69**) states that schedule credibility is determined by monitoring key indicators within the IMS that reflect both good and poor characteristics of schedule structure and maintenance. Examples of key indicators within the logic network that must be monitored include the following: number of missing predecessors and successors, invalid task constraints, omission of task status, improper status on future tasks, logic ties to/from summary tasks, inaccurate logic ties, and improperly reflecting tasks as milestones. These indicators are based on standard rules of logic network development utilized in critical path method (CPM) scheduling techniques. has 5 pages of health checks

B.5 LOADING THE SCHEDULE WITH COST

B.5.1 Strategy to Load Cost Data

There are several decisions to make regarding the approach to cost loading the schedule model. Ideally, the CISM and FICSM point estimate cost and durations match. Also, the total cost uncertainty for both the CISM and FICSM should approximately match before schedule uncertainty and risk register influences are added to the FICSM model. If a CISM model is used as the source for the data, the documentation and basis for much of the FICSM model is already available. To construct the example FICSM model, we used the following process:

- **Basis for the FICSM point estimate:**
  - **Cost**: We chose to use the point estimate from the CISM model that is based upon the PE_S
  - **Schedule:** To be consistent with the point estimate cost, the PE_S durations derived from the CISM model are used
- **Mapping cost to a FICSM model:**
  - **Mapping WBS to activities**: The approach for performing this mapping (often one WBS element to many activities) is determined on a WBS element by WBS element basis

- **Mapping CISM cost to TI and TD:** We chose to use the CISM model as guidance for assigning these costs

- **Mapping CISM cost uncertainty to a FICSM Model**:
  - Run the CISM with the risk register disabled
  - Review selected histogram results from the CISM model to determine distribution shape. Develop consistent approach for determining the shape parameters. Enter the parameters as a percent of the point estimate to preserve CV.
  - Match the CISM model correlation for cost uncertainties in order to match (or get close) at the total level before schedule uncertainty and the risk register is applied
- **Assign duration uncertainties:** Similar process to cost
- **Apply the risk register**
- **Apply correlation**
- **Perform a health check:** Before running the simulation, check for issues with the model. This is a more important and more complex process than a similar check for a CISM model.
- **Run the model and develop reports**

B.5.2 Collect Cost Data

A properly developed and documented CISM model is a good source of the cost and cost uncertainty information required for populating the FICSM model. However, it is important that the CISM model is configured to permit schedule uncertainty and risk register elements to be disabled. Use this capability to generate results from CISM model for use as inputs to the FICSM model.

Table B-2 compares key results from the CISM example model. These are not the allocated risk dollar results that would be used to propose a budget. Rather, these are the statistical results direct from the simulation for cost uncertainty only. The TY statistical results are shown for reference, but would **not** be used to populate the FICSM model.

Table B-2 Example CISM Model Estimate Risk and Uncertainty Statistical Results

	Point Estimate (xx%) =	Program Estimate Results From Complete CISM Model					Program Estimate Results From No Schedule Uncertainty or Risk Register				
WBS	Probability of PE	**Mean**	**Std Dev**	**CV**	**50%**	**85%**	**Mean**	**Std Dev**	**CV**	**50%**	**85%**
Missile System TY$	$ 276,893 (15%)	$365,333	$78,884	0.216	$353,524	$439,934	$338,615	$62,843	0.186	$330,567	$400,359
EMD TY$	$ 88,102 (16%)	$140,417	$52,654	0.375	$130,228	$189,083	$119,952	$36,159	0.301	$113,205	$153,228
Production & Deployment TY$	$ 188,792 (19%)	$224,916	$37,027	0.165	$220,985	$261,913	$218,663	$34,869	0.159	$214,633	$254,116
*** DETAILED ESTIMATE**	**PE**	**Mean**	**Std Dev**	**CV**	**50%**	**85%**	**Mean**	**Std Dev**	**CV**	**50%**	**85%**
Missile System	$ 246,836 (15%)	$325,183	$70,260	0.216	$314,740	$391,724	$302,070	$56,779	0.188	$294,754	$357,505
Engineering and Manufacturing Dev	$ 83,539 (17%)	$130,683	$47,692	0.365	$121,567	$174,317	$112,971	$34,025	0.301	$106,567	$144,172
Air Vehicle	$ 14,944 (29%)	$28,615	$21,080	0.737	$22,954	$44,749	$23,886	$14,921	0.625	$20,138	$36,195
Design & Development	$ 12,000 (33%)	$24,380	$20,409	0.837	$18,752	$39,874	$19,795	$14,287	0.722	$16,053	$31,387
Prototypes	$ 2,944 (23%)	$4,235	$1,516	0.358	$3,975	$5,805	$4,091	$1,452	0.355	$3,837	$5,597
Software	$ 31,500 (41%)	$44,497	$23,620	0.531	$39,641	$65,970	$40,302	$22,164	0.550	$35,269	$60,480
System Engineering	$ 17,500 (17%)	$27,113	$8,293	0.306	$25,693	$35,496	$22,659	$5,078	0.224	$21,861	$28,639
Program Management	$ 15,000 (26%)	$20,528	$5,395	0.263	$19,672	$26,158	$17,155	$2,813	0.164	$16,776	$20,436
System Test and Evaluation	$ 1,766 (9%)	$3,654	$1,683	0.461	$3,289	$5,267	$3,529	$1,614	0.457	$3,194	$5,084
Training	$ 897 (20%)	$2,038	$1,551	0.761	$1,618	$3,215	$1,701	$1,104	0.649	$1,413	$2,581
Data	$ 1,196 (20%)	$2,714	$2,054	0.757	$2,156	$4,278	$2,266	$1,469	0.648	$1,893	$3,456
Peculiar Support Equipment	$ 736 (9%)	$1,524	$703	0.461	$1,379	$2,209	$1,472	$675	0.459	$1,331	$2,135
											$0
Production & Deployment	$ 163,297 (19%)	$194,499	$31,984	0.164	$191,119	$226,478	$189,100	$30,121	0.159	$185,619	$219,735
Air Vehicle	$ 104,826 (23%)	$127,291	$25,142	0.198	$124,490	$152,834	$122,955	$23,622	0.192	$120,399	$146,789
Airframe	$ 21,651 (35%)	$25,477	$7,692	0.302	$24,570	$33,903	$25,477	$7,692	0.302	$24,570	$33,903
Propulsion	$ 21,849 (32%)	$24,056	$4,541	0.189	$23,617	$28,319	$24,056	$4,541	0.189	$23,617	$28,319
Guidance	$ 27,810 (27%)	$37,322	$10,274	0.275	$35,949	$48,294	$32,986	$7,143	0.217	$32,058	$41,295
Payload	$ 17,246 (31%)	$19,607	$4,195	0.214	$19,204	$23,913	$19,607	$4,195	0.214	$19,204	$23,913
Air Vehicle IAT&C	$ 16,269 (41%)	$20,830	$11,501	0.552	$18,270	$31,108	$20,830	$11,501	0.552	$18,270	$31,108
System Engineering	$ 12,000 (40%)	$12,442	$1,363	0.110	$12,347	$13,990	$12,442	$1,363	0.110	$12,347	$13,990
Program Management	$ 10,000 (40%)	$10,369	$1,136	0.110	$10,289	$11,658	$10,369	$1,136	0.110	$10,289	$11,658
System Test and Evaluation	$ 5,000 (33%)	$5,369	$690	0.128	$5,296	$6,162	$5,369	$690	0.128	$5,296	$6,162
Training	$ 4,193 (10%)	$6,587	$1,972	0.299	$6,295	$8,602	$6,364	$1,885	0.296	$6,061	$8,323
Data	$ 4,193 (10%)	$6,036	$1,475	0.244	$5,855	$7,529	$5,830	$1,396	0.239	$5,653	$7,221
Peculiar Support Equipment	$ 7,634 (50%)	$7,634	$2,039	0.267	$7,635	$9,747	$7,634	$2,039	0.267	$7,635	$9,747
Initial Spares and Repair Parts	$ 15,451 (37%)	$18,771	$6,090	0.324	$17,821	$24,795	$18,137	$5,839	0.322	$17,257	$23,930

B.5.3 Mapping cost to a FICSM model

There are two steps in mapping the point estimate from a CISM model to FICSM. Before extracting data from the CISM model, make sure that the point estimate is based upon the PE_S. The costs from CISM need to be mapped to the appropriate activities and then divided into TI and TD values. **Table B-3** illustrates both steps in a single template. There are many ways to arrive at the factors - the only guidance offered is to establish a consistent approach that is defendable, repeatable and well documented.

Table B-3 Mapping CISM Costs to a FICSM Model

CISM WBS Element	Point Estimate
Design & Development	$12,000
Software	$31,500

FICSM Activity Name	CISM Result	Factor	FICSM PE	TI%	TD%	TI$	TD$
EMD HW Des & Dev	$12,000		**$12,000**				
HW Sys Des		10%	$1,200	0%	100%	$0	$1,200
HW Initial Des		20%	$2,400	0%	100%	$0	$2,400
HW Detailed Des		50%	$6,000	0%	100%	$0	$6,000
HW Final Des		20%	$2,400	0%	100%	$0	$2,400
EMD Software	$31,500		**$31,500**		100%		
SW Sys Des		10%	$3,150	100%	0%	$3,150	$0
SW Initial Des		20%	$6,300	100%	0%	$6,300	$0
SW Detailed Des		40%	$12,600	100%	0%	$12,600	$0
SW RR#1 EMD Detailed Des Complications							$0
SW Des Refinements		30%	$9,450	100%	0%	$9,450	$0

CISM Data	Point Estimate
Air Vehicle	$105,839
Airframe	$21,860
Propulsion	$22,060
Guidance	$28,079
Payload	$17,413
Air Vehicle IATC	$16,426

FICSM Activity Name	CISM Result	Factor	FICSM PE	TI%	TD%	TI$	TD$
Prod Mfg	$105,838		$105,838				
Air Vehicle Minus IATC	$89,412						
Procure Materials		40%	$35,765	100%	0%	$35,765	$0
Air Vehicle Manufacturing		60%	$53,647	90%	10%	$48,282	$5,365
AV RR#2 Prod Guidance Sys Mfg Problem							
IAT&C	$16,426	100%	$16,426	100%	0%	$16,426	$0

B.5.4 Mapping CISM Cost Uncertainty to a FICSM Model

At this point, we have created a schedule using point estimate durations (no duration uncertainty) and mapped the point estimate costs from the CISM model to the FICSM model. It is recommended that cost uncertainty be applied before applying duration uncertainty or the risk register. In that way, we can set a goal to have the FICSM cost uncertainty match, at the total level, the CISM cost uncertainty when schedule and the risk register are turned off. This provides a point of departure where the models matched under similar conditions. It will be the last time any such match will be possible.

In general, CERs cannot be used in FICSM models to estimate TI or TD costs. Almost every FICSM tool requires TI and TD costs to be entered as throughputs. This makes it necessary to create a cost risk and uncertainty model in order to obtain the convolved cost uncertainty elements for the FICSM model. When using a CISM model, it is important to turn off schedule and risk register uncertainty in order to obtain the baseline uncertainty distributions suitable for entry into the FICSM model. It is recommended that you begin by creating histograms of the associated elements. In most cases, the results will look like the left image of **Figure B-9**. Generally the mean and standard deviation will be sufficient to replicate the distribution in the FICSM model. However, since we have to apply one WBS element distribution to several activities, it is recommended that the distribution parameters be reduced to factors of the point estimate.

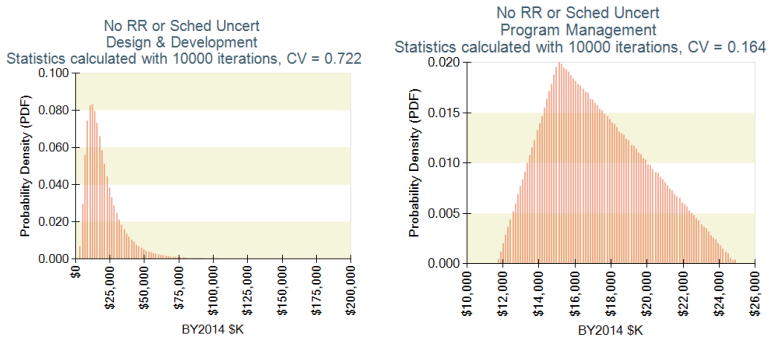


Figure B-9 Example CISM Cost Histograms

Using this approach allows us to apply the same relative uncertainty to all activities, TI and TD. The problem with this approach is that the sum of TI and TD cost uncertainty will not equal the original total cost uncertainty for that CISM element. There are a few ways to address this issue:

- Find the correlation required using trial and error to make the sum of TI and TD uncertainty match the total. This is not a satisfactory method as it interferes with correlation to anything else in the FICSM model.
- Apply the scaled parameters, measure the correlation in the CISM model as a whole and apply it to the FISCM cost elements

This recommended approach should achieve our goal of matching the cost uncertainty in the FICSM with the CISM model (before schedule uncertainty is applied in FICSM). As evidence that such a goal is feasible we offer **Figure B-10** where a CISM estimate of our example model (in Crystal Ball, @RISK or ACE) is compared to a FICSM estimate in the Joint Analysis of Cost and Schedule (JACS, an ACEIT tool).

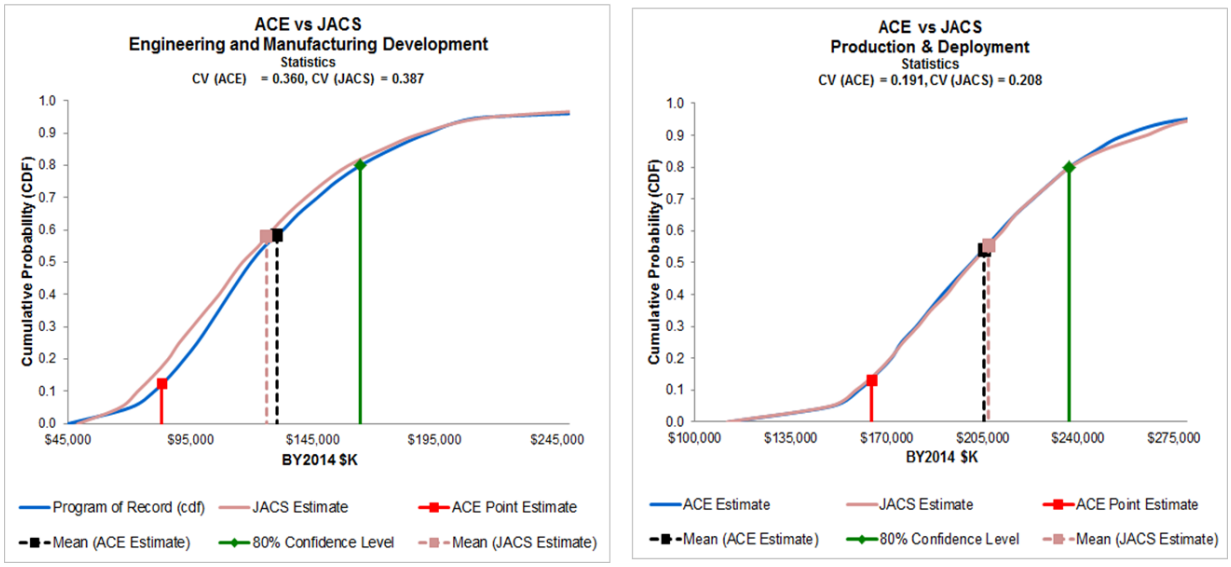


Figure B-10 Comparing Only Cost Uncertainty Between a CISM and FISM Model

B.5.5 Assign the Schedule Uncertainty

To this point, point estimate durations have been entered. A source is required to define the duration uncertainty. For our example we will draw our information from the CISM model. There are two sources of duration uncertainty in the example model, each modeled using a triangular distribution as illustrated in **Figure B-11**. It is important to ensure that the duration uncertainties obtained from the CISM model do not include the impact of the risk register as they will be added later.

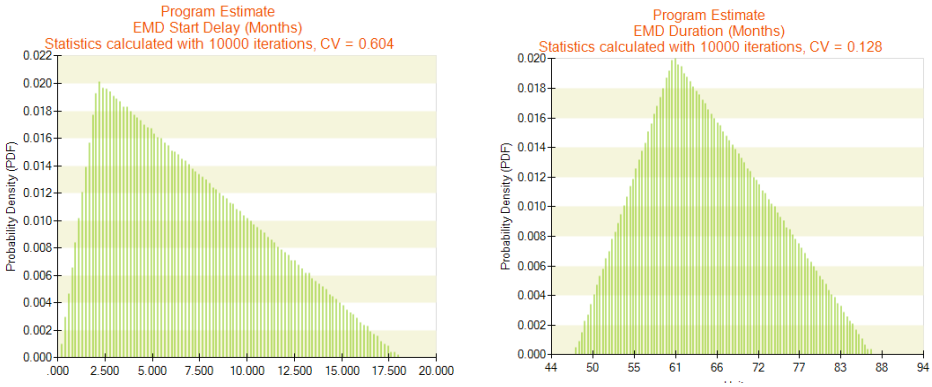


Figure B-11 Example Model EMD Duration Uncertainties

In our case, the EMD PE_S is the mode of the duration uncertainty. Often, however, the PE_S is in fact estimated as an average (mean), A common mistake is to treat it as the mode anyway. Unless the triangular

distribution is symmetrical, the mean will not be the same as the mode. It is, however, simple to calculate the mode of a triangular distribution if the minimum, mean and the maximum are known: Mode = 3*Mean – Min – Max. See **Appendix A.6** for the mathematics of the triangular and other common distributions.

Correctly obtaining the Production duration uncertainty from the CISM (or similar) model requires a complete understanding of the source model. In our case, the model combines the Production start date uncertainty with the production rate uncertainty. **Figure B-12** illustrates the combined effect. This is not the data we should use to model Production duration uncertainty in FICSM.

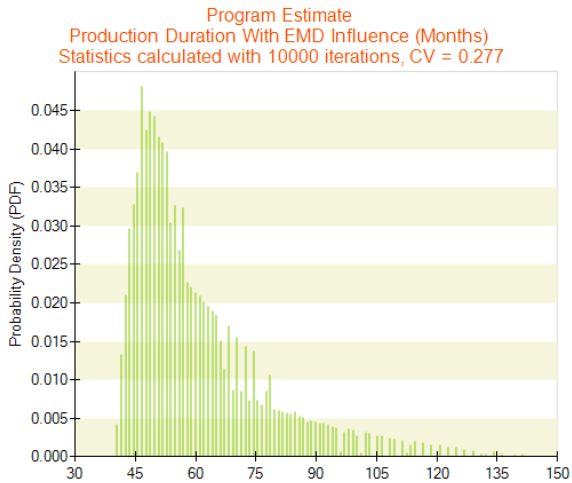


Figure B-12 Production Duration Uncertainty With the Influence of EMD

Production duration uncertainty was not developed in the example CISM model delivered with this handbook (as directed). However, in a previous version we explored introducing production duration uncertainty by creating a production rate variable. Dividing the point estimate production total quantity by a the point estimate duration for production gave a production steady state (SS) rate. Using an uncertain SS rate (left image of **Figure B-13**) we could have the model generate the uncertain duration (12 * TotalQty/ProdSSRate). The image on the right of **Figure B-13** is the result. This is the image we used to assign duration uncertainty to Production activities in the FICSM model. Note that dividing by a triangular distribution does not result in a triangular distribution. This is a situation where an empirical distribution may be appropriate (see **Section 2.4.3.6**).

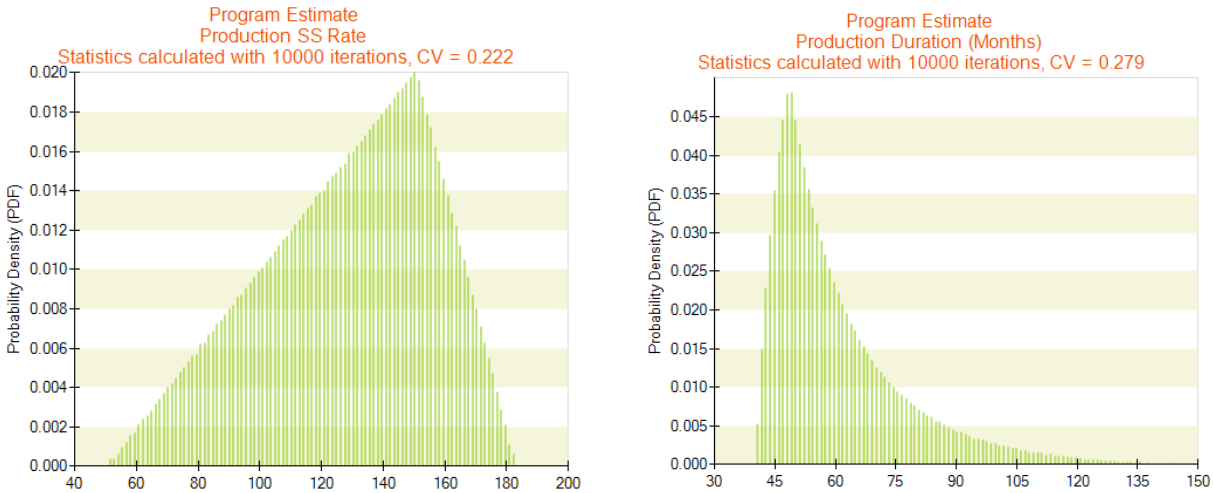


Figure B-13 Converting Production Rate Uncertainty to Duration Uncertainty

B.5.6 Apply the Risk Register

It is common practice to create the risk register in a FICSM model as a single list to facilitate cross checking with the risk management team. An alternative is to weave the risk register (RR) items directly into the schedule model structure. This has two advantages: linkages are easier to see and elements affected by the RR item are more clearly visible. We used this approach in the example model, but either approach is acceptable. Two common methods for modeling the RR event include:

- **Discrete events**: A single RR event may impact many activities. Each impact affects a specific activity and its successors. The event impacts are built directly into the network and assigned a point estimate of zero for cost and duration. If the event is triggered, the event takes on an uncertain cost and/or duration. Modeling risk events in this way is straight forward. Modeling cost opportunities is slightly complicated since uncertainties are negative numbers. In CISM models, one can simply model the uncertainty as positive and subtract it from the element in question. Functional relationships tend not to be available in FICSM models as they are schedule tool based. Modeling schedule opportunities requires some effort as the affected activity needs to be broken into multiple segments. Most FICSM tools handle risk events easily. Few, if any, provide a simple way to implement opportunities in this fashion.
- **Risk Driver**: This popular method allows a single event to trigger the application of uncertain factors to one or more activities. It can be looked at as a different way to apply uncertainty. Using a risk event has the advantage of making it easy to address both risks and opportunities. Its disadvantage is that tools allowing this method only permit a single factor that may be applied to many activities (albeit a different one for duration, TD and TI cost). Justifying that the event will have the identical impact on a variety of activities may be troublesome to defend. Nevertheless, it is widely used and defined in detail in **Reference 79** and **94**.

For our example CISM and FICSM model we used the discrete event method as shown in **Figure B-14**.

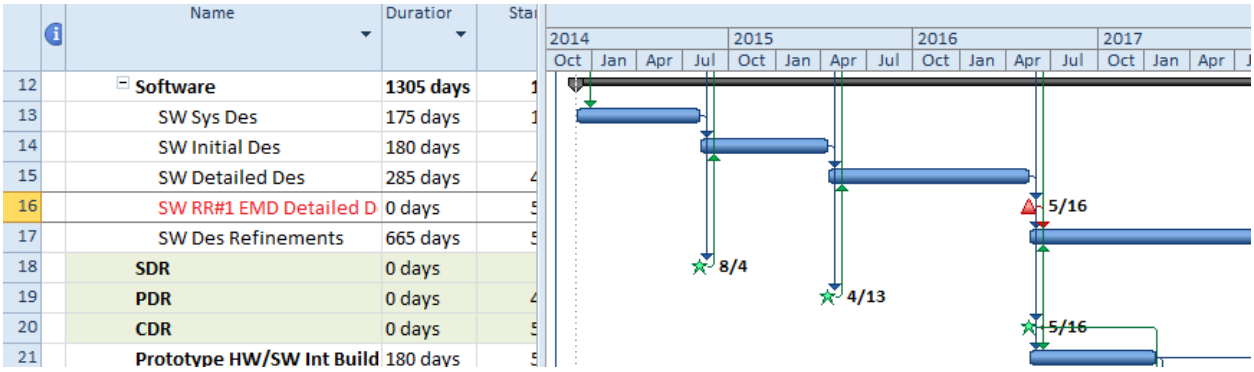


Figure B-14 Inserting a Risk Register Event into the Schedule Network

This particular RR event adds duration to the EMD phase. In the CISM model, this RR event influenced the EMD duration variable which in turn, was applied to all EMD WBS elements. It is not that simple in a schedule model. But there is also more flexibility. Every activity that precedes the RR event is unaffected when it occurs. The events that immediately follow are not affected either since it is set up with F-S logic. The event adds cost and duration to software development as a total. Other activities like System Engineering and Project Management will stretch since they are linked to the start and finish of software. Take care not to double count risks.

B.5.7 The Impact of Applied Correlation on a Schedule Network

Cost models are built on the premise that the total cost is the sum of the lower-level costs. Schedule networks are built on the premise that the total duration is based on the longest path through the lower-level

activities. The total of a summary level in a cost model is the sum of the immediate subordinates in the WBS. The duration of summary task in a schedule, however, is the maximum duration of all the subordinate tasks, not the sum. There is no simple analytical solution for calculating the mean or standard deviation in this situation. The table in **Figure B-15** emulates a schedule where the total is a summary task with five subordinate activities; all five durations are uncertain and are running in parallel. The total (summary task) is calculated by using the MAX( ) function in Excel. The mean and standard deviation are obtained through a simulation. In the table on the left, the activity duration uncertainties are independent of each other. The table on the right illustrates the impact of applying correlation. In a cost model, where elements sum, applying positive correlation across lower-level elements causes the variation of the parent level to increase and has little or no effect on the parent mean. As shown in the table to the right, the standard deviation does increase with correlation in a schedule model. However, the mean actually got noticeably smaller.

No Correlation		Parameters			Simulation	
Total is the max		Std Dev	Min	Max	Mean	Std Dev
Total	100				159.61	26.69
Lognormal	100	40			100.00	40.00
Triangular	100		75	200	125.00	27.00
BetaPert	100		75	200	112.50	21.65
Normal	100	35			100.00	35.00
Uniform	100		75	200	137.50	36.09

0.3 Correlation		Parameters			Simulation	
Total is the max		Std Dev	Min	Max	Mean	Std Dev
Total	100				153.50	30.63
Lognormal	100	40			100.00	39.98
Triangular	100		75	200	125.00	27.00
BetaPert	100		75	200	112.50	21.65
Normal	100	35			100.00	35.01
Uniform	100		75	200	137.50	36.09

Figure B-15 Impact of Correlation on Parallel Activities

B.5.8 Apply Correlation

Correlation was applied across the FICSM cost elements in **Appendix B.5.4** as a first, not final application. Now that all the risk and uncertainty has been applied, it is time to revisit correlation. There are some interesting aspects of the model to consider.

- **Duration and TD Cost**: When at least some of the cost is time dependent, we recommend that correlation between cost and duration be avoided unless there is compelling reason to do otherwise. For instance, if there is evidence that as duration changes, so should the cost per unit duration (the cost rate, e.g., dollars/day) then correlation between them may be appropriate.
- **TI and TD Costs**: Unless there is compelling reason to do otherwise, it is recommended that TI and TD elements be correlated in a single matrix
- **By Phase**: The analyst should carefully consider whether correlation should be applied within a each phase of the project rather than across all phases. In the example model, EMD elements were correlated with each other as were Production (rather than correlating EMD elements with Production).

B.5.9 FICSM Model Health Check

The model is complete. But before running the simulation, it is essential to perform at least some validation checks. We already performed the DCMA checks on the schedule (see **Appendix B.4.6**). At this point we need to revisit these checks plus others to verify uncertainty specifications. Most FICSM tools will provide for this step and it should not be skipped. The health check tests criteria can be grouped into 10 categories:

- **Schedule Software**: Check for schedule software issues (e.g. No status date defined)

- **Conflict**: Check Actual Start Dates or Actual Finish Dates do not conflict with the project Status Date
- **Linkage and Logic**: Check task linkage (e.g. Task with no predecessors)
- **Structural**: Check the structure of the tasks to ensure validity (e.g. milestone should have 0 duration)
- **Constraint**: Review activities for constraints on Start and/or Finish Dates
- **Status**: Check status date against task parameters (start, finish, % complete, etc.)
- **Cost Loading**: Check the logic and syntax of TI and TD costs loaded onto activities
- **Uncertainty**: Check syntax of uncertainty applied to task resources (TI cost, TD cost, and duration)
- **Risk Register Specification**: Check the logic and syntax of discrete risk events in the schedule
- **Syntax Error**: Syntax check on various FICSM inputs (e.g., max is less than mode)

Every FICSM tool provides some measure of health checking over and above the DCMA checks. By way of an example, the Health Check report from one tool is provided. It is divided into three sections to assist the analyst in finding and fixing the errors: Summary, Issue Counts, and Issue List. Generally the tools will provide feedback in a series of reports similar to those shown in **Table B-4** and **Table B-5**. A final report is not shown that would direct the user to the specific task and error. While it is best practice to clear all errors, it is common to ignore many of the minor issues flagged by the software.

Table B-4 Example Health Check Report

Report Date 05 Jan 2013

Project Name: CSRUH Analysis Schedule MSP2010 rev8 21Dec2012_w_shortnames.mpp

Schedule Status

Description	Current
Current Start	01 Dec 2013
Current Finish	01 Dec 2022
Estimated Workdays Remaining	2584
Is this schedule is linked to other schedules?	N
Resource Loaded (Y/N)	N
Satus Date	05 Jan 2013

Task and Milestone Count *(Note: excludes summary tasks)*

Description	Count	% of Total
Total Tasks and Milestones	40	
Completed Tasks and Milestones	0	0%
To Go Tasks and Milestones	40	100%
Tasks with Time-Independent (TI) Cost Specified	15	38%
Tasks with Time-Dependent (TD) Cost Specified	20	50%
Tasks and Milestones with Estimated Duration	0	0%
Tasks and Milestones Without Predecessors*	1	3%
Tasks and Milestones Without Successors*	13	33%
Summaries with Logic Ties	0	0%
Constraints (other than ASAP)*	1	3%

** These counts exclude summary and started/completed tasks*

Critical JCL Issues

Description	Count	% of Total
Improper Specification	0	N/A
Actual Finish after Status Date	0	0%
Actual Start after Status Date	0	0%
Actual Finish Task with Uncertainty	0	0%
Summary Task with Successor	0	0%
Summary Task with Predecessor	0	0%
Tasks with Zero (0) Duration Containing Cost specificatic	0	0%
TD Cost and no Spending Profile	0	0%
TI Cost and no Spending Profile	0	0%
Tasks marked as Milestones with Cost Specification	0	0%
Risk Missing Uncertainty Declaration	0	0%
Risk Missing Prob of Occurrence Specification	0	0%
Risk with Prob of Occurrence <0	0	0%
Risk with Prob of Occurrence >1	0	0%
Risk with Relative Duration Uncertainty	0	0%
Risk with Relative Cost Uncertainty	0	0%
Risk with TD Cost Specification	0	0%

Breakdown of Schedule Issues by Category

Description	Warning		Info	
	Count	% of Total	Count	% of Total*
Project	1	3%	0	0%
Conflict (potential or actual)	0	0%	0	0%
Linkage and Logic	14	35%	0	0%
Structural	0	0%	27	68%
Constraint	1	3%	1	3%
Status	0	0%	0	0%
Cost Loading	1	3%	1	3%
Uncertainty Specification	0	0%	1	3%
Risk Specification	0	0%	0	0%
Syntax Error	0	0%	0	0%
Uncategorized	0	0%	0	0%
Total	17		30	

** These percentages may exceed 100% due to multiple issues reported on single task*

Table B-5 Example Health Check Issue Counts Report

Issue	Category	Severity	Count	Tasks
No Status Date Defined	Project	Warning	1	-1
Constraint Defined	Constraint	Info	1	3
Task With Soft Constraint	Constraint	Warning	1	3
Task With No Predecessors	Linkage	Warning	1	3
Short Total Float	Structural	Info	19	3,4,5,8,9,10,13,14,15,17,18,19,20,43,45,47,50,63,71
Task Without Correlation	Uncertainty	Info	1	4
Long Total Float	Structural	Info	8	11,21,23,24,25,26,60,62
Task With Uncertainty Missing TD Cost	Cost Loading	Info	1	21
Task With No Successors	Linkage	Warning	13	27,30,33,37,40,51,54,57,62,63,65,68,71
Hammock Without TD Cost	Cost Loading	Warning	1	33

B.6 TYPICAL FICSM MODEL REPORTS

B.6.1 Overview

FICSM models are rich in reports. There are so many, it is easy to get lost in the wide array of report options and their many variations. It is also easy to forget that the cost from a FICSM model most often represents the "as expended" rather than "as obligated" estimate to be consistent with the schedule. There are a few ways to approach this disconnect.

- **Convert to TY**: Extract the FISCM phased results to develop an as expended profile. Use raw inflation indices to inflate to the current year and sum the total to arrive at a single total in TY dollars that are obligated at the beginning of the phase. This becomes more complicated if there are to be multiple obligations through any specific phase. The schedule would have to be re-worked to ensure there is an as expended string of activities associated with each planned obligation.
- **Convert to Obligation**: There are some presentations in the public domain that propose various methods to convert an expenditure profile to an obligation profile such as **Reference 35**
- **Build an Obligation Schedule:** Instead of an expenditure schedule, build one based on obligations directly. This is often done in cost estimating. The problem with this option is that most schedules are not created this way.

At the time of this report, virtually every FICSM tool has ignored the implementation of inflation. The assumption is that all costs are entered in a specific constant year (base year). This is how we loaded the example FICSM model, therefore we will apply the first approach.

The next most common aspect of the model that is ignored is the number of trials (iterations) required to converge. For the CISM model, 7,000 trials were required for EMD and 5,000 for Production (see **Section 3.4.3** for details). **Figure B-16** illustrates that for this example FICSM model, EMD required 5,000 trials while only 1,500 trial were necessary for Production. We will use 1500 trials to generate the Production reports from the example FICSM model.

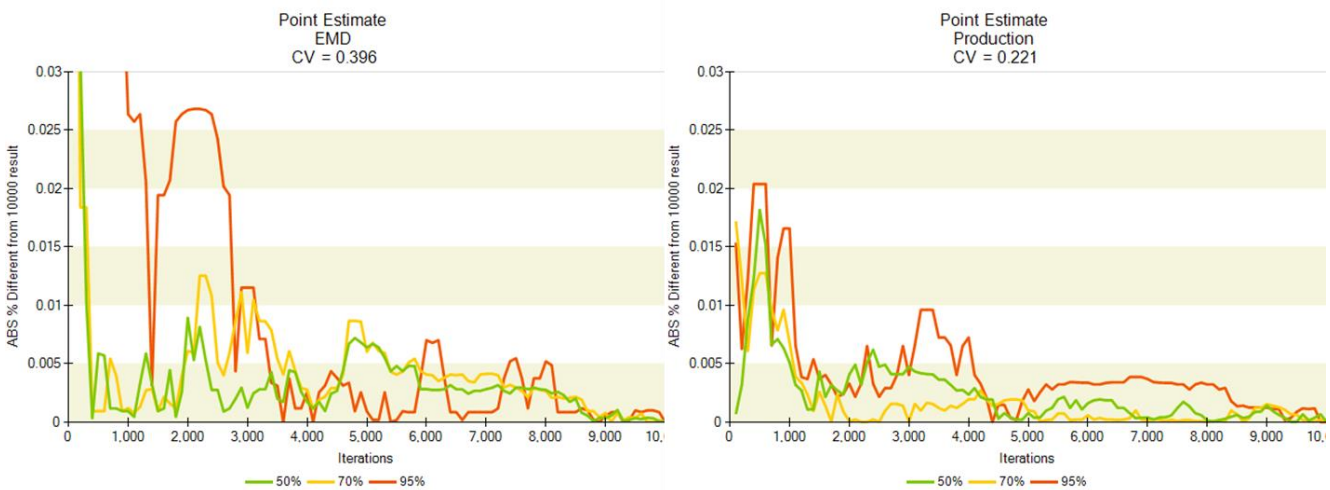


Figure B-16 Trials Required for a Converged FICSM Cost Result

B.6.2 FICSM Project Level Reports

A primary reason to build the FICSM model is to identify the relationship between cost and schedule. Consequently one of the first reports analysts will examine is the scatterplot where cost is on the y-axis and either duration or finish date is on the x-axis. Each point on **Figure B-17** represents the result of a specific trial (iteration). In this case, they represent the 1,500 results from the simulation. The cross hairs on the left image are set to the point estimate cost and schedule. 0.8% of the trial results fall under the point estimate cost and before the point estimate finish date. This is known as the Joint Confidence Level (JCL) for the project. The yellow line is the 50% frontier, meaning if the cross hairs are centered anywhere on the yellow line, the JCL will be 50%. In the image on the right, the cross hairs are set to 60% for the date and 60% for cost. For this example project, this results in a JCL of 43.4%. These colorful charts have been in use for over five years, but are seldom used when reporting to management. They are useful for the analyst to validate simulation behavior; visualize the JCL; and discuss within an estimating team's technical review. But there are many other FICSM charts that are more useful for reporting to management.

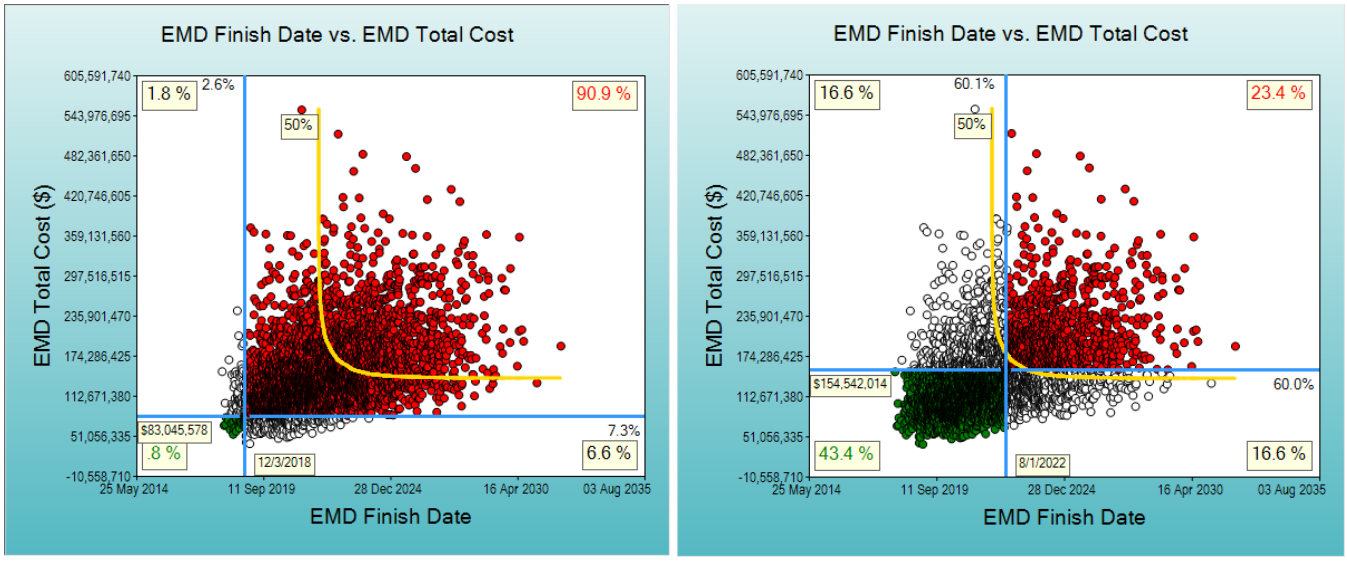


Figure B-17 Scatterplot of EMD Finish Date vs. Cost

While it may seem unlikely to find close agreement amongst the various FICSM-like tools on the market, Booz Allen and Hamilton, in support of a study for NASA Glenn Research Center, released **Figure B-18** that compares three different FICSM-like models. As **Figure B-18** shows, reasonably close comparison between tools is possible.

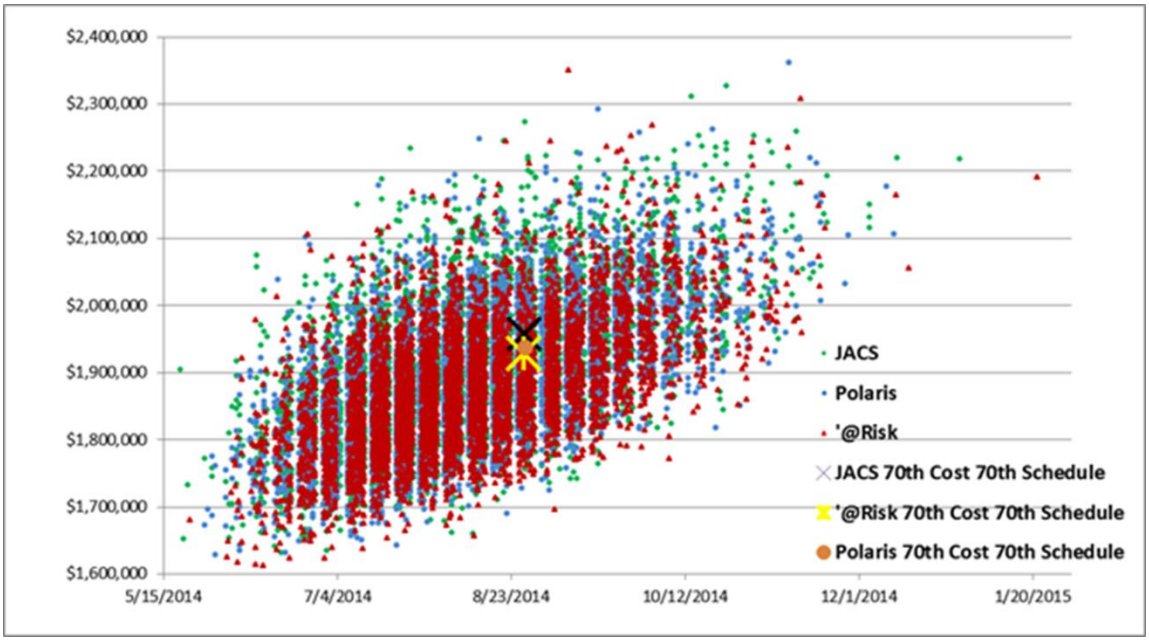


Figure B-18 Different Model Compared Across Three Tools

A GANTT[18] chart depicts progress in relation to time and is used in planning and tracking a project. It shows activities as a series of bars and illustrates information such as work complete, durations, milestones and linkages. There are several variations on the GANTT that are possible from FICSM model results. **Figure B-19** illustrates the PE_S in the upper left. The lower left shows a point estimate based on using the activity mean durations. But the more interesting charts are on the right. The top right illustrates the uncertainty of the activity start and the bottom right shows the potential finish dates.

[18] Named after Henry Laurence Gantt (1861-1919)

Figure B-19 GANTT Charts From the FICSM Example Model

In addition to GANNT charts, FICSM models provide cost uncertainty by year as illustrated in **Figure B-20**. The image on the left shows the uncertainty by year, with the cumulative on the right. The PE and the mean cost are also shown for reference.

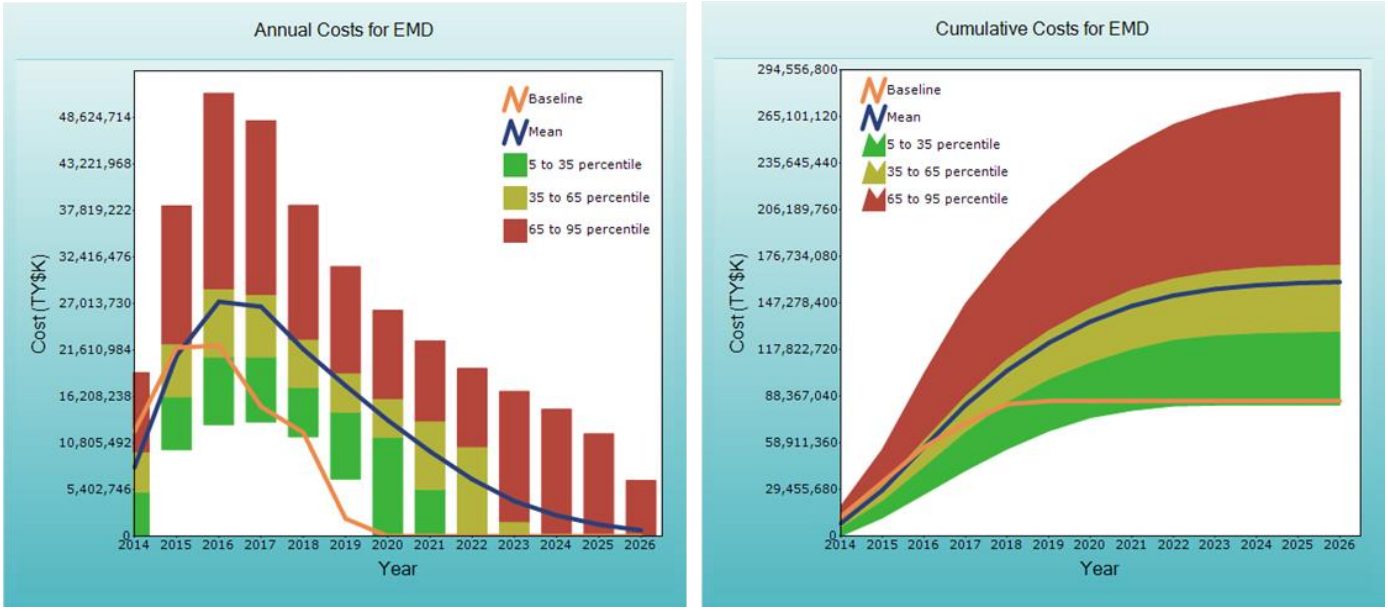


Figure B-20 Cost Uncertainty by Year in TY $K

The final project level chart we will introduce is called an "overlay" chart. This chart is an effort to put a variety of information on a single chart. **Figure B-21** illustrates the concept. The planned dates for various milestones are plotted against the cost expended to that point (yellow points). The various clusters are the simulation results for these key events in the project. Additionally, the risk register events are plotted where the size is a relative measure of impact and the color a relative measure of probability of occurrence (green

for low, red for high). This chart provides a high level, overall view of how much the project might cost, how long the project may take and when the risk register events are expected.

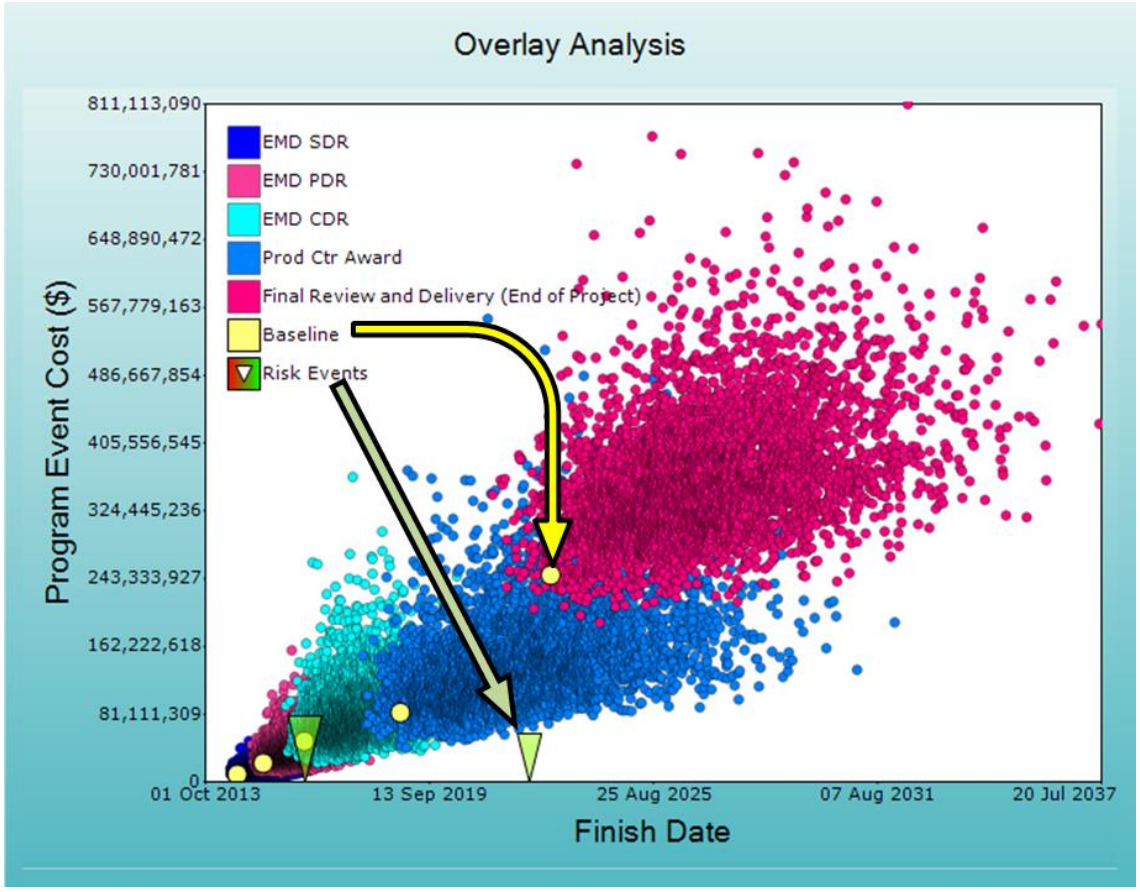


Figure B-21 Overlay Chart

B.6.3 FISCM Sensitivity Reports

For sensitivity analysis, there are a wide range of reports to consider. Every FICSM tool and every discussion of FICSM like reports (see **Reference 94** as an example) will have a variation of the following types of reports:

- **Cost Sensitivity**: Correlation between task cost and total program cost; see **Figure B-22**
- **Duration Sensitivity**: Correlation between task duration and total program duration; see **Figure B-22**

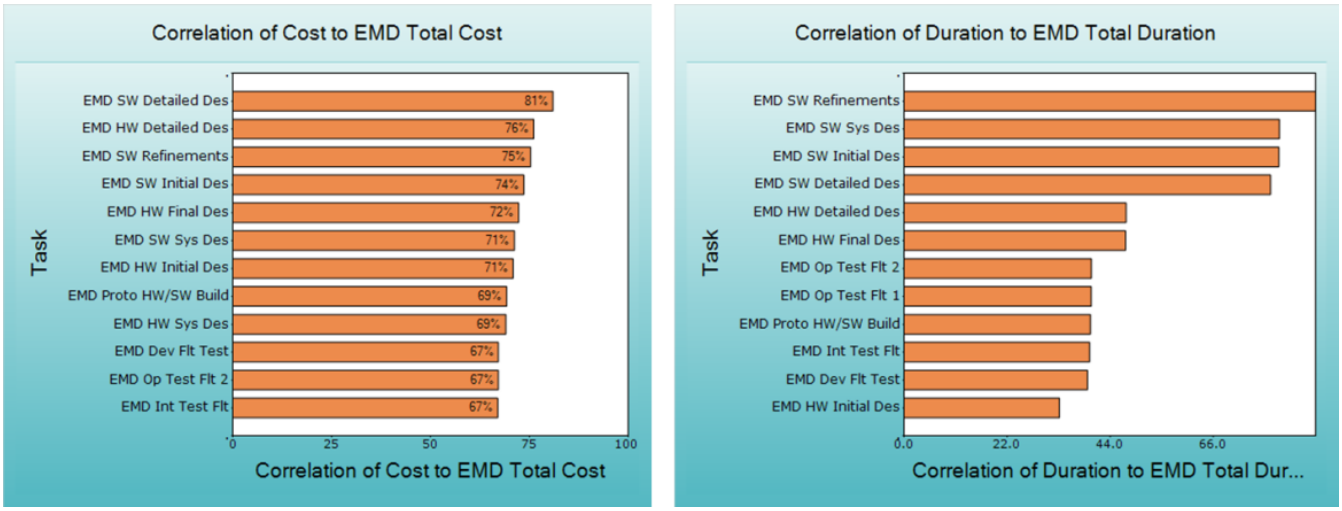


Figure B-22 FICSM Cost and Duration Sensitivity

- **Duration to Cost Sensitivity**: Correlation between task duration and total program cost; see **Figure B-23**
- **Criticality Index**: Probability that a specific task lies on the schedule's critical path; see **Figure B-23**

Figure B-23 FICSM Duration Sensitivity to Cost and Criticality Index

- **Cruciality Index**: the product of the criticality index and the selected sensitivity. For instance, cruciality index for duration would be the product of a task's probability on the critical path times the correlation of the task's duration to the total project duration. In **Figure B-23** SW Initial Design and SW System Design are about the same when considering the duration uncertainty's impact on the total EMD cost uncertainty. However, SW System Design is not on the critical path as often (49%) as SW Initial Design (54%). SW Initial design is more *crucial* than SW System Design.
- **Risk Register Criticality**: the probability that a risk register event will be on the critical path if it occurs. **Figure B-24** shows that the Software risk register item is on the critical path 92% of the time when the risk is realized for the example model (left image). On the other hand, the Guidance item almost never is on the critical path. The right image (provided by NASA) is another example of this report from a project with multiple risk register elements.

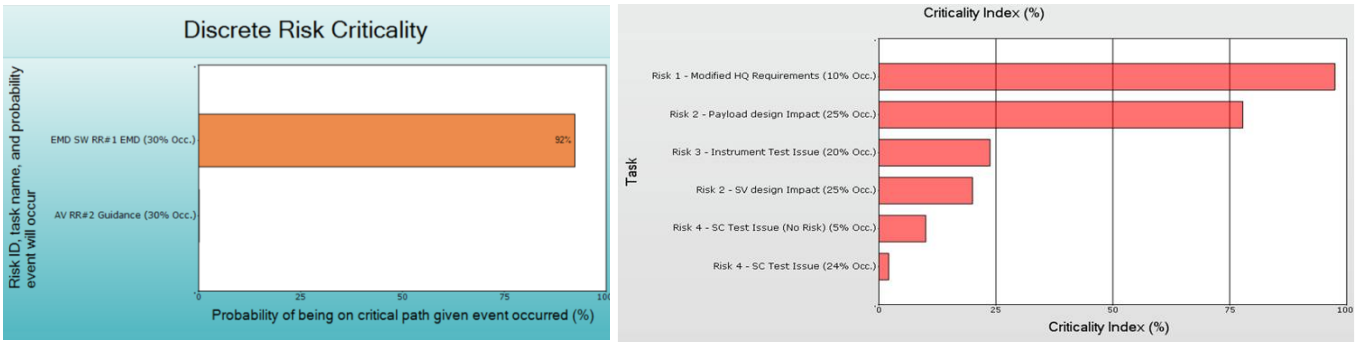


Figure B-24 FICSM Discrete Risk Criticality

- **Sensitivity Index**: The task cruciality times the ratio of the task standard deviation to the total project standard deviation. Some tools may use the variance ratio rather than the standard deviation ratio.
- **Cost Contributors**: **Figure B-25** is a special form of the Pareto chart. Two forms of this chart are shown. The top chart sorts task durations relative to the point estimate and the bottom sorts based upon duration the task start exceeds the point estimate. In each case the sort is by their 95% value. These charts provide a compact visual illustration of the additional duration required to reach the next percentile. Note that if the top chart in **Figure B-25** was sorted by the 70% value, EMD SW Detailed Design would be first rather than second. The chart helps identify those tasks with the greatest potential overrun on duration.

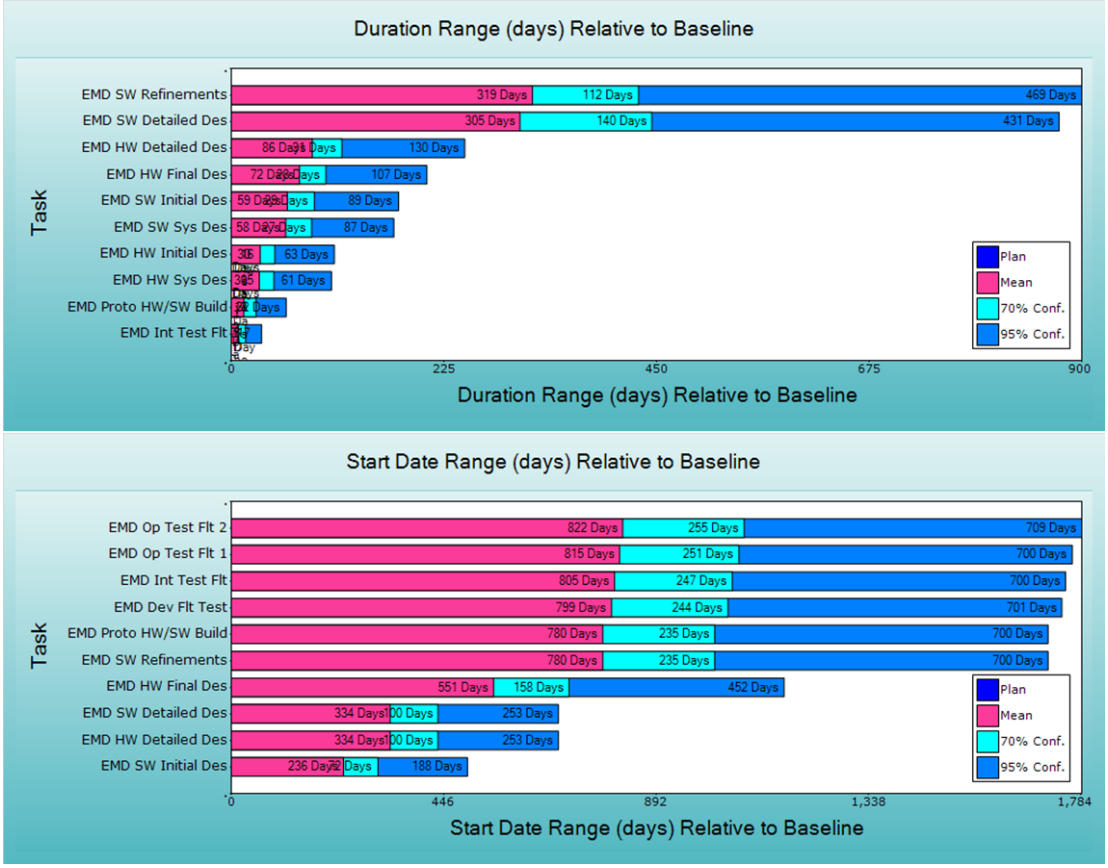


Figure B-25 EMD Task Duration and Task Start Date Relative to Baseline

The first three reports (cost, duration and duration to cost sensitivity) are important products of the FISCM process because they identify the cost and schedule uncertainty drivers. But there is much dispute on how to perform the measurement. Measuring the correlation between a target activity and every other activity in the

network could yield spurious results (see **Section 4.1.6** for details on this issue). Creating these charts with applied correlation enabled and disabled will help identify drivers that in fact are not influencing the total of interest. As an alternative, many analysts have resorted to turning uncertainty off one activity at a time to directly measure the impact of each uncertain task cost or duration on the total of interest.

There are many variations on FICSM driver charts, the ones shown here are just a few. These and other reports are only possible from a FICSM model. They provide significantly more information than the CISM approach. Industries such as oil and gas have used these tools for over a decade. DoD and NASA, on the other hand, are just getting started.

B.7 FICSM CONCLUDING COMMENTS

FICSM is a cohesive integration of cost uncertainty, schedule uncertainty, and the risk register. This appendix described one approach for incorporating cost uncertainty, schedule uncertainty, and the risk register into a fully integrated model. This FICSM integrated framework delivers a rich array of reports that can be used show the impacts of risk events to a project as well as highlight the relationship between cost and schedule uncertainty, establish projected cost and schedule requirements at various probability levels, and to identify programmatic cost and schedule uncertainty drivers. Finally, FICSM is the only way to estimate the probability of achieving both cost and schedule targets: the Joint Confidence Level (JCL).